PHYSICS

THEORETICAL AND DESCRIPTIVE

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PREFACE

The preparation of a serviceable and efficient Physics text-book has engaged the serious thought and effort of so many of the best workers in this field that the publication of another book implies some good reason for doing so, and demands, perhaps, some explanation.

The art of science teaching is progressive. The time and place of Physics in the curriculum, the scope and treatment of topics, and the method of teaching the subject in the High School have come to be more clearly defined within the past five years than ever before. This has been due largely to the influence of the College Entrance Board, the discussions of the National Educational Association, and the careful preparation of syllabuses in many school systems. All of these influences, moreover, have tended to unify the work of teaching Physics and thereby to increase its worth as a study.

Reason and experience point to several conclusions which should be taken into careful consideration in preparing a text-book in this science:

It is recognized by all as a sound pedagogical principle that laboratory work performed by the pupil should constitute his introduction to each new subject or principle. It is also generally conceded that the experiments and all that pertains to the individual laboratory work which the pupil is expected to do, should be eliminated from the text-book, and placed in the pupil's hands in the form of a Laboratory Manual. This Manual should contain a concise statement of the purpose of the experiment, explicit directions for procedure, and rational questions that provoke close observation and clear thinking about the results obtained.

It is not possible for the pupil to acquire in a single school year by laboratory work alone the fullness and breadth of knowledge of

Physics which the high school course should give him; hence the need and function of the text-book. This should set before the pupil the theoretical and descriptive aspects of elementary Physics, and should introduce him to the long-accumulated fund of knowledge relating to physical phenomena, carefully formulated and logically arranged. The quantitative nature of Physics as a study cannot be ignored; but the beauty of the subject lies in the fact that these exact relations may be so patently and convincingly illustrated from the pupil's own experience when that experience is appealed to in the right way. A Physics text-book must contain clear and concise statements of those principles indispensable for the comprehension of the manifold physical phenomena of daily life — principles which the pupil cannot readily master unless presented to him in the printed page.

To fix securely in mind the quantitative relations derived from the more or less specific and isolated conditions in the laboratory, and from the generalizations in the text-book, a considerable amount of drill in numerical problems is absolutely essential. To that end, a number of carefully compiled, practical problems have been appended to each chapter.

The authors believe that the pupil should feel that his text-book contains no superfluous matter, but that everything in the book is worthy of being studied and mastered.

An examination of the text-book will show that it meets fully the College Entrance Requirements as promulgated by the College Entrance Examination Board; it also covers all the topics of the syllabus recently adopted by the New York State Board of Regents, and of that adopted for the high schools of New York City.

A few discussions have been collected in the appendix and may prove useful in supplementary work where time and conditions warrant.

Recognizing the value of good illustrations, the authors have taken great care in the preparation of large, accurate, and in many cases, much-simplified diagrams, and in providing a few photographs and color prints which it is believed will add interest and clearness to the text.

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PHYSICS

CHAPTER I

INTRODUCTION

1. Physics Defined. — Physics treats of *energy* and *matter* and their *relations to each other* in so far as there is no change in the identity of the matter.

Energy may be provisionally defined as that which may cause a change in matter, and matter as that which occupies space.

2. Body and Substance. — A *body* is a *distinct portion of matter*, as a nail, a hammer, a car, a ship, etc.

A *substance* is a *particular kind of matter*, as iron, sugar, water, oxygen, etc.

3. Fundamental Quantities in Physics. — In dealing with energy and matter the fundamental quantities are *length*, *mass*, and *time*. To construct an exact science it is necessary to make accurate measurements, to do which requires the adoption of units of measurement.

4. Measurement. — *To measure* any quantity is to determine its value in terms of a *definite portion of the same kind of quantity*; the definite portion thus employed is called a *unit of that quantity*.

Thus to measure a *length*, a certain definite *length* called a *unit of length* must be employed; to measure a *surface*, a certain definite *surface* called a *unit of area* must be employed; to measure a *space*, it must be compared with

a certain definite *space* called a *unit of volume.* A *unit of mass, i.e.* a certain definite *mass*, must be employed to measure *mass*, and a certain definite duration of *time* called a *unit of time*, to measure *time.*

Since *surface* has *two* dimensions, a *unit of area* is readily derived from the unit of length by constructing a *square* having the unit of length for its side; and as *space* has *three* dimensions, a *unit of volume* is also derived from the unit of length by constructing a *cube* having the unit of length for its edge.

The three *fundamental units* of measurement are therefore those of *length*, of *mass*, and of *time.*

5. Two Systems of Units. — There are two systems of units in use: (1) the metric or centimeter-gram-second (C.G.S.) system, (2) the English or foot-pound-second (F.P.S.) system.

6. Units of Length. — The C.G.S. unit of length is a *centimeter;* the F.P.S. unit of length is a *foot.*

A centimeter is $\frac{1}{100}$ of the *length* of a certain platinum bar when the bar is at the temperature of 0° C. (32° F.). This bar, known as the *Metre des Archives*, is kept by the French government. A foot is one third of the *length* of a certain bronze bar when the bar is at a temperature of 62° F. This bar, known as the Standard Yard, is kept in the Standards' office of the English government.

7. Equivalents of Linear Units.

1 meter = 10 decimeters	= 100 centimeters = 1000 millimeters
1 yard	= 3 feet = 36 inches
1 kilometer	= 1000 meters = 0.62 mile
1 meter (m.)	= 1.0936 yd. = 3.28 ft. = 39.37 in.
1 centimeter (cm.)	= 0.3937 in.
1 inch	= 2.54 cm.
1 foot	= 30.48 cm.

8. Units of Mass. — The *mass* of a body may be provisionally defined as the *quantity of matter* in it. In considering the *mass* of a body the question is simply *how much* matter is in it, irrespective of its weight, volume, or substance. The C. G. S. unit of mass is a *gram;* the F. P. S. unit of mass is a *pound.*

A *gram* is the $\frac{1}{1000}$ part of the *mass* of a certain piece of platinum, called a kilogram, which is kept in the archives at Paris.[1] A *pound* is the *mass* of a certain piece of platinum kept in the Standards' office of the English government.

9. Equivalents of Units of Mass.

1 kilogram (kgm.)	= 1000 grams (gm.)	= 2.2 lb.
1 pound (lb.)	= 16 ounces (oz.)	= 453.59 gm.
1 gram	= 0.035 oz.	
1 ounce	= 28.35 gm.	

10. Unit of Time. — The unit of time in both systems is a *second,* which is $\frac{1}{86400}$ $\left(\frac{1}{24 \times 60 \times 60}\right)$ of the average time from noon to noon, called a *mean solar day.*

11. Units of Area. — The *area* of a surface is the number of *units of area* contained in the surface.

The C. G. S. unit of area is one *square centimeter* (sq. cm.); the F. P. S. unit of area is one *square foot* (sq. ft.).

12. Equivalents of Units of Area.

1 sq. ft.	= 144 sq. in.
1 sq. m.	= 10,000 sq. cm.
1 sq. in.	= 6.451 sq. cm.
1 sq. cm.	= 0.155 sq. in.

[1] For all practical purposes the mass of one cubic centimeter of water is equal to one gram. This, however, should not be considered as the definition of a gram.

13. Units of Volume. — The *volume* of a body is the number of *units of volume* in the space it occupies.

The C. G. S. unit of volume is a *cubic centimeter* (cc.); the F. P. S. unit of volume is a *cubic foot* (cu. ft.).

14. Equivalents of Units of Volume.

1 cu. ft. = 1728 cu. in.

1 cu. m. = 1,000,000 cc.

1 cu. in. = 16.38 cc.

15. Determination of the Volume of a Regular Solid. — To determine the volume of a body of regular geometric shape a formula obtained mathematically for the volume of the given shaped solid may be used. Measurements of the dimensions involved in the formula are made and substituted in the formula.

For example, the volume of a right cylinder is expressed by the formula $\frac{\pi d^2 h}{4}$. To find the volume of any right cylinder it is only necessary to make measurements directly of the diameter, d, and the altitude, h, and substituting these values in the formula, calculate the volume required.

16. Determination of the Volume of Any Solid. — The volume of any solid may be determined by submerging it in a *known* volume of a liquid in which it does not dissolve or undergo chemical action, contained in a vessel graduated in terms of a unit of volume, and then reading the *joint* volume. The difference between the volume readings before and after submergence is the volume of the solid.

17. Determination of the Mass of a Body. — The mass of a body may be found by balancing it upon the scale pans of

a beam balance (Fig. 1), with known masses which are multiples or submultiples of the gram or of the pound.

18. Measurement of Time. — Time is conveniently measured by means of a pendulum. If the length of the pendulum is such that the *duration* of each swing to *or* fro is *one second*, the pendulum is called a *seconds pendulum.*

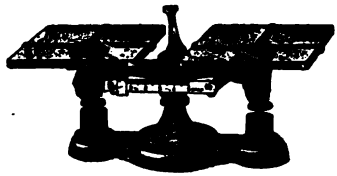


FIG. 1.

The length of a seconds pendulum for numerous places on the earth's surface has been carefully determined, the length in New York being 99.32 cm.

19. Density and its Determination. — If the mass of a body of any given substance is divided into as many equal parts as there are *units of volume* in the space it occupies, then the mass per unit volume thus obtained, *i.e.* the quantity of matter in each unit of volume, is the *density* of that substance.

Density is a property of each substance in general and not simply of a particular body of that substance. A small piece of iron has the same density as any piece of the same kind of iron.

20. Relation of Mass and Volume to Density. — If M represents the mass of a body, V its volume, and the Greek letter ρ (pronounced *rho*) its density, then

$$\rho = \frac{M}{V}, \; M = V\rho, \text{ or } V = \frac{M}{\rho}.$$

If the mass is expressed in grams and the volume in cubic centimeters, the denomination of the density is grams per cubic centimeter. This matter of expressing

the denomination is very important, and must invariably be given in full.

21. Variation. — In general, a scientific law is most concisely and completely expressed in the form of a *variation*, or its equivalent, a *proportion*.

Thus: The quantity of water flowing from a faucet *varies directly as* the time of flow; or, The quantity of heat produced in a furnace is *directly proportional to* the quantity of coal burned.

22. Kinds of Quantities. — Quantities are of two kinds: *constants* and *variables*. A *constant* is a quantity whose value is fixed, as the length of a meter, the mass of a body, the duration of a year; or whose value remains fixed in a given discussion, *e.g.* the *area* of the pages of a notebook being *constant*, the quantity of record that may be made on a page depends upon the coarseness of the writing.

A *variable* is a quantity whose value changes, as the length of an elastic band, the weight of a body, the duration of the afternoons, or, in the illustration just given, the quantity of record per page and the coarseness of the writing.

23. Ratio. — A change in value of a variable is always related to a change in value of one or more other variables.

Thus, the number of steps being fixed, the *distance* one walks is related to the *length of step* taken. If, taking the same number of steps each time, one walks several distances, say 20 ft. (D_1), 25 ft. (D_2), and 30 ft. (D_3), the length of step being 2 ft. (L_1) the first time, 2.5 ft. (L_2) the second, and 3 ft. (L_3) the third time, then the *distance* walked is one variable and the *length of step* taken is a related variable. Furthermore, the relation between any two values of one of these variables is the same as the relation between the two *corresponding* values of the other variable. These relations are mathematically expressed as ratios.

A *ratio* is the quotient obtained by dividing the value of one quantity by the value of another quantity *of the same denomination*, and is itself an abstract number, *i.e.* has no denomination.

Thus the ratio of a 20 ft. distance to a 25 ft. distance is $\frac{20 \text{ ft.}}{25 \text{ ft.}}$, or, as commonly written, 20 ft.: 25 ft., which equals $\frac{4}{5}$ or .8, an abstract number. And again, the ratio of a 2 ft. step to a 2.5 ft. step is $\frac{2 \text{ ft.}}{2.5 \text{ ft.}}$, or 2 ft.: 2.5 ft., which equals ^{4} or .8, a number having no denomination.

24. Proportion. — A *proportion* is an equality of two ratios. While each ratio in itself is always the quotient of two quantities of the same kind, the two equal ratios constituting a proportion are generally of different kinds of quantities.

For example, the ratio of 6 gm. to 3 gm. is 2, and the ratio of 10 cm. to 5 cm. is 2; and since the two ratios are equal they may constitute the proportion: —

$$6 \text{ gm.} : 3 \text{ gm.} = 10 \text{ cm.} : 5 \text{ cm.}$$

The *terms* of a ratio or of a proportion are the several quantities constituting it.

In the problem given in § 23, the ratio of *any two* distances walked, *e.g.* 20 ft. : 30 ft. (= $\frac{2}{3}$), is equal to the ratio of the *corresponding* lengths of step taken, 2 ft. : 3 ft. (= $\frac{2}{3}$), hence these ratios may constitute the proportion: —

20 ft. (D_1) : 30 ft. (D_3) = 2 ft. (L_1) : 3 ft. (L_3). Likewise 20 ft. (D_1) : 25 ft. (D_2) = 2 ft. (L_1) : 2.5 ft. (L_2), and 25 ft. (D_2) : 30 ft. (D_3) = 2.5 ft. (L_2) : 3 ft. (L_3).

25. Direct Proportion. — The relation between these two variables, *viz.* the distance walked and the length of step taken, is expressed by saying "the distance one walks (the number of steps being constant) *varies directly* as the length of step taken," or "the distance one walks (the

number of steps being constant) is *directly proportional to* the length of step taken."

In general, one quantity is said to *vary directly as* or *to be directly proportional to* another quantity when the ratio of *any two* values of one of the variables equals the ratio of the two *corresponding* values of the other variable.

If D_1 and D_2 are *any two* values of a variable D, and L_1 and L_2 are the two corresponding values of another variable L so related to D that

$$\frac{D_1}{D_2} = \frac{L_1}{L_2} \text{ or } D_1 : D_2 = L_1 : L_2,$$

then D is said to vary directly as, or to be directly proportional to, L. The sign of variation is $\propto$, and is read "varies directly as"; thus the expression, $D \propto L$, is read "D varies directly as L."

26. Inverse Proportion.

Again, if one walks a fixed distance, say 30 ft., several times, taking a different length of step each time, as 2 ft. (L_1), 2.5 ft. (L_2), 3 ft. (L_3), etc., it is evident that the *number of steps* taken is different each time, being 15 (N_1), 12 (N_2), 10 (N_3), etc., respectively. In this case the *length of step* is one variable and the *number of steps* is a related variable; but as the value of the one increases that of the other decreases. If, however, the ratio of *any two* values of one of the variables is compared with the ratio of the *reciprocals* of the *corresponding* values of the other variable, the ratios are found to be equal. Thus,

$$\frac{2 \text{ ft. } (L_1)}{2.5 \text{ ft. } (L_2)} = \frac{\frac{1}{15}\left(\frac{1}{N_1}\right)}{\frac{1}{12}\left(\frac{1}{N_2}\right)} = \frac{12\ (N_2)}{15\ (N_1)}, \text{ because}$$

the ratio of the reciprocals of two quantities equals the reciprocal or inverted ratio of the two quantities. Hence 2 ft. (L_1) : 2.5 ft. (L_2) = 12 (N_2) : 15 (N_1), *i.e.* the length of step taken the *first* time is to the length of step taken the *second* time as the number of steps taken the *second* time is to the number of steps taken the *first* time. Because of

the inverted order of the terms in the second ratio the relation between these two variables is expressed by saying: "In walking a fixed distance the length of step *varies inversely as* the number of steps taken," or "In walking a fixed distance the length of step *is inversely proportional to* the number of steps taken."

In general, one quantity is said to *vary inversely as* another quantity when the ratio of *any two* values of one of the variables equals the *reciprocal* ratio of the two *corresponding* values of the other variable.

If L_1 and L_2 are *any two* values of the variable L, and N_1 and N_2 are the two *corresponding* values of another variable N so related to L that

$$\frac{L_1}{L_2}=\frac{\frac{1}{N_1}}{\frac{1}{N_2}}=\frac{N_2}{N_1},$$

then N is said to vary inversely as, or to be inversely proportional to, L, and is written $N \propto \frac{1}{L}$.

27. Compound Proportion or Joint Variation. — A quantity is said to *vary jointly as* two or more other quantities when it varies as the *product* of the other quantities.

In the given problems the distance one may walk depends both upon the length of step and upon the number of steps taken, *i.e.* D varies jointly as L and N, or $D_1 : D_2 = L_1N_1 : L_2N_2$.

In the same problem it is evident that the number of steps taken varies *directly* as the distance and *inversely* as the length of step taken, or

$$N_1 : N_2 = \frac{D_1}{L_1} : \frac{D_2}{L_2}, \text{ or } = D_1L_2 : D_2L_1.$$

As a variation this is written $N \propto \frac{D}{L}$, which is read, "N varies directly as D and inversely as L."

28. Relation between Mass and Density, the Volumes being Equal. — *The densities, ρ_1 and ρ_2, of two bodies of different substances, having equal volumes, V, are directly proportional to their masses, M_1 and M_2.*

For $$\rho_1 = \frac{M_1}{V} \text{ and } \rho_2 = \frac{M_2}{V}.$$

Therefore $$V = \frac{M_1}{\rho_1} = \frac{M_2}{\rho_2},$$

or, $$\rho_1 : \rho_2 = M_1 : M_2.$$

29. Relation between Volume and Density, the Masses being Equal. — *The densities, ρ_1 and ρ_2, of two bodies of different substances, having equal mass, M, are inversely proportional to their volumes, V_1 and V_2.*

For $$\rho_1 = \frac{M}{V_1} \text{ and } \rho_2 = \frac{M}{V_2}.$$

Therefore $$M = V_1\rho_1 = V_2\rho_2,$$

or, $$\rho_1 : \rho_2 = V_2 : V_1.$$

30. Graphical Representation of Direct Proportions. — Direct proportions are conveniently represented in the following way on cross-section paper (Fig. 2). Suppose the relation of the data tabulated below is to be graphically represented :—

Substance	Mass	Volume	Density
Alcohol	800 gm.	1000 cc.	0.8 gm. per cc.
Ice	910 gm.	1000 cc.	0 91 gm. per cc.
Water	1000 gm.	1000 cc.	1.0 gm. per cc.
Glycerine	1260 gm.	1000 cc.	1.26 gm. per cc.
Sulphuric Acid	1840 gm.	1000 cc.	1.84 gm. per cc.
Glass	2600 gm.	1000 cc.	2.6 gm. per cc.

From the lower left-hand corner of the cross-section paper, taken as zero, lay off to the right on the horizontal axis the *masses* of the different substances on some convenient scale, say 1 cm. $\propto$ 400 gm. From the same corner lay off on the vertical axis the *densities* of the substances on a scale, say 1 cm. $\propto$ .50 gm. per cc.

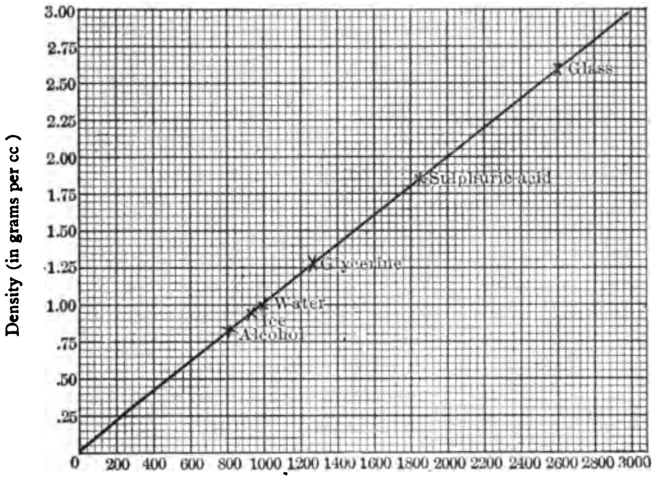


FIG. 2. — Graph showing the *direct* variation, $\rho \propto M$, V being constant.

On the 800 gm. line, representing the mass of the alcohol, at a distance above the horizontal axis, representing the density of alcohol, *viz.* .8 gm. per cc., mark a cross thus, X. In a similar way, indicate the densities of the other substances on the lines which represent their corresponding masses. Having plotted the six values, a line passing through all of the crosses is found to be a straight line which passes through the origin 0 (zero). This straight line, sloping upward from the origin, indicates that the *densities* of the substances are *directly proportional* to the *masses* having *equal volume* (1000 cc.).

The fact may be better understood if it is observed that six *similar triangles* have been formed whose *altitudes* are *proportional* to their bases, the altitudes representing the densities and the bases representing the masses, respectively.

31. Graphical Representation of Inverse Proportion.

Substance	Mass	Volume	Density
Lead	1000 gm.	88.5 cc.	11.3 gm. per cc.
Iron	1000 gm.	135 cc.	7.4 gm. per cc.
Aluminum	1000 gm.	360 cc.	2.78 gm. per cc.
Oak	1000 gm.	1250 cc.	0.8 gm. per cc.
Pine	1000 gm.	2000 cc.	0.5 gm. per cc.
Cork	1000 gm.	4166 cc.	0 24 gm. per cc.

If, in a similar way (Fig. 3), the relation between the volumes of equal mass and the densities of the various substances given in the above set of data is represented by laying off on the horizontal axis the *volumes* on a scale of 1 cm. ≎ 600 cc., and on the vertical axis, the *densities* on a scale of 1 cm. ≎ 1.2 gm. per cc., then the line passing through the six densities thus plotted is a curve which continually approaches both the horizontal and vertical axes but never intersects them and which also is concave in a direction away from the origin. This curve, called a *hyperbola*, represents an inverse proportion between the two sets of quantities plotted, which in the present case are the *volumes of equal mass* and the corresponding *densities* of the substances.

32. Summary.—To summarize the foregoing relations:—

1. The density of a substance is the mass in unit volume, *e.g.* grams per cubic centimeter, and is denoted by the Greek letter ρ.

2. $\rho = \dfrac{M}{V}$.

3. If equal volumes of different substances are taken, the densities of the substances are directly proportional to the masses in these equal volumes.

4. If equal masses of different substances are taken, the densities of the substances are inversely proportional to the volumes of the equal masses.

33. Gravitation.—Every particle of matter attracts every other particle of matter and the value of this attraction

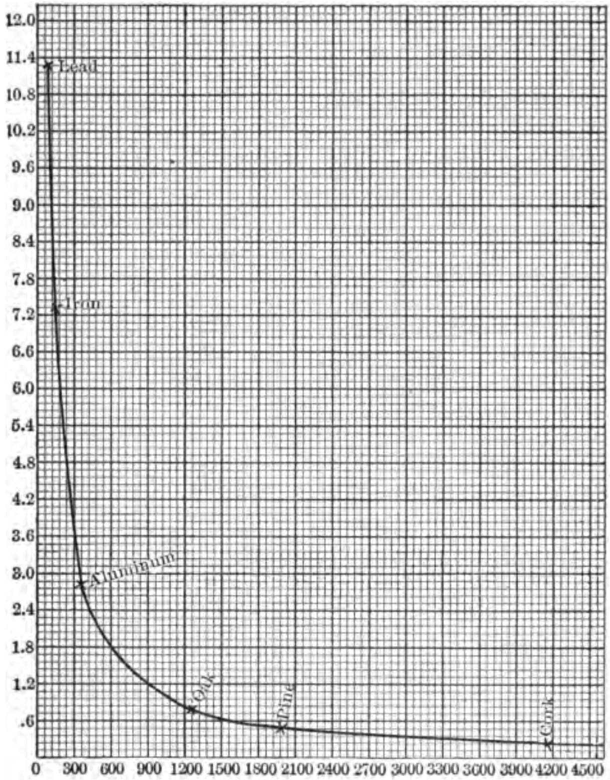


FIG. 3.—Graph showing the *inverse* variation, $\rho \propto \frac{1}{V}$, M being constant.

varies directly as the product of the attracting masses and inversely as the square of the distance between their centers. This attraction is called *gravitation* and may be ex-

pressed in the form of an equation: $F = \frac{mm'}{d^2} c$, where F is the force of attraction, m and m' are the attracting masses, d is the distance between their centers, and c is a constant factor.

34. Weight. — When one of the attracting masses is the earth and the other some body upon the earth, this attraction is called *gravity*, and the measure of this attraction is the *weight* of the body.

If m is the mass of the earth and d the distance from the center of the earth to the center of mass of the given body, then the *weight* of this body will depend entirely upon the value of its mass, m', provided it remains at the same place on the earth, because in the formula, $F = \frac{mm'}{d^2} c$, m, d, and c have the same value for the same place.

The fact is expressed by the statement: *the weight of a body is directly proportional to its mass.*

35. Units of Weight. — The C. G. S. unit of weight is the weight of one gram; the F. P. S. unit of weight is the weight of one pound. The meaning of the statement that the weight of a body is 100 grams is that the earth's attraction for the body is 100 times its attraction for one gram.

36. Variation in the Weight of the Same Mass. — Since the shape of the earth is not truly spherical, the polar diameter being 7900 mi. while the equatorial diameter is 7926 mi., the value of d decreases 13 mi. upon moving from the equator to either pole, hence a body whose mass remains constant will weigh more at either pole than at the equator.

The weight of one pound mass at the equator is, according to the formula, $\frac{m \cdot 1}{(3963)^2}c = \frac{mc}{15,705,369}$. The weight of one pound mass at either pole is $\frac{m \cdot 1}{(3950)^2}c = \frac{mc}{15,602,500}$. Therefore the weight of a body at the equator is approximately $\frac{156}{157}$ of its weight at either pole.

The point to be remembered in this connection is that the *mass* of a body, or its quantity of matter, is entirely independent of the mass of the earth or its position on the earth; while the *weight* of the body or the measure of the attraction between it and the earth, may vary with a change in its position upon the earth's surface.

37. States of Matter. — Matter exists in three states: *solid*, *liquid*, and *gaseous*, which are dependent upon the temperature of the substance and the pressure upon it. The substance whose chemical constitution is indicated by the formula H_2O is a solid at temperatures below 0° C., a liquid between 0° and 100° C., and a gas at temperatures above 100° C. when the pressure equals 760 mm. of mercury, or normal atmospheric pressure.

38. Kinetic Theory of the Constitution of Matter. — Matter is conceived to consist of extremely small particles, called *molecules*. These molecules are in continual vibratory motion, so that no two molecules are ever in permanent contact with one another.

When matter is in the solid state, its molecules move in very restricted paths and in the same portion of the body, so that a solid tends to preserve a definite shape and a definite volume.

When matter is in the liquid state, its molecules have perfect freedom of motion amongst themselves, moving about through all parts of the liquid and even at times breaking through the surface of the liquid, producing the

phenomenon known as evaporation. The molecules, however, preserve on the average the same relative distance from each other, so that in consequence, a liquid tends to preserve a definite volume. On account of the ease with which the molecules of a liquid glide over one another, the shape of a body of liquid conforms to the shape of the vessel in which it is placed.

When matter is in the gaseous state, its molecules moving with enormous velocities in every direction tend to increase constantly the distances between them, and if not confined in a vessel, a gas will occupy a space indefinitely large. If, however, it is confined in a vessel, its volume is the entire capacity of the vessel, for it fills it completely. On account of the enormous velocities with which the molecules of a gas move, their bombardment of the sides of the vessel which confines it produces a pressure upon these walls whose value is dependent upon the mass of the confined gas and upon the capacity of the confining vessel.

39. Fluid: Liquid, Vapor, Gas. — The term *fluid* is used to include both liquids and gases because of their common tendency to flow. A *vapor* is a substance in the gaseous state which under ordinary conditions of temperature and pressure is a liquid or a solid. Steam, for example, is water vapor. The term *gas* is restricted to include only those substances, such as carbon dioxid or hydrogen, which continue in the gaseous state at all ordinary temperatures or pressures. The following grouping of these terms suggests their relations: —

$$\begin{array}{c}\text{States}\\ \text{of}\\ \text{Matter}\end{array}\left\{\begin{array}{l}\text{Solid}\\ \left.\begin{array}{l}\text{Liquid}\\ \text{Gaseous}\left\{\begin{array}{l}\text{Gas}\\ \text{Vapor}\end{array}\right.\end{array}\right\}\text{Fluid}\end{array}\right.$$

40. General Divisions of Mechanics. — *Mechanics* is that branch of Physics which treats of the *action of forces upon matter.* By a *force* is meant a *push* or a *pull.* The idea of force is obtained through the muscular sense, from the sensation of exertion experienced when an attempt is made to move a body or to change its motion.

Mechanics is subdivided into (1) *Statics*, in which the body acted upon by forces is conceived to be at rest; (2) *Kinematics*, in which the motion of bodies is considered regardless of the force causing the motion or of the mass moving; (3) *Kinetics*, in which the motion of a body is considered as the effect of applied forces.

41. Pressure; Tension. — *Pressure* is that *portion of a push* exerted against a body which does not result in motion. Pressure generally acts upon appreciable surfaces and its value is the quantity of force upon unit area. *Tension* is that *portion of a pull* upon a body which does not result in motion and is measured in the same way as is pressure.

PROBLEMS

1. What is the capacity (volume) of a bottle which will hold exactly 1 kgm. of mercury whose density is 13.6 gm. per cc.? *Ans.* 73.53 cc.

2. The density of iron is 430 lb. per cu. ft. What is the volume of a flatiron whose mass is 8 lb.? *Ans.* 32.14 cu. in.

3. The mass of 32 cc. of aluminum is 90 gm. What is the density of aluminum? *Ans.* 2.8 gm. per cc.

4. What is the mass of 50 cc. of sulphuric acid whose density is 1.84 gm. per cc.? *Ans.* 92 gm.

5. The density of aluminum is 2.7, lead 11.3, copper 8.8 gm. per cc. How long a bar of each, having a cross section of 1 sq. cm., will have a mass of 1000 gm.? *Ans.* 370.4 cm., 88.5 cm., and 113.6 cm., respectively.

6. Find the volume of 1000 gm. of iron whose density is 7.2 gm. per cc. *Ans.* 138.9 cc.

7. If the density of brass is 7.1 gm. per cc., what is the volume of 1000 gm. of brass? *Ans.* 140.8 cc.

8. The area of a circle is πr^2. What is the ratio of the areas of two circles one of which has twice the diameter of the other? *Ans.* 4 : 1.

9. What is the ratio of the diameters of two circles one of which has twice the area of the other? *Ans.* $\sqrt{2}:1$.

10. The distance a train moves in a given time varies as its average velocity. If the velocity of a train is 50 mi. per hr., how far will it travel in the time that it moves 4 mi. with a velocity of 20 mi. per hr.? *Ans.* 10 mi.

11. The time it takes a train to move a certain distance varies inversely as its velocity. If the velocity of a train is 15 mi. per hr., how long will it take to move the same distance it moves in 35 min. with a velocity of 40 mi. per hr.? *Ans.* $93\frac{1}{3}$ min.

12. The volume of a gas is inversely proportional to its pressure. If a certain mass of hydrogen occupies a volume of 75 cc. under a pressure of 750 mm., what volume will it occupy under a pressure of 760 mm.? *Ans.* 74.01 cc.

13. 180 gm. of iron has the same volume as 25 gm. of water; the density of water is 1 gm. per cc. What is the density of iron? *Ans.* 7.2 gm. per cc.

14. How many grams of glycerine whose density is 1.26 gm. per cc. can be put into a bottle which will hold 500 gm. of sulphuric acid whose density is 1.84 gm. per cc.? *Ans.* 342.4 gm.

15. The density of coal is 1.6 gm. per cc. How large is a piece of coal that has the same mass as 120 cc. of brass whose density is 8.2 gm. per cc.? *Ans.* 615 cc.

16. What is the density of a substance 100 cc. of which has the same mass as 60 cc. of mercury whose density is 13.6 gm. per cc.? *Ans.* 8.16 gm. per cc.

CHAPTER II

STATICS OF LIQUIDS

42. Causes of Liquid Pressure. — Liquid pressure is due to two causes: (1) an externally applied force; (2) the weight of the liquid itself. For the sake of clearness in the discussion of these two causes it will be assumed that but one of them is operative at any one time although, in reality, both causes act simultaneously.

43. Transmission of Pressure externally Applied. — To consider the pressure exerted by a liquid from the first cause, suppose a closed vessel, as shown in Fig. 4, to be filled with a liquid and that at two different places, *A* and *B*, in the containing walls the surfaces can be moved like pistons in or outward. If a force is applied externally to piston *A*, pushing it in against the liquid, the liquid will be under pressure and, since the molecules of a liquid move over one another with perfect freedom, this pressure will cause the liquid molecules to move in all directions with the same force, and in the transmission of this pressure there is no loss due to internal friction. Every molecule is then exerting the same force and exerting it in every direction. If we denote the value of the pressure, *i.e.* the pressure upon unit area, by the small letter p, the value of p will be the same in all parts of the liquid, for there will be

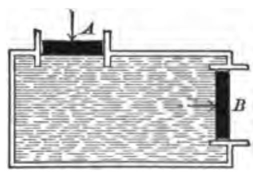


FIG. 4.

as many molecules pushing against a unit surface in one part of the liquid as in another.

44. Proportionality of Pressures and Areas. — Since the value of p is constant throughout the liquid, the *total* pressures upon the surfaces of the pistons, A and B, are directly proportional to the areas of those surfaces; for p equals the *total* pressure upon A divided by the area of A, and also equals the *total* pressure on B divided by the area of B.

$$\frac{\text{Pressure on } A}{\text{Area of } A} = \frac{\text{Pressure on } B}{\text{Area of } B} = p.$$

Therefore,

Pressure on A : Pressure on B = Area of A : Area of B.

45. Pascal's Law. — This mode of transmitting pressure is equally true for *gases* and is termed transmission of pressure by fluids. The above relation is expressed in what is known as Pascal's Law.

A pressure exerted upon any part of a fluid inclosed in a vessel is transmitted undiminished in all directions, equally to equal areas and at right angles to them, and the total pressures upon any two surfaces exposed to the fluid under pressure are directly proportional to the areas of those surfaces.

Illustration. — If the area of surface A is 2 sq. cm. and a force of 100 gm. is applied to A, every square centimeter of surface within the fluid or in contact with it is under a pressure of 50 gm. If the area of B is 100 sq. cm., the total pressure against B is $50 \times 100 = 5000$ gm.

Stating the same by proportion,

$$100 \text{ gm.} : x \text{ gm.} = 2 \text{ sq. cm.} : 100 \text{ sq. cm.}$$

$$2x = 10{,}000. \qquad x = 5000 \text{ gm.}$$

This is the force which would prevent B from moving outward, or is the force B would exert in moving outward.

All that is necessary, then, that a force of 100 units at any surface of a fluid shall produce a pressure of 5000 units at some other surface is that the first surface shall be $\frac{1}{50}$ as large as the second.

46. Application of Pascal's Law. — This principle is applied in hydraulic presses (Fig. 5), jacks or elevators, in compressed-air machinery, steam boilers, engines, and in every other case of fluids under pressure.

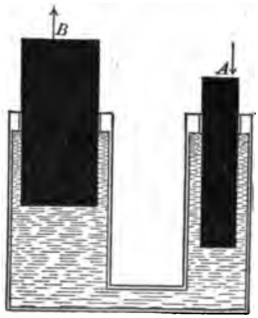


FIG. 5.

47. Gravity Pressure of a Liquid. — Since there is an attraction in a vertical direction between the earth and all bodies on the earth, it follows that any surface beneath a liquid receives a pressure due to the pull of the earth on the liquid *above* it.

To determine upon what this pressure depends, suppose that a horizontal surface (Fig. 6) whose area is a sq. cm. is placed h cm. below the surface of the liquid, as shown in the diagram. The pressure upon a is the *weight* of the column of liquid vertically above it. The volume of this column of liquid equals $h \times a$ cc. and, if the density of the liquid is ρ gm. per cc., the weight of this column of liquid is $ha\rho$ gm.

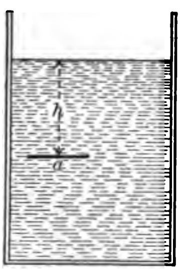


FIG. 6.

Hence the pressure of the liquid due to gravity on any horizontal surface whose area is a sq. cm. when placed at a depth of h cm. in a liquid whose density is ρ gm. per cc. equals $ha\rho$ gm.

Since gravity pressure depends in the same manner upon the three quantities, *depth*, *area*, and *density*, a variation in any one of these factors must produce a corresponding variation in the pressure. From this are derived the following laws: —

1. The gravity pressure of a liquid upon any surface is directly proportional to the depth of that surface in the liquid.

2. The gravity pressure of a liquid is directly proportional to the density of the liquid.

3. The total gravity pressure of a liquid is directly proportional to the area of the surface pressed upon.

48. Equal Pressure in All Directions at Same Depth. — From the principle of fluid transmission of pressure, previously discussed, it follows that if any layer of liquid *a* (Fig. 7) is pressed upon by a column of liquid *h* directly above it, this layer of liquid transmits this pressure equally in all directions, so that any equal surfaces, *b* and *c*, *at the same level* will have exerted upon them the same pressure, *hap*, whether they be horizontal as *b* or vertical as *c*. Hence the gravity pressure of a liquid is equal at the same depth in all directions.

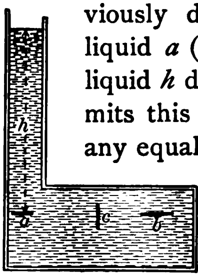


FIG. 7.

It must be clearly understood that the pressure upon any surface beneath a liquid is entirely independent of the depth of liquid *below* that surface. The only depth of liquid that determines a pressure is the depth of liquid *above* the level of the surface.

49. Pressure Independent of Shape of Vessel. — It also follows from the foregoing that the total pressures on bases of equal areas of differently shaped vessels which contain liquid of the same density and depth are equal.

The vessels shown in Fig. 8, (*a*), (*b*), and (*c*), are supported by the ring stands, and a disk whose area is A sq. cm. is held by the upward force P applied to the cord which is attached to this disk, against the lower end of the

vessel. Liquid of the same density is poured into each until the depth is h cm. above A. The force P required to hold the disk against the lower end will in each case be $hA\rho$ gm.

In vessel (*a*) the weight of the liquid just equals this pressure. In vessel (*b*) the weight of the liquid is more than $hA\rho$ gm., but the outward sloping sides pressing up-

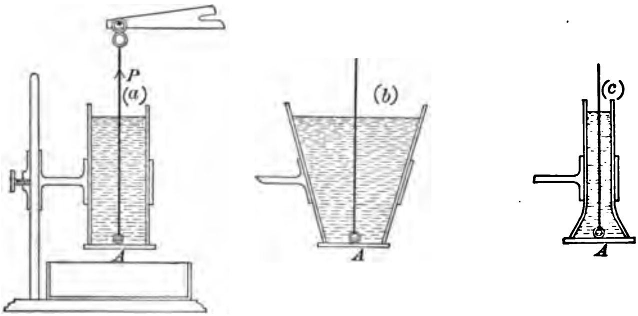


FIG. 8.

ward against the liquid support the excess weight. In vessel (*c*) the weight of the liquid is less than $hA\rho$ gm., but the inward sloping sides pressing downward against the liquid make up the deficiency.

Hence it follows that the pressure upon the base of a vessel is independent of the shape of the vessel, or the quantity of liquid in it provided the values of h, A, and ρ remain unchanged.

50. Gravity Pressure of a Liquid upon a Vertical Rectangular Surface. — If the surface is vertical, the depth of each successive portion of the surface increases as the lower edge is approached, hence the pressure increases proportionally.

The *average* of the series of pressures upon *unit* area

beginning with $h_1\rho$ (Fig. 9) at the upper edge a of the surface ab, and ending with $h_2\rho$ at the lower edge b, is the pressure at the *center* c of this surface. The total pressure upon a vertical surface therefore equals the weight of a column of liquid whose depth is the depth of the *center of the surface*, whose base is the area pressed upon, and whose density is that of the given liquid. Letting h_c represent the depth of the center of the given surface below the surface of the liquid, the total pressure $P = h_c a\rho$.

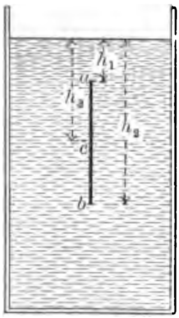


FIG. 9.

51. Liquid Pressure on Any Rectangular Surface. — It follows also that if the surface ab (Fig. 10) is rotated in a vertical plane about c as a center so that *c always remains at the same depth in the liquid*, the pressure upon the surface is unchanged as ab takes the successive positions $a'b'$, $a''b''$, $a'''b'''$. Hence the pressure upon any rectangular surface, whether vertical, horizontal, or oblique, equals $h_c a\rho$.

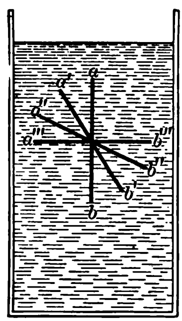


FIG. 10.

52. Problems. — To show the application of the foregoing theory of liquid pressure, solutions of the following problems are given: —

1. A cylindrical can, 10 cm. in diameter and 20 cm. high, is filled with kerosene, whose density is .8 gm. per cc. Find the pressure of the kerosene upon the bottom and sides of the can.

Solution. — (1) The depth of liquid h_c pressing on the bottom of the can is 20 cm. The bottom being circular its area, πr^2, is $3.1416 \times 5^2 = 78.54$ sq. cm. The pressure on the bottom, $h_c a\rho$, $= 20 \times 78.54 \times .8 = 1256.66$ gm.

(2) The depth h_c of the *center* of the sides of the can below the surface of the kerosene is $\frac{1}{2}$ of 20 cm. = 10 cm. The area of the sides is the area of a rectangle one side of which is the *circumference* of the can, $2\pi r$, and the other side is the height of the can, 20 cm., $= 2 \times 3.1416$

$\times 5 \times 20 = 628.32$ sq. cm. The pressure on the sides of the can, $h_c a \rho = 10 \times 628.32 \times .8 = 5026.66$ gm.

2. A rectangular closed vessel (Fig. 11) 40 cm. long, 25 cm. wide, and 60 cm. high is half full of naphtha, whose density is .75 gm. per cc.; in the space above the naphtha is compressed air under a pressure of 2000 gm. per sq. cm. Find from both causes operating simultaneously the pressure upon the bottom and one end of the vessel.

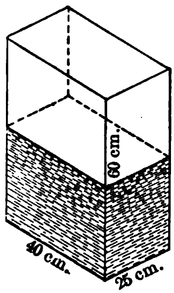


FIG. 11.

Solution. — (1) Since the compressed air exerts a pressure of 2000 gm. upon each square centimeter, and since the liquid transmits this pressure undiminished equally in all directions, the pressure on the bottom of the vessel due to the compressed air $= 2000 \times$ the area of the bottom $(40 \times 25 = 1000$ sq. cm.$) = 2,000,000$ gm. The depth h_c of liquid above the bottom is $\frac{1}{2}$ of 60 cm. $= 30$ cm. The pressure of the liquid due to its weight, $h_c a \rho$, $= 30 \times 1000 \times .75 = 22,500$ gm. Therefore, the total pressure on the bottom from both causes $= 2,022,500$ gm.

(2) The pressure on the end, 60×25 cm., due to the compressed air, $= 2000 \times 1500 = 3,000,000$ gm. The area of that half of the end in contact with the liquid $= 750$ sq. cm. The depth, h_c, of the *center* of this portion below the surface of the liquid $= \frac{1}{2}$ of $30 = 15$ cm. The pressure on the end due to the weight of the liquid, $h_c a \rho$, $= 15 \times 750 \times .75 = 8437.5$ gm. The total pressure on the end from both causes $= 3,008,437.5$ gm.

53. Liquid in Communicating Vessels. — The surfaces of a liquid at rest in communicating vessels lie in the same horizontal plane if the surfaces are under the same pressure from above.

To show why this is so, consider the pressure upon the plane *a* (Fig. 12) in the liquid in the communicating tube. It is pressed toward the right by the liquid in vessels *A*, *B*, and *C*, and toward the left by the liquid in *D*. The plane *a* can be at rest only if these pressures to the left and to the right are equal.

The surface a is common to both pressures, the density of the liquid is the same, therefore *the height of liquid above* a *must be the same on both sides of it* in order that $h_c ap$ shall be equal on both sides. Hence the liquid surfaces in A, B, C, and D lie in the same horizontal plane.

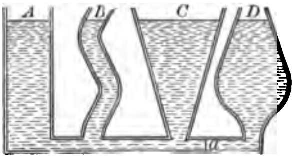


FIG. 12.

54. Free Surface of a Liquid at Rest. — For the same reason as the foregoing the free surface of a liquid at rest is *horizontal.*

If this were not so, two surfaces, a and b (Fig. 13), at the same level, would be at different depths below the surface of the liquid, and would in consequence be under different pressures. The greater pressure at b would force liquid toward a until the pressures become equal, or until the depth of a below the surface of the liquid equals the depth of b, which condition requires that the surface of the liquid shall be horizontal.

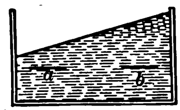


FIG. 13.

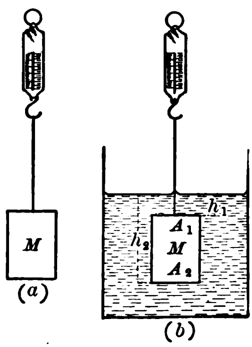


FIG. 14.

55. Archimedes' Principle. — Since the pressure of a liquid due to its weight increases as the depth of the liquid *above* the surface pressed upon increases, it follows that if a body (Fig. 14 b) is suspended from a spring balance and submerged in a liquid, the *upward* pressure, $h_2 ap$, of the liquid upon the under surface of the body being greater than the *downward* pressure, $h_1 ap$, of the liquid upon the upper surface of the body will cause the spring balance to register a less

pull on it than before submerging the body (Fig. 14 *a*), or the body is said to lose weight.

This loss of weight must equal the difference of the upward and downward pressures of the liquid upon the submerged body, or $h_2 a\rho - h_1 a\rho = (h_2 - h_1)a\rho$ gm. But $(h_2 - h_1)$ equals the altitude of the submerged body, and the product, $(h_1 - h_1)a$, equals the volume of that body. If this volume is multiplied by the density ρ of the liquid, the product, $(h_2 - h_1)a\rho$, equals the weight of that volume of liquid, or is the weight of liquid the body displaces when completely submerged.

Hence, the loss of weight of a body submerged in a fluid equals the weight of the fluid it displaces.

This principle was first stated by the Greek, Archimedes, as follows: *A body submerged in a fluid is buoyed up by a force equal to the weight of the fluid it displaces.*

56. Principle of Flotation. — If a body is buoyed up by a fluid with a force *equal to its weight*, the body *floats* in the fluid.

Since the force with which a body is buoyed up by a fluid equals the weight of the displaced fluid, it follows that *a floating body displaces a weight of fluid equal to its own.*

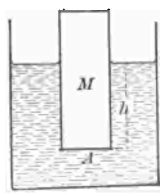


FIG. 15.

If the length of the portion of the floating body, M (Fig. 15), which is below the liquid surface is h cm., and the area of the under surface of the body is a sq. cm., the *upward* pressure of the liquid on the under surface of the body is $ha\rho$ gm. But ha equals the volume of the submerged portion of the body, or the volume of the displaced liquid, and $ha\rho$ equals the weight of the displaced liquid.

Therefore, whenever a body floats in a liquid, the weight of the liquid displaced equals the weight of the body.

57. Conditions of Sinking and of Floating. — Whether a body sinks or floats in a liquid depends therefore upon three things: (1) the weight of the body, (2) the volume of the body, (3) the density of the liquid.

Let W gm. = the weight of the body, V cc. the volume of the body, and ρ gm. per cc. the density of the liquid. Then $V\rho$ = the weight of liquid the body displaces when *completely* submerged. If $W > V\rho$, the body will sink, for its weight is greater than the buoyant force of the liquid. If $W = V\rho$, the weight of the body and the buoyant force are equal, and the body will remain at rest at any point within the liquid at which it is placed. If $W < V\rho$, the body will float, for it will displace less than its volume, V, of liquid when displacing its own weight of liquid.

58. Specific Gravity and Relative Density. — *The specific gravity of a substance is the ratio of the weights of equal volumes of that substance and of water.*

If the specific gravity of iron is 7, any piece of iron weighs 7 times as much as the same volume of water.

Since the weight of a body is proportional to its mass, the *ratio* of two masses of equal volumes equals the *ratio* of the weights of equal volumes of the same substances. When discussing the densities of substances, it was proved that the ratio of the masses of equal volume of any two substances equals the ratio of the densities of the same substances. Therefore, the specific gravity of a substance is the same *number* as the relative density of the given substance and of water. It was also proved that the densities of substances are *inversely* proportional to the volumes of equal mass of those substances. Hence the specific

gravity of a substance also equals the *inverse* ratio of the volumes of equal mass of the given substance and of water.

59. Basis of Specific Gravity Methods. — All of the methods employed to determine the specific gravity of substances are based upon these two relations: (1) the ratio of masses of equal volume; (2) the ratio of volumes of equal mass.

For example, if the mass of a certain piece of metal is 90 gm. and the mass of the same volume of water is 32 gm., the specific gravity of the metal is the ratio 90 : 32 = 2.8. Again, if the volume of a certain mass of a given liquid is 21 cc. and the volume of the same mass of water is 16 cc., the specific gravity of the liquid is the inverse ratio of these volumes, 16 : 21 = .76.

To find the specific gravity of a substance, it is necessary, then, to determine either the mass of the substance and the mass of an equal volume of water, *or* the volume of the substance and the volume of an equal mass of water.

If M_s is the mass of the substance, and M_w is the mass of an equal volume of water, the specific gravity of the substance, sp. gr., $=\frac{M_s}{M_w}.$

If V_s is the volume of the substance, and V_w is the volume of an equal mass of water, the specific gravity of the substance, sp. gr., $=\frac{V_w}{V_s}.$

60. Specific Gravity of Solids. — 1. *Solids that sink in water.* — By Archimedes' principle the loss of weight of a solid in water equals the weight of water of the same volume as the solid. Therefore, *the specific gravity of a solid that will sink in water equals its weight divided by its loss of weight in water.*

2. *Solids that float in water.* — In this case the solid must be entirely submerged in water by tying a sinker to it and the *loss* of weight in water of the combined solids found. Then, if the loss of weight in water of the sinker alone is found, *the difference between these losses* equals the weight of water displaced by the floating solid when completely submerged.

The specific gravity of the solid that will float in water equals its weight divided by the weight of water it displaces when completely submerged.

3. If the floating solid is of regular geometric shape, its specific gravity may be found by floating it in water and calculating the volume of the submerged portion. Since a floating body displaces its weight of liquid, a volume of liquid equal to the volume of the submerged portion of the body must have the same mass as the floating body.

The entire volume of the body is then calculated, and since the volume of the body and the volume of the water it displaces when floating have the same mass, the specific gravity of the body is the inverse ratio of these volumes, $V_w : V_s$, where V_w is the volume of displaced water and V_s is the volume of the body.

61. Specific Gravity of Liquids. — 1. The mass of a given liquid and the mass of water which a certain flask will contain is found. Since the volumes are equal, the specific gravity of the liquid is the ratio, $M_s : M_w$.

2. If the *loss of weight* of some sinking solid in the given liquid and the *loss of weight* of the same solid in water is found, these two losses equal the weights of the liquid and of water which have the same volume as the sinking solid. The specific gravity of the liquid is the ratio of the loss of weight of the solid in the liquid to the loss of weight in water.

3. If a light solid of regular shape is floated in the given liquid and in water and the volume of the submerged portion of the solid in each liquid is found, these volumes are the volumes of the liquid and of water which the floating solid displaces and therefore have the same mass as that of the solid. The specific gravity of the liquid is the inverse ratio of these volumes, $V_w : V_s$. Such a floating solid used to determine the specific gravity of liquids is called a *hydrometer*. Hydrometers are usually provided with a scale so that the specific gravity of the liquid in which they are immersed may be read directly from the position of the liquid surface on the scale.

4. If the liquid will not mix with water, it may be poured into one arm of a U-tube half filled with water, as shown in Fig. 16. If h_1 is the height of the column of liquid *ab*, which exerts the same pressure on surface *b* as the column *cd* of water, whose height is h_2, $h_1 a \rho_1 = h_2 a \rho_2$ where ρ_1 is the density of the given liquid, and ρ_2 the density of water. Since the area *a* is common to both pressures, $h_1 \rho_1 = h_2 \rho_2$, or $\rho_1 : \rho_2 = h_2 : h_1 =$ the relative density of the liquid and water = the specific gravity of the liquid.

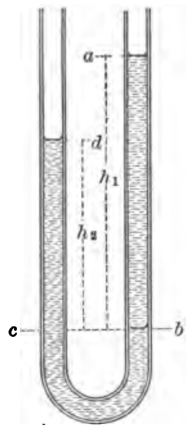


FIG. 16.

MOLECULAR FORCES

62. Cohesion; Adhesion; Tenacity. — The force acting between the molecules of solids and, to a less degree, of liquids, which holds the molecules in such relative positions that they tend to form a body of constant volume, is called

cohesion. The force acting between molecules of different substances causing them to adhere, one to the other, is called *adhesion.* These forces act only if the particles of the body are brought so close together that their distances apart are immeasurably small.

In virtue of the molecular force of cohesion a body offers resistance to being pulled apart. This property of solids and liquids is called *tenacity;* the term, however, is usually limited in its application to solids.

63. Surface Tension of a Liquid. — Because of the existence of these molecular forces in liquids it follows that the layer of molecules forming the free surface of a liquid is in a different condition from that of any other layer of particles in the liquid. The particles within the body of the liquid are attracted by particles on all sides of them as shown in Fig. 17; while at the surface these particles are attracted only toward the body of the liquid, there being no opposing attractions above this surface layer. The condition produced in the liquid surface from this cause is *a state of tension* as is indicated by the following facts determined by experiment: —

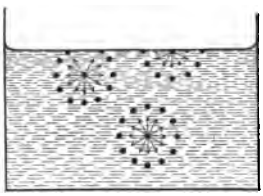


FIG. 17.

A body of liquid when free from the action of external forces assumes a spherical shape. This is shown approximately in falling raindrops or falling drops of molten lead. A better demonstration is afforded if a small quantity of olive oil is floated completely submerged in alcohol mixed with water until it is of the same density as the oil which then assumes the shape of a perfect sphere. A soap bubble is a liquid film which takes a spherical shape when blown;

when the pipe is withdrawn from the mouth, the air within escapes, because the tension of the film contracts the bubble until it returns to the interior of the pipe bowl.

Since the surface of a sphere is less than the surface of any other shaped body of the same volume, these phenomena indicate that the surface of a liquid tends to contract like a stretched membrane until its extent is the smallest possible.

64. Capillarity. — If a solid, such as a glass plate, is placed in water or any other liquid that wets it, the force of adhesion between the glass and the liquid being greater than the cohesion between the particles of the liquid, the surface of the liquid is drawn up slightly along the sides of the plate, the tension of the surface rendering it curved concave upward. These liquid particles attracted upward along the glass drag along some of the rest of the liquid. The liquid thus raised above the level of the horizontal portions of the surface is supported by the surface tension or contracting force of this curved portion of the surface. If the glass plate is placed in mercury or other liquid which does not wet it, the cohesion of the liquid being greater than the adhesion between the glass and the liquid, the liquid is drawn down at the part where the liquid touches the glass, the surface tension of the liquid rendering it curved convex upward.

This phenomenon is more pronounced when a tube of small bore is placed in a liquid. If the liquid is one that wets the tube, the contracting tendency of the liquid surface in the bore of the tube renders it concave upward (Fig. 18), and as the part in contact with the glass moves upward along its surface it raises the liquid in the tube above the level of liquid outside until the weight of the liquid

raised equals the tension of the liquid surface in the tube. If the diameter of the tube is reduced one half, the length of the circumference of the surface, $2\pi r$, exerting the supporting tension, is also reduced one half; but the weight of the liquid in the tube, varying as the area of the cross section, πr^2, is reduced to one fourth, the height of the column being the same; hence the height of the raised

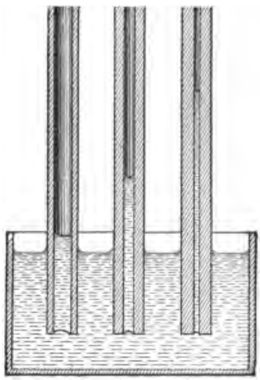


FIG. 18.

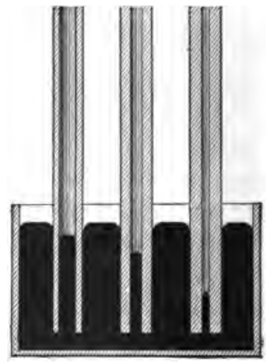


FIG. 19.

column must be doubled in order that its weight shall equal the surface tension. Therefore, *the height to which the liquid is raised in a tube is inversely proportional to its diameter.*

If a small bore glass tube is dipped in mercury, the surface of the mercury within the tube is convex upward (Fig. 19), and on account of the contracting force of the surface the liquid is depressed until this force is in equilibrium with the pressure at that level in the mercury outside. *The depression is inversely proportional to the diameter of the bore of the tube.*

65. Illustrations of Capillarity. — This phenomenon of capillary ascension of liquids is illustrated in the rise of oil

in lamp wicks, and in the thorough wetting of a towel hanging over the side of a bath tub with one end of the towel below the water in the tub. In taking readings of the volume of liquids in burettes or graduates, the reading of the lowest point of the concave surface, or in the case of mercury the uppermost point of the convex surface, is taken, because in tubes of fairly large bore this capillary ascension or depression is mainly confined to the edges.

PROBLEMS

1. One piston of an hydraulic press is 4 sq. cm. in area and the other is 480 sq. cm. What pressure on the smaller piston will produce a pressure of 5000 kgm. on the large piston? *Ans.* 41.6 kgm.

2. The diameter of the valve stem of a bicycle tire is 8 mm.; the diameter of the piston of a bicycle pump is 20 mm. What pressure on the valve is produced by a force of 40 kgm. on the pump piston? *Ans.* 6.4 kgm.

3. The diameter of a safety valve of a steam boiler is $\frac{3}{8}$ inch. What must be the outside pressure on top of the valve to prevent its "blowing off" until the pressure in the boiler is 125 lb. per sq. in.? *Ans.* 55 lb.

4. Find the pressure on the bottom of a cubical vessel 10 cm. on a side, when filled with kerosene whose density is .78 gm. per cc. *Ans.* 780 gm.

5. Find the pressure on one *end* of a rectangular vessel 40 cm. long, 25 cm. wide, and 15 cm. high, when filled with dilute sulphuric acid whose density is 1.2 gm. per cc. *Ans.* 3600 gm.

6. A can whose shape is that of a frustum of a cone is 12 cm. high. The base is 170 sq. cm. in area, the top is 120 sq. cm. The can is filled with maple sirup whose density is 1.1 gm. per cc., and is then closed at the top. (*a*) What is the pressure on the base when the can is resting on that end? (*b*) What is the pressure on the top when the can is turned upside down and rests on the top? *Ans.* (*a*) 2244 gm.; (*b*) 1584 gm.

7. A regular hollow pyramid 10 cm. high and 12 cm. square at the base is filled with water. (*a*) What is the pressure on the base? The

volume of the water is 480 cc. (*b*) How can the difference between its weight and the pressure on the base be accounted for? *Ans.* (*a*) 1440 gm.

8. What is the pressure on a dam 40 ft. long and 20 ft. high, the density of water being 62.5 lb. per cu. ft.? *Ans.* 500,000 lb.

9. The surface of the water in a city reservoir is 250 ft. above sea level. Find the pressure on a sq. in. of a faucet in a pipe connected with this reservoir, the faucet being 210 ft. above sea level. The density of water is 62.5 lb. per cu. ft. *Ans.* 17.36 lb.

10. (*a*) Find the loss of weight of 40 cc. of iron when weighed in water. (*b*) If the iron weighs 290 gm. in air, what will it weigh in water? *Ans.* (*a*) 40 gm.; (*b*) 250 gm.

11. (*a*) Find the loss of weight of 40 cc. of gold when weighed in water. (*b*) If the gold weighs 732 gm. in water, what is the density of gold? *Ans.* (*a*) 40 gm.; (*b*) 19.3 gm. per cc.

12. Eight cubic centimeters of silver weighs in air 83.2 gm. (*a*) What will it weigh in water? (*b*) What will it weigh in oil whose density is .9 gm. per cc.? *Ans.* (*a*) 75.2 gm.; (*b*) 76 gm.

13. A piece of iron weighs in air 100 gm.; in alcohol whose density is .8 gm. per cc. it weighs 89 gm. What does it weigh in water? *Ans.* 86.25 gm.

14. A cubical solid 3 cm. on an edge is suspended in water with its lower face horizontal and 15 cm. below the surface of the water. What is the pressure on each face, and what is the effect of these pressures?

If the cube is lowered until its lower surface is 25 cm. below the water surface, what are the pressures on each face and their effects?

		1st position	2d position
Ans.	Pressure on top,	108.0 gm.	198.0 gm.
	Pressure on each side,	121.5 gm.	211.5 gm.
	Pressure on bottom,	135.0 gm.	225.0 gm.

15. A cube $4 \times 4 \times 4$ ft. is submerged until its top surface is 12 ft. below the surface of a liquid whose density is 1.8 times that of water. Find the pressure in pounds, (*a*) on the top surface; (*b*) on the bottom; (*c*) on one side.

Ans.	Pressure on top,	21,600 lb.
	Pressure on side,	25,200 lb.
	Pressure on bottom,	28,800 lb.

16. What is the tension of a cord supporting a block of iron whose density is 7 gm. per cc. and whose volume is 200 cc. which is submerged in a liquid whose density is 1.5 gm. per cc.? *Ans.* 1100 gm.

17. A cube of wood 10 cm. on an edge is held submerged in water so that the upper face is 20 cm. below the water surface. (*a*) What is the pressure of the water on the top of the cube? (*b*) On the bottom? (*c*) If the cube weighs 800 gm. in air, what does it weigh in water? *Ans.* (*a*) 2000 gm.; (*b*) 3000 gm.; (*c*) −200 gm.

18. A piece of wood floats in water $\frac{2}{3}$ submerged; and when 10 gm. is placed on top of the wood it floats with the wood completely submerged. (*a*) What is the weight of the wood in air? (*b*) What is the volume of the wood? *Ans.* (*a*) 20 gm.; (*b*) 30 cc.

19. A lighter whose hull is rectangular, 70 ft. long by 20 ft. wide, floats in water with 4 ft. of its hull under water. (*a*) When a weight of 43.75 tons is placed on the lighter, how much of the hull will be under water? (*b*) What is the weight of the lighter? *Ans.* (*a*) 5 ft.; (*b*) 175 tons.

20. A stick floats in water $\frac{2}{3}$ submerged. How much of it will be submerged when it is floated in kerosene whose density is .78 gm. per cc.? *Ans.* .854.

21. A log that floats in fresh water half immersed is just completely immersed when a man weighing 125 lb. stands upon it. What is the volume of the log? *Ans.* 4 cu. ft.

22. A rectangular stick having a base of 4 sq. cm. and a height of 22 cm. floats in water with 4 cm. of its height out of water. (*a*) What is the weight of the stick? (*b*) How many cc. of lead whose density is 11 gm. per cc. placed on top of the stick will just submerge it? *Ans.* (*a*) 72 gm.; (*b*) 1.46 cc.

23. A piece of copper weighs 200 gm. in air and 177 gm. in water. What is the sp. gr. of copper? *Ans.* 8.7.

24. A cork weighs 5 gm. in air; when tied to a sinker which weighs in air 100 gm. and both are submerged in water, they weigh 71 gm.; the sinker alone in water weighs 86 gm. What is the sp. gr. of cork? *Ans.* .25.

25. A rectangular cherry block 7.5 cm. long, 7.5 cm. wide, and 3.75 cm. thick floats in water with 2.6 cm. of its thickness submerged. What is the sp. gr. of cherry wood? *Ans.* .693.

26. An empty bottle weighs 42 gm.; when filled with water it weighs 108 gm.; when filled with alcohol it weighs 95 gm. What is the sp. gr. of alcohol? *Ans.* .8.

27. A piece of lead weighs 140 gm. in air, 127.5 gm. in water, and 125 gm. in dilute sulphuric acid. What is the sp. gr. of the acid? *Ans.* 1.2.

28. A stick of uniform section, loaded at one end, floats upright in water with 25 cm. of its length submerged; when floated in nitric acid 18 cm. of its length is submerged. What is the sp. gr. of nitric acid? *Ans.* 1.39.

29. Into one arm of a U-tube half full of water, oil is poured until the column of oil is 16 cm. long; the height of water in the other arm above the level of the surface separating the two liquids is 14 cm. What is the sp. gr. of the oil? *Ans.* .875.

30. A cube 2 cm. on an edge weighs 20 gm. in water. (*a*) What does it weigh in air? (*b*) What does it weigh in oil whose sp. gr. is .8? *Ans.* (*a*) 28 gm.; (*b*) 21.6 gm.

31. A body whose substance has a density of 3.5 gm. per cc. is 4 cm. × 5 cm. × 3 cm. (*a*) Find its mass, (*b*) its weight in water, (*c*) its weight in alcohol whose density is .8 gm. per cc. *Ans.* (*a*) 210 gm.; (*b*) 150 gm.; (*c*) 162 gm.

32. In a tube whose diameter is 1 mm. the capillary ascension of water is 30 mm. What will be the ascension in a tube .8 mm. in diameter? *Ans.* 37.5 mm.

33. In a tube 1 mm. in diameter, the capillary depression of mercury is 18 mm. What allowance should be made for capillarity in reading the height of a column of mercury in a tube 9 mm. in diameter? *Ans.* 2 mm. should be added to the reading.

CHAPTER III

STATICS OF GASES

66. Causes of Gas Pressure. — Gases exert pressure from two causes: (1) because of their weight, (2) because of the impact of their rapidly vibrating molecules against the walls of the containing vessel, as stated in the kinetic theory of matter.

67. Gravity Pressure of a Gas. — Since the density of gases is a very small quantity (the mass of 1 cc. of air is .001293 gm.), it is only in the case of a very large volume of a gas, such as the atmosphere, that the pressure due to its weight becomes an appreciable force. The atmosphere is a layer of a mixture of gases, variously estimated at from 50 to 200 miles in thickness, which completely surrounds the earth.

As in liquids, the gravity pressure of the atmosphere varies with its depth and density, but there is this important difference: Liquids are but very slightly compressible, so that 1 cc. of a liquid near the bottom of even a very large body of it has practically the same mass as 1 cc. of the liquid near the surface. Gases, on the contrary, are very compressible, so that in the layers of the atmosphere nearest the earth which are compressed by the weight of the large mass of air above them the density is greatest. It follows then that a column of air 100 feet high nearest the earth contains a greater mass than any column of the

same length higher up in the atmosphere. Therefore, the gravity pressure of the atmosphere increases *with* the depth of the atmosphere, but *not in proportion* to the depth.

68. Atmospheric Pressure and Height above Sea Level. — The pressure of the atmosphere at high altitudes is less than at sea level, because of the less depth of atmosphere pressing at these high levels, but the pressure at sea level is *more* than proportionally greater.

Altitude in Miles	Pressure in mm. of Mercury	Altitude in Miles	Pressure in mm. of Mercury
18.63	9	4.66	301
12.42	51	3.11	416
9 32	108	1.56	565
7.77	153	0.00	760
6.21	217	(sea level)	

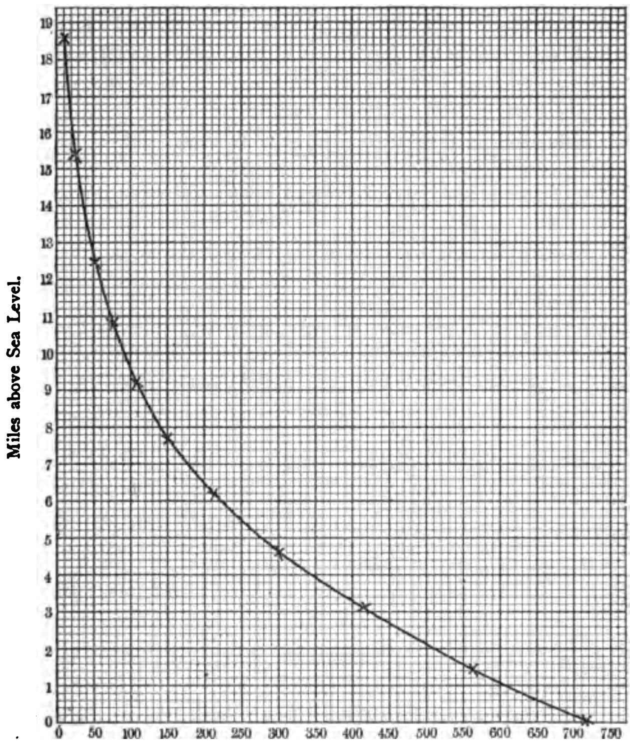
By plotting these data on cross-section paper (Fig. 20), laying off on the vertical axis the altitudes in miles above sea level on a scale 1 cm. ≈ 2 mi., and on the horizontal axis the pressure of the atmosphere in millimeters of mercury on a scale 1 cm. ≈ 100 mm., the curve shown is obtained.

As appears from the curve, half of the mass of the atmosphere is within $3\frac{1}{2}$ mi. of sea level.

Starting at the altitude 18.63 mi. above sea level as the upper surface of the atmosphere, since the pressure there is but 9 mm. of mercury, it is seen that the pressure of the first 6 mi. of atmosphere below this assumed upper surface is $51 - 9 = 42$ mm.; in the second 6 mi. the pressure increases to $240 - 51 = 189$ mm.; and in the last 6 mi. the pressure increases to $760 - 240 = 520$ mm.

69. Causes of Variation of Atmospheric Pressure. — The density of the atmosphere varies not only because of its compressibility, but also because of the presence, to a greater or less extent, of water vapor whose density is .62

that of dry air. At places, then, where the air is moist, or just before a storm, the pressure of the atmosphere de-



Pressure of the Atmosphere in mm. of Mercury.

FIG. 20.—Graph showing the decrease of atmospheric pressure with an increase in the altitude above the sea level.

creases because of the presence of a large quantity of this less dense vapor.

The greater number of the storms that cross our country in an easterly direction are cyclonic in character, that is,

a large volume of the atmosphere comes to have a whirling motion. Whenever a mass of fluid is rotated, it tends to heap up on the outside of the whirl and form a depression in the center. That part of the earth's surface which is below the center of one of these storms will then be the point of lowest pressure owing to a least depth of atmosphere above it, and there will be a ridge of high pressure on the outside of this whirl, which progresses as a wave over the surface of the earth.

70. Measurement of Atmospheric Pressure. — A *barometer* is an instrument devised for measuring atmospheric pressure. There are two general types of barometers: *mercurial* and *aneroid* (meaning without liquid).

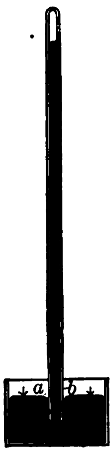


FIG. 21 *a*.

A simple mercurial barometer consists of a glass tube over 80 cm. long (Fig. 21 *a*), closed at one end, which has been filled entirely with mercury (thus removing all of the air from the tube), and inverted in a cistern of mercury. The mercury in the cistern transmits the pressure of the atmosphere on its surface to the open end of the tube, so that a column of mercury extending above the level of the mercury outside is supported in the tube whose weight equals the pressure transmitted to the open end of the tube. The normal height of this column of mercury in the tube above the level of the mercury in the cistern is 760 mm. at sea level.

The pressure of the mercury on 1 sq. cm. at the point *b* within the tube is the same as the pressure of the atmosphere on 1 sq. cm. at the same level *a* on the outside mercury surface. The pressure of a liquid upon 1 sq. cm. is

hp, and since the height of the mercury in the tube above b is 76 cm. and the density of mercury is 13.6 gm. per cc., the pressure of the atmosphere on 1 sq. cm. at a equals $76 \times 13.6 = 1033.6$ gm.

Converting centimeters into inches and grams per cubic centimeter into pounds per cubic inch,

$$1 \text{ in.} = 2.54 \text{ cm.}, \quad 1 \text{ cu. in.} = (2.54)^3 = 16\,386 \text{ cc.}, \quad 1 \text{ lb.} = 453.6 \text{ gm.}$$

$$76 \text{ cm.} = \frac{76}{2.54} = 29.92 \text{ in.}$$

$$13.6 \text{ gm. per cc.} = \frac{13.6}{453.6} \times 16.386 = .491 \text{ lb. per cu. in.}$$

Therefore the atmospheric pressure on 1 sq. in. $= 29.92 \times .491 = 14.69$ lb. This is usually stated in whole numbers, as 15 lb. per sq. in.

A mercurial barometer mounted for actual use is shown in Fig. 21 *b*. The directions for reading such an instrument are given in the appendix, § 451.

Since the mean barometric height at sea level is 76 cm., or, as usually given, 760 mm., the barometer tube must be at least that long, and should be a little longer to allow for any increase above normal pressure. The space above the mercury column in the tube, containing nothing except possibly a small amount of mercury vapor, is known as the Torricellian Vacuum, named from an Italian, Torricelli, a pupil of Galileo, who first measured the air pressure by this method in 1643.

FIG. 21 *b*.

71. Aneroid Barometer. — An aneroid barometer (Fig. 22) consists essentially of a thin metal box partially exhausted of air. The cover of this

box moves in and out, like the chest in breathing, as the pressure of the atmosphere upon it varies. This slight motion of the cover, multiplied by a system of levers, is shown by an index moving over a scale which is calibrated by comparison with a standard mercurial barometer.

72. The Barometer and Weather Changes. — The changes in the height of the barometer with a change in the weather depends, as stated, to some degree upon the change in the density of the atmosphere due to a variation in the quantity of water vapor present; but these changes in the barometer depend, to a very large extent, upon the cyclonic whirls in the atmosphere common to all the general storms, which, so far as the United States is concerned, usually originate either somewhere northwest of Puget Sound or in the Gulf of Mexico. These whirls have a general easterly course, and the pressure at any place varies as these atmospheric whirls pass over it. Each whirl piles up the atmosphere on the outside of it, forming there a ridge of high pressure and producing a region of low pressure in the center.

FIG. 22.

The height of the barometer does not indicate any particular kind of weather; but the *change* in the height, the *direction* of the change, and the *rate* of change indicate the characteristics of the *change* in the weather. A rapidly falling barometer foretells a sudden violent storm. A grad-

ually falling barometer foretells a more moderate storm and one of longer duration. On the other hand a rising barometer foretells clearing weather in a corresponding manner.

73. Barometric Height as a Measure of Altitude. — From the table of altitudes and corresponding atmospheric pressures given in § 68 a rule may be derived for a rough estimate of altitude by means of a barometer.

In the first mile and a half (8200 ft.) above the earth's surface the pressure changes 195 mm., or 7.7 in., which indicates a fall of the barometer of about 1 in. for each 1000 ft. of ascent within this distance. However, in the first 1000 ft. the barometer falls a little more than 1 in., in the last 1000 ft. of the 8000 ft. of ascent it falls a little less than 1 in.

74. Gas Pressure due to Molecular Motion. — The second cause of gas pressure arises from the fact that the molecules of a body of gas are moving with great velocities, and if the gas is confined in a vessel, the impact of these moving molecules against the walls of the vessel creates a pressure which represents the tendency of the gas to expand.

The pressure which a given mass of gas will exert from this cause varies with the space the gas is compelled to occupy. For, as the volume of the gas is diminished, the number of impacts of the molecules in the same time upon 1 sq. cm. of surface increases.

If in a bent tube (Fig. 23), having the end of one arm closed, a certain mass of some gas is confined by pouring in sufficient mercury to fill the bend of the tube, and if the liquid surfaces, a_1 and b_1, are at the *same* level, the pressure which the confined gas exerts upon surface a equals the

pressure upon surface b, which in this case is the atmospheric pressure.

If now more mercury is poured into the open arm, the liquid surface, a_2, will be raised, causing the gas to occupy less space, the mercury surface, b_2, will also be raised, and will be higher than surface a_2, which fact indicates that the pressure of the confined gas upon surface a_2 is more than the pressure of the atmosphere upon surface b_2 by as much as the pressure of the mercury column h above the level of a_2.

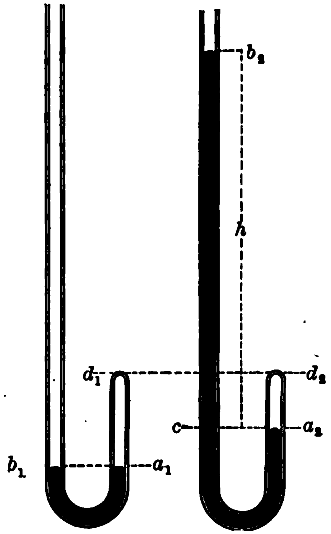


FIG. 23.

When the volume of the confined gas is d_1a_1, the pressure is equal to *one atmosphere;* when the volume of the confined gas has been decreased to d_2a_2, the pressure is increased to *one atmosphere plus* the pressure of h.

75. Boyle's Law. — The first experimental results, showing the change in pressure accompanying a change in the volume of a gas, were furnished by an Englishman, Robert Boyle, in 1662, who used a tube similar to that shown in the accompanying diagram, and mercury as the liquid. The following table is taken from the report of Boyle, entitled "Defence of the Doctrine touching the Spring and Weight of the Air."

$\frac{1}{4}''$ *A*	1'' *A*	In. *B*	*C*	*D*	*E*
48	12	00		$29\frac{2}{16}$	$29\frac{2}{16}$
46	$11\frac{1}{2}$	$01\frac{7}{16}$		$30\frac{9}{16}$	$30\frac{6}{16}$
44	11	$02\frac{13}{16}$		$31\frac{15}{16}$	$31\frac{12}{16}$
42	$10\frac{1}{2}$	$04\frac{6}{16}$		$33\frac{8}{16}$	$33\frac{1}{7}$
40	10	$06\frac{3}{16}$		$35\frac{5}{16}$	35
38	$9\frac{1}{2}$	$07\frac{14}{16}$		37	$36\frac{15}{19}$
36	9	$10\frac{2}{16}$		$39\frac{4}{16}$	$38\frac{7}{8}$
34	$8\frac{1}{2}$	$12\frac{8}{16}$		$41\frac{10}{16}$	$41\frac{2}{17}$
32	8	$13\frac{1}{16}$		$44\frac{1}{16}$	$43\frac{11}{16}$
30	$7\frac{1}{2}$	$17\frac{15}{16}$		$47\frac{1}{16}$	$46\frac{3}{5}$
28	7	$21\frac{3}{16}$		$50\frac{5}{16}$	50
26	$6\frac{1}{2}$	$25\frac{3}{16}$		$54\frac{5}{16}$	$53\frac{10}{13}$
24	6	$29\frac{11}{16}$		$58\frac{13}{16}$	$58\frac{2}{8}$
23	$5\frac{3}{4}$	$32\frac{3}{16}$	$29\frac{1}{8}$ inches	$61\frac{5}{16}$	$60\frac{18}{23}$
22	$5\frac{1}{2}$	$34\frac{15}{16}$		$64\frac{1}{16}$	$63\frac{6}{11}$
21	$5\frac{1}{4}$	$37\frac{15}{16}$		$67\frac{1}{16}$	$66\frac{4}{7}$
20	5	$41\frac{9}{16}$		$70\frac{11}{16}$	70
19	$4\frac{3}{4}$	45		$74\frac{2}{16}$	$73\frac{11}{19}$
18	$4\frac{1}{2}$	$48\frac{12}{16}$		$77\frac{14}{16}$	$77\frac{2}{3}$
17	$4\frac{1}{4}$	$53\frac{11}{16}$		$82\frac{12}{16}$	$82\frac{4}{17}$
16	4	$58\frac{2}{16}$		$87\frac{14}{16}$	$87\frac{3}{8}$
15	$3\frac{3}{4}$	$63\frac{15}{16}$		$93\frac{1}{16}$	$93\frac{1}{5}$
14	$3\frac{1}{2}$	$71\frac{5}{16}$		$100\frac{7}{16}$	$99\frac{6}{7}$
13	$3\frac{1}{4}$	$78\frac{11}{16}$		$107\frac{13}{16}$	$107\frac{7}{13}$
12	3	$88\frac{2}{16}$		$117\frac{9}{16}$	$116\frac{4}{8}$

A A.—The number of equal spaces in the shorter leg that contained the same parcel of Air diversely extended.

B.—The height of the Mercurial Cylinder in the longer leg that compressed the Air into those dimensions.

C.—The height of a Mercurial Cylinder that counterbalanced the pressure of the Atmosphere.

D.—The Aggregate of the two last Columns, *B* and *C*, exhibiting the pressure sustained by the included Air.

E.—What the pressure should be according to the Hypothesis that supposes the pressures and expansions to be in reciprocal proportion.

The above results indicate that it is very approximately, if not absolutely, true that the pressure of a gas of constant mass and temperature is inversely proportional to the volume it is compelled to occupy. For the first volume 12 cc. : the 13th volume 6 cc. = the 13th pressure $58\frac{13}{16}$ in. : the first pressure $29\frac{2}{16}$ in. The products, V_1P_1 ($349\frac{1}{2}$) and $V_{13}P_{13}$ ($352\frac{3}{4}$), are equal within 1 %. These results of Boyle are interesting, not because of their ac-

curacy, for they are not as accurate as many a modern schoolboy is able to obtain, but solely from a historical point of view, since they are the results upon which was based the relation known as Boyle's law: —

The volume of a given mass of gas at a constant temperature varies inversely as the pressure exerted upon it.

76. Limitations of Boyle's Law. — The pressure of an easily liquefied gas, calculated by using Boyle's law, for a given change in volume deviates somewhat from the true pressure, and the deviation increases as that pressure is approached at which, for the given temperature, the gas changes to liquid.

77. Open Tube Manometer. — Appliances for measurements of pressure, especially gas pressure, are called *manometers*. If the pressure to be measured is a low one, such as the pressure in the illuminating gas pipes, an open tube manometer, shown in Fig. 24, may be used. The pressure of the gas, *i.e.* its excess of pressure above atmospheric pressure, upon the liquid surface *a* equals the pressure of the column *h* of the liquid, or the difference in the level of *c* and *a*.

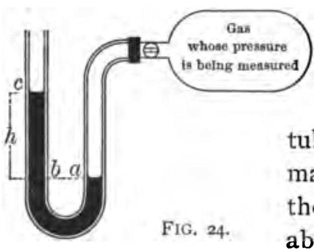


FIG. 24.

78. Pressure of Compressed Air. — It is, of course, evident that if the volume a gas can occupy is kept constant and additional quantities of gas are pumped into this space, the pressure will increase proportionally.

For example, if into a space occupied by 10 gm. of air under atmospheric pressure 10 gm. more air are pumped, the pressure becomes 2 atmospheres, and if 90 gm. of additional air are pumped into the space, the pressure becomes 10 atmospheres or approximately 150 lb. to the

square inch. It is in this manner that reservoirs of compressed air are obtained to which are connected the drills, hammers, and similar tools whose motive power is compressed air.

79. Closed Tube Manometer. — To measure such large gas pressures a closed tube manometer, shown in Fig. 25, may be employed. Air is confined in the closed arm by the mercury or other liquid which fills the bend of the tube. The closed tube is graduated to enable the pressures to be read directly from the volumes of this confined air which varies inversely as the pressure upon it.

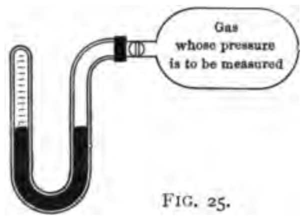


FIG. 25.

80. Exhausting Air Pump. — This pump, shown in Fig. 26, removes air from within a receiver R, by means of a reciprocating, *i.e.* up and down or back and forth, motion of a tightly fitting piston in the pump cylinder, aided by the action of three valves, V_1, V_2, and V_3, placed at the opening of the pipe leading into the cylinder, in the opening through the piston, and in the top of the cylinder respectively.

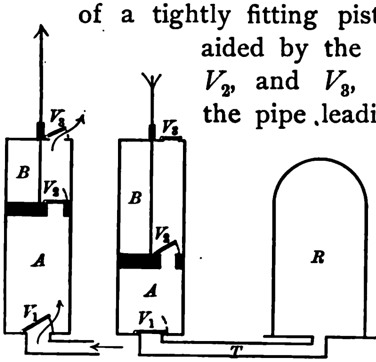


FIG. 26.

A valve is defined as a lid, cover, ball, disk, plug, or plate, lifting, oscillating, rotating, or sliding in connection with a port or aperture, so as to *permit or prevent* the passage of a fluid through the port which it guards.

Suppose all the valves to be closed; then if the piston is raised, the volume of the air in A is increased and the air pressure in that portion is proportionally decreased, therefore the greater air pressure in R and the connecting tube T opens valve V_1 and the air expanding rushes into A until the pressures are equalized. Meanwhile the air in portion B being compressed keeps valve V_2 tightly closed but opens valve V_3 and this air is forced out of the cylinder through V_3. When the piston is lowered, the air in A is compressed and the pressure in B is reduced so that valves V_1 and V_3 are held tightly shut, but valve V_2 is opened and the air in A is forced by the moving piston into portion B of the cylinder. Upon the next upstroke of the piston this air in B is forced out of the cylinder through valve V_3 and upon the opening of V_1 air from R again rushes into portion A of the cylinder. Thus at each successive upstroke of the piston some air is removed from R until the pressure of the remaining air is so diminished that it is unable to lift valve V_1.

81. Compressing Air Pump. — This pump, shown in Fig. 27, differs from the exhausting pump in two respects: (1) in the direction in which the valves move to open or close the ports which they guard and (2) in the absence of a valve in the upper port i. By the reciprocating motion of the piston additional quantities of air are forced into the reservoir R which thereby becomes a reservoir of compressed air.

Suppose both valves are closed and that the piston is drawn outward; owing to the reduced pressure in A, valve V_1 is closed and V_2 opened and the air in B rushes into A. When the piston is pushed inward, valve V_2 is closed, the air in A is compressed, V_1 is opened, and

the air in A is forced by the moving piston into the reservoir R. This operation repeated at each successive stroke forces each time into R the quantity of air which the pump cylinder contains under atmospheric pressure.

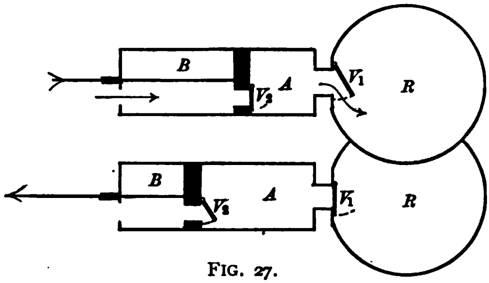


FIG. 27.

82. Lift Pump for Liquids. — In this pump, shown in Fig. 28, the position and movement of valves V_1 and V_2 are the same as in the exhausting air pump. By the reciprocating motion of the piston, aided by the atmospheric pressure upon the surface of the liquid in the reservoir R, liquid is raised from the reservoir R to the level of the spout N.

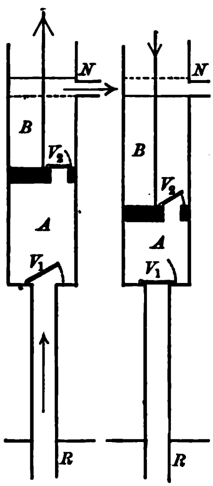


FIG. 28.

Suppose both valves are closed and that the piston is raised; owing to the reduction of the pressure of the air in A, valve V_1 is opened, valve V_2 is closed, and some of the liquid in R is forced by the atmospheric pressure on its surface into the cylinder until the air pressure in A plus the pressure of the raised liquid equals the atmospheric pressure. When the piston is lowered, valve V_1 is forced shut, valve V_2 is opened, and the air and water in A is forced by the moving piston into portion B of the cylinder. When the piston

is again raised, the water in B is lifted to the level of the spout N, where it flows out; meanwhile water is again forced into A by the pressure of the atmosphere upon the liquid in the reservoir.

It is evident that if the height of valve V_2 at its lowest position above the level of the liquid in the reservoir R is greater than the height of the column of this liquid the atmosphere can support, no liquid will come into that port of the cylinder above the piston and consequently none will be lifted to the spout even if the piston and valves are air tight.

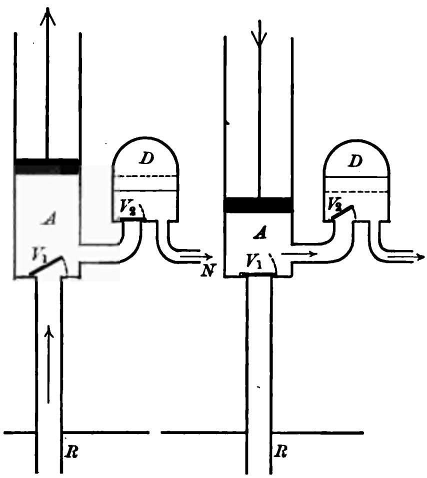


FIG. 29.

83. Force Pump for Liquids. — By this pump (Fig. 29) liquids can be raised from the reservoir R into the pump cylinder A and from there forced into the air dome D and out through the outlet N. The liquid is raised into the pump cylinder by the atmospheric pressure on the surface of the liquid in the reservoir R when the air pressure in A is reduced by the raising of the piston. When the piston is lowered, valve V_1 is closed, valve V_2 is opened, and the liquid is forced into the air dome D from which some is forced through the outlet N. The outlet is of smaller diameter than the pipe supplying liquid to the air dome D; hence during the

downstroke of the piston liquid is supplied to the air dome faster than it can be forced through the outlet, and the air in the dome is compressed by this accumulating liquid. While the piston is again being raised and more liquid is lifted by atmospheric pressure into the pump cylinder, the air in dome D expands and forces the liquid in it through the outlet. Thus by means of this air dome a continual stream of liquid is delivered at the outlet.

84. The Siphon. — An inverted U-tube filled with a liquid and having one end immersed in a vessel of the liquid and the other end either open to the air, or else immersed in another vessel of the liquid the surface of which is at a lower level than that of the first, constitutes a *siphon*.

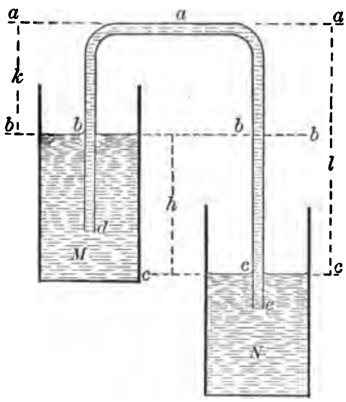


FIG. 30.

Such a tube is shown in Fig. 30. The lengths of the two *arms of a siphon* are the *vertical distances*, k and l, from the highest point in the bend of the tube down to the liquid surfaces respectively (or down to the end of the tube, e, if that end is open to the air). Careful distinction must be made between the length of an *arm of a siphon* and the length of the tube.

The liquid flows from vessel M to vessel N at a gradually decreasing rate, the flow ceasing altogether when the liquid surfaces in the two vessels reach the same level.

The force acting upon the liquid in the siphon tending to move it toward N is the atmospheric pressure *minus* the

pressure of the liquid column in arm *ab*, while the force tending to move the liquid in the siphon toward *M* is the atmospheric pressure *minus* the pressure of the liquid column in arm *ac*. Since the atmospheric pressure is the same on both sides, the force tending to move the liquid toward *N* is greater than that tending to move it toward *M* by an amount equal to the *difference* of the pressures of the two liquid columns, which equals the pressure of a liquid column whose length is *h*. The value of *h* may be determined from the levels of the two liquid surfaces, or from the length of the siphon arms, being the difference in either case. As the flow continues this difference decreases and when it reaches a zero value, the flow ceases because the forces acting are then in equilibrium.

It is evident that the length, *k*, of arm *ab* cannot be longer than the height of a column of liquid which the atmospheric pressure can support, *i.e.* *k* must not be more than 30 in. for mercury, or more than 30 in. $\times$ 13.6 = 408 in. = 34 ft. for water.

PROBLEMS

1. What is the pressure of the atmosphere per square centimeter when the height of the mercurial barometer is 750 mm., the density of mercury being 13.6 gm. per cc.? *Ans.* 1020 gm.

2. What is the height of a water barometer at the place where the mercurial barometer reading is 30.2 in.? *Ans.* 410.72 in.

3. What would be the height of the atmosphere, considered of *uniform* density, when the mercurial barometer stands at 760 mm., the density of air being .001293 gm. per cc.? *Ans.* 7993.8 m.

4. (*a*) If in place of the layer of atmosphere upon the surface of the earth a layer of water was substituted, what should be its depth in order to give the same pressure? (*b*) If a layer of mercury was substituted, what should be its depth? (Assume normal atmospheric pressure.)

5. A certain mass of air in the closed arm of a Boyle's law tube has a volume of 20 cc. when the mercury surface in the open arm is 190 mm. higher than that in the closed arm. What will be the volume of this air when the tube is adjusted so the mercury surface in the closed arm is 190 mm. higher than that in the open arm, the barometer reading being 760 mm.? *Ans.* $33\frac{1}{3}$ cc.

6. Why is air near the sea level denser than air on the top of a high mountain?

7. 30 cc. of air is confined in the closed arm of a Boyle's law tube. Mercury is poured into the open arm until its level is 1140 mm. above that in the closed arm. What is the volume of the confined air, the barometer reading being 760 mm.? *Ans.* 12 cc.

8. A gas cylinder having a cross section of 1 sq. ft. and a height of 4 ft. is filled with oxygen at a pressure of 225 lb. per sq. in. How many cubic feet of oxygen under normal atmospheric pressure are in the cylinder? *Ans.* 60 cu. ft.

9. The volume of a certain mass of hydrogen is 250 cc. under a pressure of 800 mm. of mercury. What is its volume under standard pressure, 760 mm.? *Ans.* 263.16 cc.

10. A bicycle pump cylinder is 30 cm. long. If the tube leading from the lower end is kept closed, what force is needed to push the piston whose area is 5 sq. cm. from the top of the cylinder to 10 cm. from its lower end, neglecting friction? Assume the atmospheric pressure to be 1 kgm. per sq. cm. *Ans.* 10 kgm.

11. When the height of a mercurial barometer is 30 in., what is the greatest height of the piston valve at its lowest position of a perfect lift pump above the water level in the cistern in order that water may be pumped into the cylinder? *Ans.* 34 ft.

12. When the mercurial barometer is 760 mm., what is the greatest possible length of the short arm of a siphon for sulphuric acid (sp. gr. 1.84)? *Ans.* 5.617 m.

CHAPTER IV

STATICS OF SOLIDS

85. Graphical Representation of a Force. — In the study of the conditions of equilibrium of solids under the action of forces it is often convenient to make diagrams representing these conditions. In order to completely describe a given force the three *elements of a force* must be known: (1) its point of application, (2) its direction, (3) its magnitude. In Physics, the *magnitude* of a quantity is its value expressed in units of that quantity.

Since a straight line has three similar elements any force may be represented by a straight line as follows: (1) the point of application of the force by the origin of the line, (2) the direction of the force by the direction of the line which for convenience is called the line of action of the force, and (3) the magnitude of the force by the length of the line drawn to a scale. The point of application of a force may be transferred along its line of action without changing the effect of the force, consequently the origin of the line drawn to represent a force may be any point in its line of action.

86. General Conditions of Equilibrium. — In order that a solid may be at rest two motions must be prevented, *viz.* *translatory* and *rotary*. *Translatory* motion is motion in which all the parts of a body move in straight lines parallel to one another (Fig. 31). *Rotary* motion is motion in

which the parts of a body move in circular paths about some stationary point (Fig. 32).

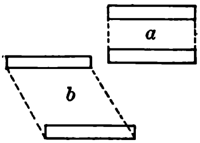


FIG. 31.

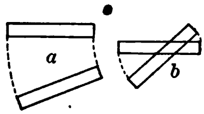


FIG. 32.

87. Simplest Case of Equilibrium. — To produce equilibrium there must be at least two forces acting upon the body. The conditions for equilibrium of two forces are: (1) *the two forces must be equal in magnitude*, (2) *they must act in opposite directions*, (3) *they must act in the same straight line.*

In Fig. 33 it is evident that if forces *A* and *B* are of equal magnitudes and act in opposite directions, the body *M* cannot have a translatory motion.

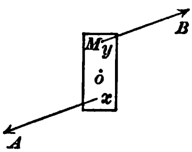


FIG. 33.

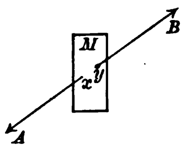


FIG. 34.

In Fig. 34 it is evident that if the forces *A* and *B* are equal and act in opposite directions, the body *M* cannot have *translatory* motion; but since they do not act in the same straight line, the two forces will *rotate* the body about the point *o* midway between *x* and *y*, the points of application of the forces.

88. Mechanical Couple. — Two equal and opposite parallel forces not in the same straight line constitute what is known as a *mechanical couple* and their joint effect always is rotation.

89. Resultant; Equilibrant.—A *resultant* is a *single* force which if substituted for two or more forces produces the same effect as the forces acting jointly. An *equilibrant* is a *single* force which when acting with other forces produces equilibrium.

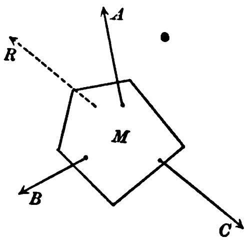


FIG. 35.

The resultant of a number of forces is equal in magnitude to the equilibrant of those forces, acts in the same straight line as the equilibrant but in the *opposite* direction.

For example, the equilibrant of forces A and B (Fig. 35) is the force C since the body M is assumed to be at rest while under the action of these three forces. The resultant of forces A and B is represented by the dotted line R, since it would have their joint effect, *viz.* to produce equilibrium with force C, the force R being equal to force C, and acting in the same straight line but in the opposite direction.

90. Conditions of No Translation by Three Parallel Forces. —If a body is acted upon by three parallel forces, in order that there shall be no *translatory* motion two of the forces must act in the same direction, and the third in the opposite direction, and the sum of the two forces in the same direction must equal the third opposing force.

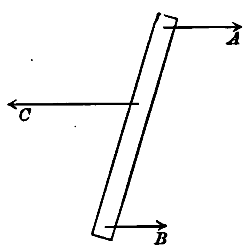


FIG. 36.

If the three parallel forces A, B, and C, shown in Fig. 36, are in equilibrium, the entire translatory effect of A and B is directly opposed to that of force C, and the sum of the effects of A and B equals that of force C.

91. Rotating Effect of a Force.—The tendency of a force to produce rotation depends upon two conditions: (1) the magnitude of the force, and (2) its point of application with reference to the axis of rotation.

That the point of application of the force is important may be shown by trying to swing an open door by pushing first at the hinge edge, then at the middle, and finally at the outer edge. In the first case it is impossible to produce any motion; and the effectiveness of the force to produce rotation increases as the distance from the hinge edge increases.

92. Moment of a Force.—*The moment of a force is its effectiveness in producing rotation* and its *value* is the *product* of the *magnitude* of the force and the *perpendicular distance from the axis to the line of action* of the force.

A moment is considered positive (+) when the tendency is to produce clockwise rotation, and negative (−) when the rotary tendency is counter clockwise.

93. Conditions of No Rotation by Three Parallel Forces.—To determine the conditions under which three forces, as A, B, and C (Fig. 37), will produce no rotation, let l be the axis and k, l, m, the points of application of the forces respectively. Force A has a tendency to rotate the body counter clockwise, *i.e.* in a negative direction about l, the value of its moment being $A \times lf$. The force C has no moment about l, for its distance from l is zero; $\therefore C \times 0 = 0$. The moment of force B about l is clockwise, *i.e.* in a positive direction, and its value is $B \times el$.

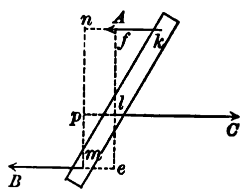


FIG. 37.

If the body is to have no rotation, the negative moment of A must equal the positive moment of B, i.e. $A \times lf = B \times el$, hence

$$A : B = el : fl.$$

Therefore to prevent rotation about the point of application of force C, the magnitude of the forces A and B must be inversely proportional to their respective distances, fl and el, from that point.

If the assumed axis passes through m, the point of application of force B, the moment of force A about m is negative and equal to $A \times mn$; the moment of force C about m is positive and equal to $C \times mp$; the moment of force B about m is zero. To prevent the rotation of the body about this axis the negative moment of B must equal the positive moment of C, or

$$A \times mn = C \times mp;$$

hence

$$A : C = mp : mn.$$

Therefore, to prevent rotation about the point of application of force B the magnitude of forces A and C must be inversely proportional to their respective distances, mn and mp, from this point.

Again, by assuming k, the point of application of force A, to be the axis, it may be shown in a like manner that to prevent rotation the magnitude of forces B and C must be inversely proportional to their respective distances, mn and pn, from this point.

From this it follows that *when three parallel forces are in equilibrium, any two of the forces are inversely proportional to their respective distances from the point of application of the third force.*

94. Point of Application of the Opposing Force. — It is evident also that the point of application of force C must

lie somewhere *between* the points of application of forces A and B; for if, as shown in Fig. 38, force C were not so applied, forces A and B might produce rotation about some point between l and k, since the moment of each force about such a point is in that case positive, and there being no moment in a contrary direction, rotation must ensue.

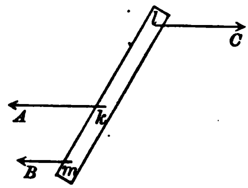


FIG. 38.

95. Conditions of Equilibrium of Three Parallel Forces. — Summarizing the conditions of equilibrium of three parallel forces: —

1. Two of the forces must act in the same direction and the third in the opposite direction.

2. The point of application of the opposing force must lie *between* the points of application of the two forces acting in the same direction.

3. The sum of the magnitudes of the two forces acting in the same direction equals the magnitude of the opposing force.

4. The magnitudes of any two of the three forces are inversely proportional to their respective distances from the point of application of the third force.

96. Equilibrium of a Weightless Lever. — A *lever* is a rigid bar supported on a line or axis, called the *fulcrum*, about which it can rotate.

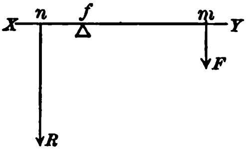


FIG. 39.

Suppose the rod, XY (Fig. 39), to have no weight, and to be supported at f in such a way that it can rotate freely about f as an axis. If the forces F, called the *effort*, and R, called the *resistance*, act, as

shown, upon this lever at the points m and n, the conditions of equilibrium of three parallel forces are represented; for there will be a force at the fulcrum f upward, and equal to the sum of F and R. Also, to prevent rotation, the distances mf and nf (called the *arms* of the lever) are *inversely* proportional to the forces F and R, or $F:R = nf$ (the arm of R) $: mf$ (the arm of F).

Suppose the fulcrum, f, of the weightless lever, XY, to be near one end of the lever, and the two forces, F and R, to act upon it, as shown in Fig. 40; the conditions of equilibrium of three parallel forces are again represented. The force at the fulcrum is downward, corresponding to R, and the sum of R and the force at the fulcrum is equal to F. Also, to prevent rotation, $F \times mf = R \times nf$, or $F:R = nf$ (the arm of R) $: mf$ (the arm of F).

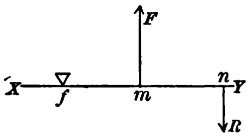


FIG. 40.

Levers are sometimes divided into three classes: (1) with the fulcrum between F and R, (2) with R between F and the fulcrum, (3) with F between R and the fulcrum.

There is no difference in the application of these fundamental principles to each class of lever. It must be borne in mind, however, that the *arm* of a force, or, as it is sometimes called, its *leverage*, is always the *distance from that force to the fulcrum.*

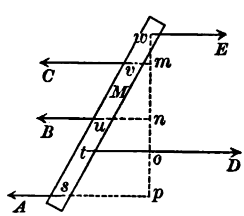


FIG. 41.

97. Equilibrium of Any Number of Parallel Forces. — In Fig. 41, let A, B, C, D, and E be five parallel forces acting upon the body M. To prevent translation, at least one of the forces must act in a direction opposing the others, and, also, the sum of the

forces in one direction must equal the sum of the forces in the opposite direction.

For example, $A+B+C=D+E$, or $A+B+C-D-E=0$. One direction may arbitrarily be chosen as positive $(+)$ and the opposite as negative $(-)$; then the *algebraic sum* of all the forces is *zero*.

To prevent rotation, the moment of at least one of the forces about an axis through *any* point must be in the opposite direction to that of the moments of the other forces, or, in other words, some of the moments must be *positive* and some *negative*, and the algebraic sum of all these moments about the chosen axis must equal zero.

If, for example, the chosen axis passes through w, the moments

$$(A \times pw) - (D \times ow) + (B \times nw) + (C \times mw) + (E \times o) = 0.$$

98. The Weight of a Body. — Thus far in discussing the action of forces upon solids it has been assumed for the sake of simplicity that the bodies had no weight. This of course is not true, so that this factor must now be considered in the conditions of equilibrium.

Since every particle of matter in a body is attracted by the earth, there will be as many gravitation forces acting upon the body as there are particles in it, and since the lines of action of these forces meet only at the center of the earth, 4000 miles away, they may be considered as practically parallel.

By the term *weight of a body* is meant a *single force equal to the sum of the weights of all the particles of the body*; or, in other words, *the weight of a body is the resultant of the weights of all its particles.*

99. Point at which the Weight of a Body Acts. — To find where this single force, called the *weight* of a body,

must be considered as acting, let the body M (Fig. 42) be supported at the point o by the force E; it will come to rest in a certain position with the line of action of force E vertical, because there are here but two forces, *viz.:* the supporting force E and the weight of the body M acting vertically downward, and the body is at rest; therefore by the conditions of equilibrium of two forces, the weight of the body equals force E, acts opposite to force E and in the same straight line. A vertical line drawn through point o will therefore pass through the point at which the weight of the body acts.

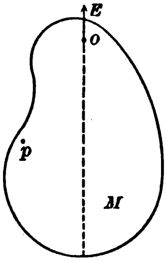


FIG. 42.

Again, let the body be supported at some other point, p, (Fig. 43), when for the same reason, the point at which the weight of the body acts lies somewhere in the vertical line drawn through the point p. Since this point lies in both lines, it must be at their intersection.

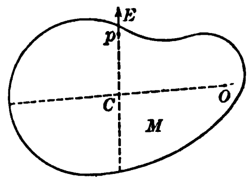


FIG. 43.

The point at which the weight of a body acts is called its *center of gravity;* and from the above statements it is evident that if a body is supported at *one point*, its center of gravity lies somewhere in the vertical line passing through the point of support.

100. Center of Gravity of a Body. — Suppose a body (Fig. 44) is made up of three masses, 2 gm., 3 gm., and 5 gm., respectively, joined together by a weightless rod, the distances between their centers being 3 cm. and 4 cm., respectively, as indicated, the center of gravity of the body may be found as follows: —

The weight of the body is 10 gm. (2 + 3 + 5). To produce equilib-

rium the equilibrant E must equal 10 gm. and be applied at such a point c that the moment of E about any axis equals the *sum* of the moments of the 2 gm., 3 gm., and 5 gm. about the same axis.

Suppose the assumed axis is at o; then the moment of the weight of the 2 gm. about o is zero, the moment of the weight of the 3 gm. about $o = 3 \times 3 = +9$, the moment of the weight of the 5 gm. about $o = 5 \times 7 = +35$. The sum of the moments of the weights 2 gm., 3 gm., and 5 gm. is $+44$; therefore the moment of E must be -44. Since E is 10 gm., its distance from $o = 4.4$ cm. The center of gravity is therefore the point c which is 4.4 cm. from o or 1.4 cm. from p.

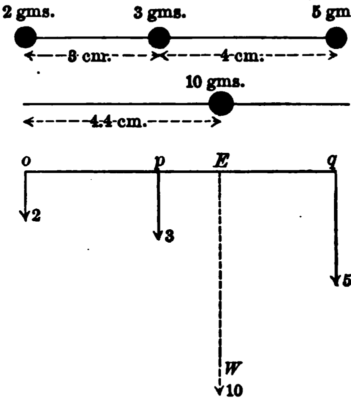


FIG. 44.

The *center of gravity* of a body is therefore the point at which the single force, called the weight of the body, acts, which force equals the sum of the weights of the parts of the body, and this point is so located that the weight of the body has the same moment about any axis as the sum of the moments of the weights of the parts of the body about the same axis.

101. States of Equilibrium. — According to the way a body behaves when disturbed from a state of rest, its state of equilibrium is *stable*, *unstable*, or *neutral*.

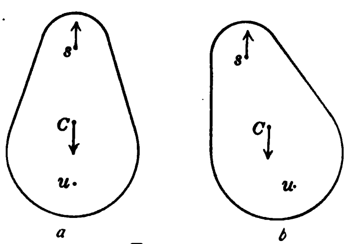


FIG. 45.

Suppose that a body whose center of gravity is point C (Fig. 45 a) is supported at point s; it will be

at rest when C is vertically beneath s. If now (Fig. 45 *b*) the lower end of the body is drawn to one side, the center of gravity is raised and the weight of the body acting at C will have a moment about s tending to bring it back to its original position of rest; so that when the lower end of the body is released, it swings back and forth, finally coming to rest in the original position with C directly below s. This state of equilibrium is called *stable*.

If, however, as shown in Fig. 46 *a*, the body is supported at u, it may be at rest when C is vertically above u. If, as shown in Fig. 46 *b*, the upper part of the body is drawn a little to one side, the center of gravity is lowered and the weight of the body acting at C will have a moment about u tending to pull the body still farther from its original position of rest; so that when released, the body swings farther away, finally coming to rest in a stable position. The state of equilibrium is called *unstable* when the body on being disturbed has its center of gravity lowered.

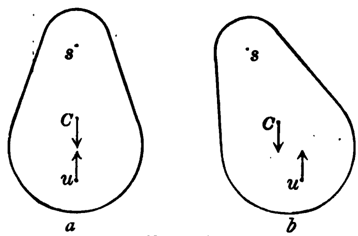


FIG. 46.

If, again, the body is supported at C, it will remain at rest in whatever position it is placed; for the weight acting at C has no moment about C. This state of equilibrium in which a disturbance neither raises nor lowers the center of gravity is called *neutral*.

102. Sensitiveness of a Beam Balance. — A beam balance is a lever xfy (Fig. 47) with scale pans resting upon it at points x and y, the fulcrum of the lever being the knife edge at f. The center of gravity, c, of the beam balance

is a short distance *below* the point of support *f*; consequently the balance is in stable equilibrium. Since *c* is near *f*, when the balance swings, the moment of the weight of the balance is very small; hence a very small mass placed in either scale pan will displace the lever from its position of equilibrium. This distance of *c* from *f* determines the *sensitiveness* of the balance.

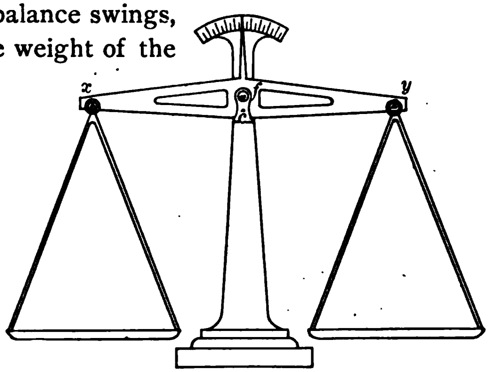


FIG. 47.

CONCURRENT FORCES

103. Parallelogram of Forces. — Suppose a body *M* (Fig. 48) is acted upon by three concurrent forces *A*, *B*, and *C*, whose lines of action meet at a point *O*; to show the condition under which the body will remain at rest in this case, represent each force by drawing from point *O* a straight line whose *direction* is that of the force, the *length* of the line representing the *magnitude* of the force to any convenient scale.

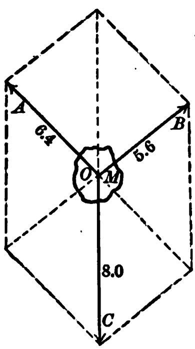


FIG. 48.

If on any two of these lines as adjacent sides a parallelogram is constructed and the diagonal which passes through the point *O* is drawn, it will be found that this diagonal is of the same length as the line representing the third force and lies in the same straight line.

104. Triangle of Forces. — To understand why the above relation exists, suppose a body at A (Fig. 49) to be acted upon by a force of 5 units sufficient to move it to B in a certain time; if then a force of 4 units acts upon it sufficient to move it to C in an equal time; if, finally, a force of 8 units acts upon the body sufficient to move it from C to A in a time equal to that of each of the two previous motions; the body will then have returned to its starting point.

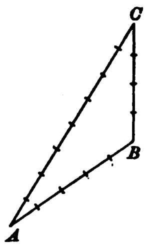


FIG. 49.

If these three forces act upon the body simultaneously, the body remains at rest because the result of the successive action of the forces is to bring the body back to the starting point, and therefore the result of their simultaneous action is to keep it at this point. It follows, then, that *if three forces can be represented by the three sides of a triangle, taken in order, the forces are in equilibrium.*

105. Equivalence of these Two Principles. — This principle of the triangle of forces and the principle of the parallelogram of forces are equivalent.

For, if from point A, Fig. 50, a line is drawn parallel and equal to BC, and the parallelogram is completed by drawing DC, and if from A the line AE is drawn equal to CA and in the same direction, then the lines AB, AD, and AE represent the forces AB, BC, and CA, respectively, and the diagonal AC of the parallelogram constructed upon AB and AD as adjacent sides is equal to the third force AE, and lies in the same straight line.

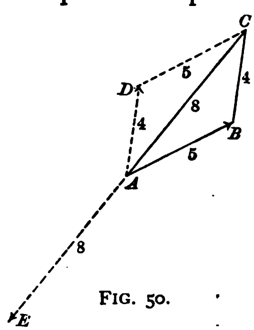


FIG. 50.

106. Resultant of Two Concurrent Forces. — It follows, then, that the resultant of two concurrent forces is equal to the diagonal of the parallelogram constructed upon these two forces as adjacent sides, the diagonal being drawn through the point common to the two forces.

The effect of the forces *AB* and *AD* is to produce equilibrium with force *AE*; but a single force, equal, opposite to, and in the same straight line as *AE*, produces equilibrium with it, and the diagonal drawn through the common point represents such a force.

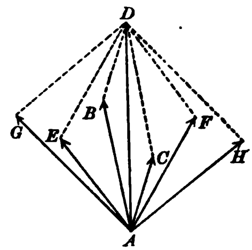


FIG. 51.

107. Resolution and Composition of Forces. — It follows from the above that the force *AD*, Fig. 51, may be the resultant of an indefinite number of pairs of forces, as *AB*, *AC*; *AE*, *AF*; or *AG*, *AH*, etc.; for *AD* is the diagonal of the parallelogram *ABCD*, also of the parallelograms *AEDF* and *AGDH*.

The process of finding the components to which a given force is equivalent is called the *resolution of a force*. The process of finding the resultant of several forces is called the *composition of forces*.

108. Resolution along Rectangular Axes. — In resolving a force, or forces, it is convenient to select components acting at right angles to each other.

The resultant of forces *OA* and *OB* (Fig. 52) may be found by resolving them into their components in the direction of the rectangular axes XX' and YY', and then finding the resultant of these four components. The components of *OA* are *OP* along the axis of *X*, and *OQ* along the axis of *Y*. The components of *OB* are *OS* along the axis of

X, and OT along the axis of Y. Since both of the Y components extend upward from O, the Y component of the resultant of OA and OB equals $OQ + OT = OT'$. Since the X components are in opposite directions from O, the X component of the resultant of OA and OB equals $OP - OS = OS'$. The resultant of OT' and OS', which is the resultant of OA and OB, equals $\sqrt{\overline{OT}'^2 + \overline{OS}'^2}$.

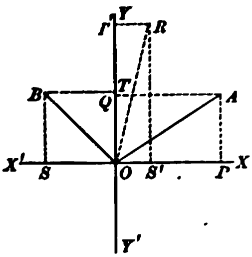


FIG. 52.

Problem in Equilibrium of Three Concurrent Forces. — The beam AB (Fig. 53 a) of a derrick is inclined at an angle of 30° with the vertical center post AC and a weight of 2 T. hangs from the upper end of the beam; find the tension of the horizontal cable BC.

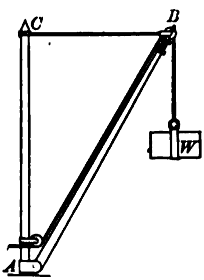


FIG. 53 a.

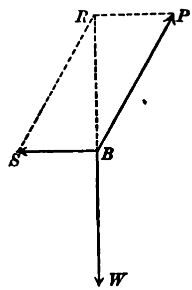


FIG. 53 b.

The three forces acting at B (Fig. 53 b) are: (1) the suspended weight, 2 T., acting vertically downward; (2) the tension of the cable BC, pulling horizontally toward the left; (3) the upward thrust of the beam AB represented by the line BP. The resultant of the tension of the cable BC and the thrust BP of the beam must equal BW (2 T.) and lie in the same straight line. Therefore draw BR vertically upward equal to BW (2 T.), and from R complete the parallelogram by drawing RS and RP parallel to BP and BC, respectively. The line BS therefore represents the tension of the cable BC.

Since $\measuredangle CAB = 30°$, $\measuredangle BRS = 30°$ (sides are parallel). $\measuredangle RBS$ is a rt. $\measuredangle$. $\therefore \measuredangle RSB = 60°$.

Therefore the line BS is one half of the line RS.

But
$$\overline{RS}^2 = \overline{BS}^2 + \overline{BR}^2$$
$$BR = 2 \text{ T. and } RS = 2\,BS$$
$$4\,\overline{BS}^2 = \overline{BS}^2 + (2)^2$$
$$3\,\overline{BS}^2 = 4$$
$$\overline{BS}^2 = 1.333$$
$$BS = \sqrt{1.333} = 1.15 \text{ T.}$$

NOTE. — For other solutions of this problem, see appendix.

PROBLEMS

1. Three parallel forces act upon a body and keep it at rest. Two of the forces pull east, the third pulls west. One of the east forces is 5 kgm., the west force is 8 kgm. and is 12 cm. distant from the east force of 5 kgm. (*a*) What is the other east force? and (*b*) how far apart are the two east forces? *Ans.* (*a*) 3 kgm.; (*b*) 32 cm.

2. A man carries a weight of 20 lb. on the end of a stick 3 ft. long placed over his shoulder; the other end of the stick he holds in his hand. (*a*) What is the pressure on his shoulder if the stick is placed so that the hand is 2 ft. from the shoulder? (*b*) What is the pressure on the shoulder if the stick is placed so that the hand is 1 ft. from the shoulder? Neglect the weight of the stick. *Ans.* (*a*) 30 lb.; (*b*) 60 lb.

3. A shovel $4\frac{1}{2}$ ft. long is held horizontally by a man with his left hand at the end of the handle and his right hand $2\frac{1}{2}$ ft. from the end of the handle. When a weight of 40 lb. is placed $\frac{1}{2}$ ft. from the other end of the shovel, (*a*) what force does the left hand exert? (*b*) What force does the right hand exert? (*c*) In what direction is the force of the left hand? (*d*) In what direction is the force of the right hand? Neglect the weight of the shovel. *Ans.* (*a*) 24 lb.; (*b*) 64 lb.; (*c*) down; (*d*) up.

4. A square top, *ABCD*, of a table, 3 ft. on a side, is supported by 4 legs, one at each corner. When a weight of 50 lb. is placed on the

table at a point 1 ft. from each of the two adjacent sides at the corner *B*, what part of the weight is borne by each leg? *Ans.* *A*, $11\frac{1}{9}$ lb. or $\frac{2}{9}$; *B*, $22\frac{2}{9}$ lb. or $\frac{4}{9}$; *C*, $11\frac{1}{9}$ lb. or $\frac{2}{9}$; *D*, $5\frac{5}{9}$ lb. or $\frac{1}{9}$.

5. A wheelbarrow is 4 ft. long from the end of the handles to the axle of the wheel in front. When a weight of 150 lb. placed on the barrow $1\frac{1}{2}$ ft. from this axle is lifted by a force at the ends of the handles, what is the pressure of the wheel on the ground? Neglect the weight of the barrow. *Ans.* $93\frac{3}{4}$ lb.

6. A weightless rod 70 cm. long rests on a fixed point 25 cm. from one end. To this end a weight of 2 kgm. is attached. What weight must be hung from the other end so that the rod may be horizontal? *Ans.* 1.11 kgm.

7. Two parallel forces when acting at the ends of a rod 75 cm. long have a resultant of 5 kgm. if they are acting in the same direction and one of 1 kgm. when acting in opposite directions. Find (*a*) the forces and (*b*) the points of application of their resultants. *Ans.* (*a*) 2 kgm. and 3 kgm.; (*b*) (1) 45 cm. from the 2 kgm. force; (2) 225 cm. from the 2 kgm. force.

8. Two parallel forces of 100 gm. and 120 gm. act in opposite directions 60 cm. apart. Find the magnitude and point of application of the force which will produce equilibrium. *Ans.* 20 gm. acting 360 cm. from the 100 gm. force and 300 cm. from the 120 gm. force.

9. A stiff pole 10 ft. long projects horizontally from a vertical wall. It would break if a weight of 30 lb. were hung at the outer end. How far out on the pole may a boy weighing 110 lb. venture with safety? *Ans.* 2.7 ft.

10. A horse and a colt capable of pulling 800 lb. and 500 lb. respectively are harnessed to a wagon. If the horse is hitched to the cross bar at a point $1\frac{1}{2}$ ft. from the tongue of the wagon, at what point should the colt be hitched in order that he may pull his share of the load? *Ans.* 2 4 ft. from the tongue of the wagon.

11. A bridge 40 ft. long is supported on stone abutments at its ends. The weight of the bridge, 20 T., may be considered as acting at its center. A horse and wagon weighing 2 T. is 18 ft. from one end of the bridge, and a man on horseback, joint weight $\frac{3}{4}$ T., is 12 ft. from the other end of the bridge. What is the pressure on each abutment? *Ans.* (1) $11\frac{13}{40}$ T., (2) $11\frac{17}{40}$ T.

12. A uniform board 8 ft. long and weighing 20 lb. rests on top of a wall 2 ft. thick so that $2\frac{1}{2}$ ft. of the board overhangs on the near side of

the wall. If a boy weighing 60 lb. hangs on the near end of the board and a boy weighing 85 lb. hangs on the far end, (*a*) about which edge of the wall will the board turn? (*b*) How much more should the near boy weigh to prevent its turning? *Ans.* (*a*) The farther edge. (*b*) $3\frac{8}{9}$ lb.

13. A stick weighing 400 gm. is balanced at a point 15 cm. from one end when 200 gm. is hung at that end. Where is the center of gravity of the stick? *Ans.* 22.5 cm. from the one end.

14. A stick 50 cm. long weighing 400 gm. balances at a point 20 cm. from one end. At what point will it balance if 100 gm. are placed at that end and 200 gm. at the other end? *Ans.* 25.7 cm. from the 100 gm.

15. A 3-part telescope is 15 cm. long when closed and 45 cm. long when drawn out. The weights of the parts, each considered uniform, are 100 gm., 75 gm., and 50 gm. respectively. Where is the center of gravity of the telescope when drawn out? *Ans.* 19.16 cm. from the larger end.

16. A 50 cm. stick has a mass of 200 gm. hung from its 5 cm. mark. A fulcrum is under its 10 cm. mark and the stick is balanced. (*a*) If the stick is uniform, what is its weight? (*b*) What is the pressure on the fulcrum? *Ans.* (*a*) $66\frac{2}{3}$ gm.; (*b*) $266\frac{2}{3}$ gm.

17. A rod weighing 2 kgm. balances at a point 60 cm. from one end; but if a body of unknown weight is hung at that end, the balancing point is 25 cm. from the end. What is the weight of the body? *Ans.* 1.43 kgm.

18. A uniform rod 3 ft. long weighing 4 lb. is fastened to the center of a uniform rod 2 ft. long weighing 3 lb. and at right angles to it. Find the center of gravity of the body. *Ans.* $10\frac{2}{7}$ in. along the 3 ft. rod from the point of attachment.

19. Find the center of gravity of a letter E made of wood, the principal pieces of which are of the same length and the central piece half that length. *Ans.* $\frac{5}{14}$ of the length of the central piece from its outer end.

20. Two forces of 6 kgm. and 8 kgm. act upon a body at an angle of 90°; (*a*) what force will keep them in equilibrium? (*b*) In what direction must it act? *Ans.* (*a*) 10 lb.; (*b*) in the same line as the diagonal of the ▱ constructed on the 6 and 8 lb. forces which passes through the common point but in the opposite direction; *or* (*b*) at an angle of 127° with the 6 lb. force and 143° with the 8 lb. force.

21. A person weighing 120 lb. sits in a hammock so that one rope (*A*) makes an angle of 30° with the vertical post to which it is tied and

the other rope (B) makes an angle of 60° with the vertical post to which it is tied. What is the pull on each rope? *Ans.* A, 103.9 lb.; B, 60 lb.

22. What is the component of a weight of 10 lb. placed on a plane inclined 60° to the horizon which is effective in moving the body along the plane? *Ans.* 8.66 lb.

23. A picture weighing 25 lb. is hung by a cord passing over a nail, the two parts of the cord making an angle of 60° with each other. What is the tension of the cord? *Ans.* 14.43 lb.

24. The saddle of a bicycle is placed at the apex of two parts of the frame, making an angle of 60° with each. Considering these two parts of the frame to be of the same length, what is the thrust along each part of the frame when a person weighing 150 lb. sits on the saddle? *Ans.* 86.6 lb.

25. A horizontal rod rests loosely at its inner end in a socket in the wall of a building and from the outer end hangs a weight of 40 lb.; from this end also is run a wire which is fastened to the wall of the building at a point above the rod such that the wire makes an angle of 45° with the wall. What is the tension of the wire? *Ans.* 56.57 lb.

26. A square space is inclosed by passing a rope around 4 posts at the corners; the tension of the rope is 10 lb. What is the pressure the rope exerts on each post? *Ans.* 14.14 lb.

27. A boat propelled by a force which would give it a velocity of 10 ft. a sec. in still water moves directly across a river 1800 ft. wide, the river current being 2 ft. a sec. (*a*) Toward what point in the opposite shore is the boat headed? (*b*) How long will it take the boat to cross the river? *Ans.* (*a*) Toward a point 367 ft. up the river from the point directly opposite; (*b*) 3 min. 3.7 sec.

28. During a rain in which the drops fall vertically a man standing on the front platform of a trolley car moving at the rate of 16 ft. per sec. finds that he must step back 2 ft. under the front end of the car roof which is 8 ft. above the platform, to avoid getting wet. What is the velocity of the raindrops? *Ans.* 64 ft. per sec.

CHAPTER V

KINEMATICS

109. Kinematics. — *Kinematics* is the name given to that portion of mechanics which treats of motion, leaving out of consideration, for the time being, the influence of the forces acting, or the mass of the body acted upon.

110. Motion Relative. — Motion is change of position. To state the position of any point some other point must be taken as reference. Motion is therefore a relative term. A man seated in a moving train is not changing his position with reference to his fellow-passengers, and therefore with reference to them is not in motion; but he is in motion with reference to objects outside of the train.

To completely describe the motion of any body at a given instant of time both the *rate* and the *direction* of the motion must be stated.

111. Velocity. — The rate of motion of a body is its velocity. The velocity of a body at any instant is the distance it *would* move during a unit of time if its rate of motion remained unchanged.

The statement that the velocity of a bullet is 2000 ft. per sec. means, that if the bullet continued to move for 1 sec. at the rate at which it is moving at the given instant, it would traverse a distance of 2000 ft. The velocity of a train at a given instant may be 40 mi. per hr., and yet the train may be brought to rest within the next five minutes.

112. Units of Velocity. — The C. G. S. unit of velocity is 1 cm. per sec.; the F. P. S. unit of velocity is 1 ft. per sec.

113. Uniform Motion. — The velocity of a body may be *uniform* or *variable*. If it is *uniform*, the body *moves the same distance in each successive unit of time.*

For example, the velocity of a train after it has "resumed full speed" may be uniform for a considerable length of time, *i.e.* it may move the same distance (say 50 ft.) in each second.

Let v be the velocity of a body; t, the number of seconds the body is moving; then the distance s the body moves in this time with uniform velocity is expressed by the equation: —

$$s = vt. \tag{1}$$

114. Acceleration. — If the velocity of a body is *variable*, it moves *unequal distances during successive seconds of its motion. The distance a body, having variable velocity, moves in each second is its average velocity for that second.*

Acceleration is the name given to the *rate of change of velocity*, *i.e.* the *change per second* of the velocity.

For example, if at the end of the third second of its motion a body has a velocity of 15 cm. per sec., at the end of the fourth second its velocity is 20 cm. per sec., at the end of the fifth second is 25 cm. per sec., the change of velocity is 5 cm. per sec. during each second, or the *acceleration* is 5 cm. per sec. per sec.

115. Units of Acceleration. — The C. G. S. unit of acceleration is 1 cm. per sec. per sec.; the F. P. S. unit is 1 ft. per sec. per sec.

116. Uniformly Accelerated Motion. — If a body, starting *from rest*, has a uniform acceleration of a cm. per sec. per sec., its velocity at the end of the first second is a cm. per sec.; at the end of the second second the velocity is $2a$ cm.

per sec.; at the end of the third second the velocity is $3a$ cm. per sec.; at the end of t sec. the velocity is at cm. per sec. Expressed in equational form:—

$$v = at. \tag{2}$$

If the velocity at the end of the first second is a cm. per sec., and the body started *from rest*, the *average* velocity during the first second is $\frac{0 + a}{2} = \frac{a}{2}$ cm. per sec.

Since the average velocity in the first second is $\frac{a}{2}$ cm. per sec., the distance the body moves during the first second is $\frac{a}{2}$ cm. At the end of the first *two* seconds the velocity has become $2a$ cm. per sec. The average velocity during these two seconds is $\frac{0 + 2a}{2} = a$ cm. per sec. A body moving for two seconds with an average velocity of a cm. per sec. traverses a distance $a \times 2 = 2a$ cm. At the end of the first three seconds the velocity has become $3a$ cm. per sec. The average velocity during this period is $\frac{0 + 3a}{2} = \frac{3a}{2}$ cm. per sec. The distance the body has moved during the three seconds is then $\frac{3a}{2} \times 3 = \frac{9a}{2}$ cm. At the end of t seconds the velocity has become at cm. per sec. The average velocity during this period is $\frac{0 + at}{2} = \frac{at}{2}$ cm. per sec. The distance s the body has moved during the t seconds is then $\frac{at}{2} \times t = \frac{at^2}{2}$ cm.

Expressed in equational form:—

$$s = \tfrac{1}{2} at^2. \tag{3}$$

From formula (3) it is evident that the distance traversed by a body, starting from rest and moving with uniform acceleration, is directly proportional to the *square* of the number of seconds it is moving, and is equal to the product of the *square of this time* and the *distance* it moved from rest *during the first second.*

Squaring equation (2), $t = \frac{v}{a}$, gives $t^2 = \frac{v^2}{a^2}$.

Substituting for t^2 in equation (3),

$$s = \tfrac{1}{2} a \left(\frac{v^2}{a^2}\right) = \frac{v^2}{2a};$$

hence
$$v^2 = 2as. \tag{4}$$

117. Motion with Initial Velocity. — If instead of starting from rest a body has impressed upon it at the start an initial velocity V_0, as, for example, when a stone is *thrown* down from the top of a building, the velocity the body has after moving for t sec., with an acceleration of a cm. per sec. per sec., is the sum of the initial and acquired velocities, or

$$v = V_0 + at. \tag{5}$$

The distance s the body will move is the *sum* of the distance $V_0 t$ it would move with the initial uniform velocity V_0 and the distance $\frac{1}{2}at^2$ it would move with the accelerated velocity given to it, or

$$s = V_0 t + \tfrac{1}{2}at^2. \tag{6}$$

Eliminating t from equations (5) and (6),

$$v^2 = V_0^2 + 2as. \tag{7}$$

If the direction of V_0 is opposed to that of the acceleration, as, for instance, when a body is thrown *upward*, the minus sign is substituted for the plus sign in formulæ (5), (6), and (7).

118. Falling Bodies. — The most familiar example of uniformly accelerated motion is that of bodies falling under the action of gravity. The acceleration of a freely falling body is 980.26 cm. per sec. per sec., or 32.128 ft. per sec. per sec., in the latitude of New York.

The approximate values, 980 and 32, are generally used. This value is commonly denoted by the small letter g, and formulæ (2) to (7) inclusive may be written to apply directly to falling bodies by substituting the letter g for a.

PROBLEMS

1. If a body moves with a uniform velocity of 10 cm. per sec. for 20 sec., how far will it have traveled? *Ans.* 200 cm.

2. In what time will a train moving at a uniform rate of 20 mi. per hr. traverse a distance of $\frac{1}{2}$ mi.? *Ans.* $\frac{1}{40}$ hr. (90 sec.).

3. With what uniform velocity will a body move 1800 ft. in 5 min.? *Ans.* 6 ft. per sec.

4. While a train is moving at the rate of 30 mi. per hr. (44 ft. per sec.), the brakes are put on, bringing the train to a full stop in 22 sec. (*a*) What is its average velocity during this time? (*b*) How far did the train move after the brakes were applied? (*c*) What was the acceleration of the train, considered uniform, in this period? (*d*) What was the velocity of the train 8 sec. after the brakes were applied? (*e*) In what time did the train move the first 100 ft. after the brakes were applied? (*f*) What was the velocity of the train after it had moved this 100 ft.?

Ans.
- (*a*) 22 ft. per sec. (15 mi. per hr.).
- (*b*) 484 ft.
- (*c*) − 2 ft. per sec. per sec. ($1\frac{4}{11}$ mi. per hr. per sec.).
- (*d*) 28 ft. per sec. ($19\frac{1}{11}$ mi. per hr.).
- (*e*) 2.4 sec.
- (*f*) 39.2 ft. per sec.

5. A baseball is thrown vertically upward with a velocity, as it leaves the hand, of 2940 cm. per sec. (*a*) How long will it rise? (*b*) How high will it rise? (*c*) How long after it leaves the hand will it return to the starting point? (*d*) When will its velocity be 1000 cm. per sec.? (g = 980 cm. per sec. per sec.)

Ans. (*a*) 3 sec. (*b*) 4410 cm. (*c*) 6 sec. (*d*) 1.98 sec. after leaving hand and again 4.02 sec. after leaving hand.

6. A stone is dropped from a bridge 90 ft. above the river flowing under it. If g is 32 ft. per sec. per sec., (*a*) after what time will the stone strike the water? (*b*) What is its velocity just as it strikes the water? (*c*) What is its velocity 2 sec. after it starts falling?

Ans. (*a*) 2.37 sec. (*b*) 75.5 ft. per sec. (*c*) 64 ft. per sec.

7. A body thrown downward with a velocity of 20 ft. per sec. from the top of a building reaches the ground in 2.5 sec.; find the height of the building. *Ans.* 150 ft.

8. A package moving upward in an elevator at a uniform rate of 10 ft. a sec. falls off and strikes the bottom of the elevator shaft 2 sec. later. (*a*) How far above the bottom of the shaft was it when it fell off the elevator? (*b*) How far above the bottom of the shaft was it $\frac{1}{4}$ sec. after falling off the elevator? *Ans.* (*a*) 44 ft. (*b*) $45\frac{1}{2}$ ft.

9. A body starting from rest acquires in 5 sec., with a uniform acceleration, a velocity of 4900 cm. per sec. (*a*) What is its average velocity? (*b*) How far did it move with this average velocity in the 5 sec.? *Ans.* (*a*) 2450 cm. per sec. (*b*) 12,250 cm.

10. A body at a given instant was moving at the rate of 10 ft. a sec.; at the end of 5 sec. thereafter its velocity was 18 ft. a sec. Assuming the acceleration to be uniform, what was its velocity at the end of (*a*) 3 sec.? (*b*) 10 sec.? *Ans.* (*a*) 14.8 ft. per sec. (*b*) 26 ft. per sec.

11. How long will it take a body, starting from rest, to fall 100 ft.? (*b*) What velocity will it have when it has fallen 100 ft.? *Ans.* (*a*) 2.5 sec. (*b*) 80 ft. per sec.

12. A ball is thrown upward with a velocity of 60 ft. a sec. at an angle of 60° with the horizon. (*a*) How fast would the thrower have to run in order to catch the ball when it comes down? (*b*) How far must he run? *Ans.* (*a*) 30 ft. a sec. (*b*) 97.3 ft.

13. A body, starting from rest, moves with an acceleration of 980 cm. per sec. per sec. (*a*) In what time will it have a velocity of 50 m. per sec.? (*b*) How far has it moved while acquiring that velocity? *Ans.* (*a*) 5.1 sec. (*b*) 127.55 m.

14. A trolley car moving at the rate of 50 ft. per sec. has the brakes so applied as to give it a retardation of 2.5 ft. per sec. per sec. (*a*) In

what time will it come to rest? (*b*) How far will it move before stopping? *Ans.* (*a*) 20 sec. (*b*) 500 ft.

15. A marble, starting from rest, rolls down a smooth incline a distance of 300 ft. in half a minute. (*a*) What is the acceleration? (*b*) What is its final velocity? *Ans.* (*a*) $\frac{2}{3}$ ft. per sec. per sec. (*b*) 20 ft. per sec.

CHAPTER VI

KINETICS

119. *Kinetics* is that portion of mechanics in which the motion of bodies is considered as the effect of applied forces. In determining the effect of a force the influence of the mass of the body in modifying its velocity is taken into account.

It is from the consideration of the effect of the action of force that the true conception of what the *mass* of a body really is may be obtained. The greater the mass of a body, the less is the change in its motion which a given force can produce in a given time; and the greater the mass of a moving body, the greater is the effect it can produce in changing the motion of other bodies.

120. First Law of Motion. — Over two hundred years ago Sir Isaac Newton published three laws of motion which were generalizations from experimental data and facts of common experience. These laws, unmodified to the present day, form the basis of the science of mechanics. The first law, commonly called the Law of Inertia, is: *A body of itself is unable to change its condition of rest or of motion.*

From this law is derived the condition under which uniform motion is obtained; *viz.* that if the force (or system of forces) which sets a body in motion ceases to act (or becomes in equilibrium), then uniform motion results from the fact that matter is helpless to change its condition of rest or of motion. Once set in motion any body if left to

itself will move on forever in a straight line. No force is required to maintain it in uniform motion. If the body has been set in motion, it cannot of itself either increase or decrease its velocity, or change its direction. This property of matter is called *inertia*.

In any case, then, in which the motion of a body is uniform, the forces which originally set it in motion either have ceased to act or have become in equilibrium and the body continues to move with uniform velocity in a straight line from sheer helplessness to do anything else.

121. Momentum. — If a body whose mass is m moves with a velocity v, the product, mv, is called the *momentum* of the body. If a body of mass m moving with velocity V_0 is acted upon by a force so as to change its velocity in t seconds to v, the change in momentum is $mv - mV_0$ or $m(v - V_0)$. The *change in the momentum per second*, or the *rate of change of momentum*, is $\frac{m(v - V_0)}{t}$. From equation (7), $\frac{v - V_0}{t} = a$ (the acceleration); therefore *the rate of change of momentum of a body equals ma, i.e. the product of its mass and acceleration.*

122. Second Law of Motion. — Newton's second law of motion which deals with the above quantity is: *The rate of change of momentum of a body is proportional to the magnitude of the force acting and is in the direction of the force.*

This law then gives a *measure of any force* (F) in the *rate of change of momentum it produces.* Expressed in equational form: —

$$F = ma, \tag{8}$$

or

$$Ft = mv - mV_0. \tag{9}$$

123. Units of Force. — From equation (8) it is evident that a *unit of force* is that force which acting upon *unit mass* gives it *unit acceleration.*

The C. G. S. unit of force, called a *dyne,* is that force which acting upon 1 gram mass gives it an acceleration of 1 cm. per sec. per sec.

The F. P. S. unit of force, called a *poundal,* is that force which acting upon 1 pound mass gives it an acceleration of 1 ft. per sec. per sec.

124. Value of a Force. — From equation (8), $F = ma$, it is evident that to produce the same acceleration in different quantities of matter, the greater the mass of the body, the greater is the force required.

For example, if a full barrel and an empty barrel are rolled along side by side, the force required to set the full barrel in motion is greater proportionally to its greater mass.

Also, to increase the acceleration given to a body whose mass remains constant, a greater force is required.

For example, in order that an automobile may be brought to a stop from full speed in a few seconds a more powerful brake, *i.e.* a greater force, must be employed than if a longer time were given to stop it, and this force must be greater in proportion to the greater rate of change in its velocity.

125. Weight Expressed in Units of Force. — Since $F = ma$, the weight w, of a mass m, is mg units of force. The weight of m gm. is 980 m dynes. The weight of m lb. is 32 m poundals. The C. G. S. unit of *weight, viz.* the weight of 1 gram mass, equals 980 dynes, and the F. P. S. unit of weight, *viz.* the weight of 1 pound mass, equals 32 poundals.

126. Gravity Acceleration Independent of Mass. — When previously discussing the conditions which determine the weight of a body (§ 34) it was shown that the

weight is directly proportional to the mass of a body and is equal to $\frac{mm'}{d^2}c$. Since weight $= m'g$ where m' is the mass of a body on the earth, g must equal $\frac{m}{d^2}c$ where m is the mass of the earth and d the distance from the surface of the earth to its center. All of these factors are constant at the same place on the earth's surface; therefore the value of g must be the same for all bodies at the same place no matter what their mass may be, which means that all bodies fall equally fast under the action of the force of gravity.

This fact may be demonstrated by dropping simultaneously various masses from the top of a tall building and noting their times of fall to the ground, just as Galileo did hundreds of years ago in one of his historic experiments.

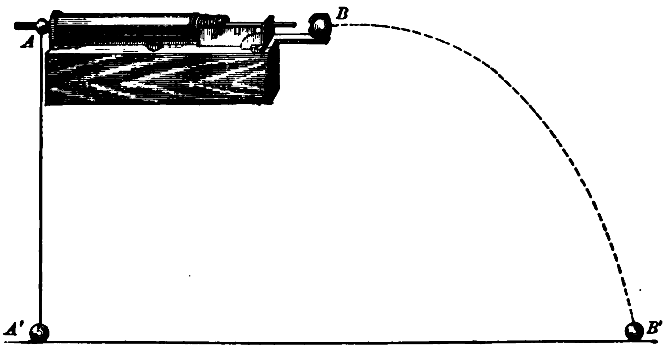


FIG. 54 *a*.

127. Illustration of the Second Law of Motion. — The Second Law implies that a given force produces the same rate of change of momentum, regardless of the mass of the body upon which it acts, or whether it is acting alone or with other forces at the same time. This fact may be

demonstrated by ***projecting*** one body ***horizontally*** and ***dropping*** another ***simultaneously*** from the same level (Fig. 54 *a*). Both bodies strike the floor at the same instant; therefore the force of gravity imparts the same momentum per second in a vertical direction to each ball regardless of the fact that one of the balls is being acted upon by another force at the same time.

If in Fig. 54 *b* the two balls *A* and *B* start at the same time from *a* the ball *B* being projected in the direction *af* while ball *A* is allowed to fall freely in the direction *aq'*, and if the ball *A* traverses the distance *aq'* in $\frac{1}{2}$ sec., in the first $\frac{1}{10}$ sec. ball *A* would move $\frac{1}{25}$ of the distance *aq'* or *am'*. Ball *B* with its uniform velocity of projection would move in the *first* $\frac{1}{10}$ sec. the distance *ab* if the earth were not attracting it; but since the earth gives it the same rate of change of momentum as ball *A*, it will also move a vertical distance *bm* equal to *am'*, *i.e.* the ball *B* will move in the path *am*. By the end of the *second* $\frac{1}{10}$ sec. ball *A* will have moved $\frac{4}{25}$ of the distance *aq'* or *an'*, therefore ball *B* will move an equal distance *cn* from the horizontal line *af* in that time, *i.e.* ball *B* will have moved in the path ***amn***. By the end of the ***third*** $\frac{1}{10}$ sec. ball *A* will have moved $\frac{9}{25}$ of *aq'* or *ao'*, therefore ball *B* will have moved an equal distance *do* below the horizontal line *af* in the same time, *i.e.* *B* has moved in the path ***amno***. By the end of the *fourth* $\frac{1}{10}$ second *A* has moved $\frac{16}{25}$ of *aq'* or *ap'*, *B* has moved in a horizontal direction the distance ***ae*** and in the vertical direction the distance ***ep*** equal to ***ap'***, ***i.e.*** *B* has moved in the path *amnop*. By the ***end of the half second A*** has fallen the distance *aq'* and reached the floor, *B* has moved horizontally the distance *af* and vertically the distance *fq*, having moved in the path ***amnopq***, and has likewise reached the floor.

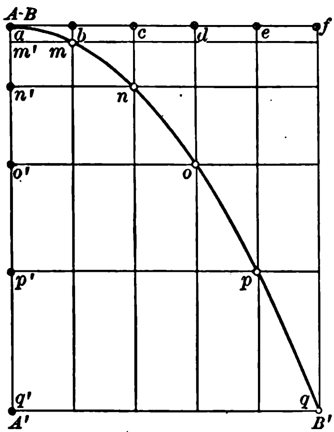


FIG. 54 *b*.

128. Illustration of Second Law of Motion: Second Method.—This law is also illustrated in an ingenious manner with the apparatus shown in Fig. 55*a*.

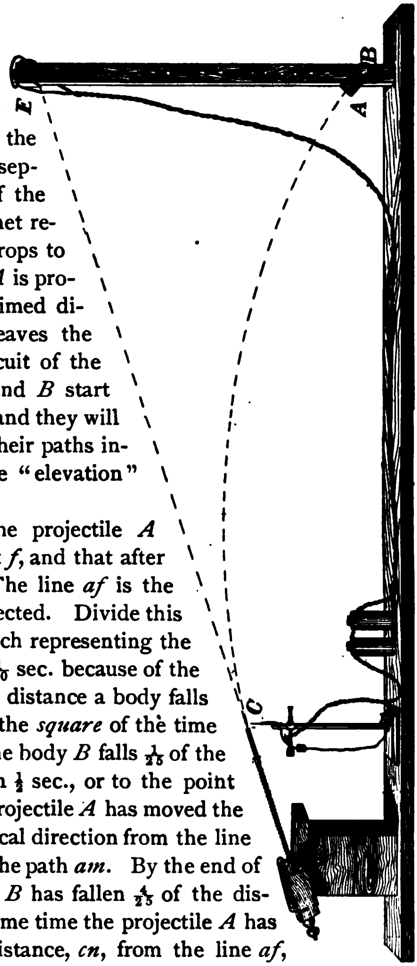


FIG. 55 *a*.

A ball *B* is supported say 10 ft. above the floor by an electro-magnet *E*. If the two wires touching at *C* are separated, the electric circuit of the magnet is broken, the magnet releases the ball *B*, and it drops to the floor. A second body *A* is projected by the spring gun aimed directly at ball *B*. As *A* leaves the gun barrel it breaks the circuit of the magnet at *C*, so that *A* and *B* start falling at the same instant, and they will collide at the point where their paths intersect, no matter what the "elevation" or "range" may be.

Suppose in Fig. 55*b* the projectile *A* starts at *a*, and the ball *B* at *f*, and that after $\frac{1}{2}$ sec. they collide at *q*. The line *af* is the direction in which *A* is projected. Divide this line into five equal parts, each representing the distance *A* would move in $\frac{1}{10}$ sec. because of the projecting force. Since the distance a body falls from rest is proportional to the *square* of the time it falls, in the *first* $\frac{1}{10}$ sec., the body *B* falls $\frac{1}{25}$ of the distance *fq* which it falls in $\frac{1}{2}$ sec., or to the point *m′*. In the same time the projectile *A* has moved the *same* distance, *bm*, in a vertical direction from the line *af*, *i.e.* it has moved along the path *am*. By the end of the *second* $\frac{1}{10}$ sec. the body *B* has fallen $\frac{4}{25}$ of the distance *fq*, or to *n′*. In the same time the projectile *A* has moved the same vertical distance, *cn*, from the line *af*, having moved along the path *amn*. By the end of the *third* $\frac{1}{10}$ sec. *B* has moved $\frac{9}{25}$ of *fq*, or to *o′*, while *A* has moved the same vertical dis-

tance, *do*, or has moved along the path *amno*. By the end of the *fourth* $\frac{1}{10}$ sec. B has moved $\frac{16}{25}$ of fq, or to p', and A has moved the same vertical distance, ep, or has moved along the path *amnop*. By the *end of the half second* the paths of the two bodies cross at q, where they collide.

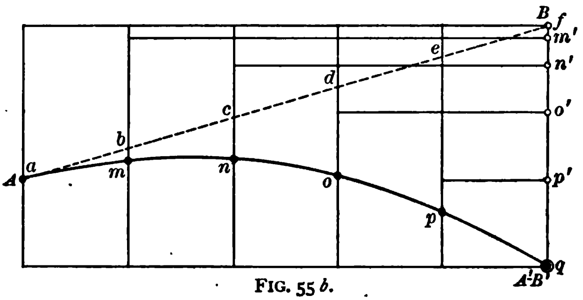


FIG. 55 *b*.

129. The Third Law of Motion. — The third law of motion stated by Newton is: *To every action there is an equal and opposite reaction.*

This implies that in every case of the exertion of force there must be *two masses* concerned and that the action is *mutual and active* for both masses, the only difference in the activity of the two masses is that the *directions are opposite.*

Since change of momentum is the measure of a force acting for a given time, the change of momentum imparted to the second body by the first is equal and opposite to the change of momentum imparted to the first body by the second.

Considering, then, *the two masses as one system*, the total change of momentum of the system due to the *interaction* of the bodies in the system is zero; therefore the momentum of the system *before* the interaction is the same as the momentum of the system *after* the interaction. Expressed in the equational form: —

$$m_1v_1 + m_2v_2 = m_1v_3 + m_2v_4 \qquad (10)$$

where m_1 and m_2 are the masses of the two bodies, v_1 and v_2 their respective velocities before interaction, and v_3 and v_4 their velocities after interaction.

These momenta are added algebraically, the momentum in one direction being considered positive, and momentum in the opposite direction negative.

For example, if a gun and the bullet with which it is loaded is considered as forming one system, the momentum of the bullet, when the gun is fired, plus the momentum of the gun (considered as negative because it is in the opposite direction) equals zero, since the sum of their momenta before firing also equals zero.

130. Reflection after Impact. — If a body A is thrown against a resisting surface BC (Fig. 56) (both being assumed perfectly elastic), the reaction of that surface equals the *normal component op* of the force of the blow *om*, while the *component pm which is parallel to the surface* remains unaffected. The direction the body takes after striking the surface is then that of the resultant *mr* of the reaction *mn*, equal and *opposite* to *op* and of *ml* equal to and in the direction of *pm*.

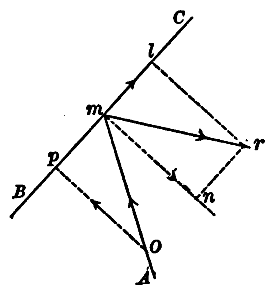


FIG. 56.

The angle, *omn*, between the *line of action of the striking body* and the *normal* to the resisting surface at the point of incidence is called the *angle of incidence*. The angle, *nmr*, between the *line of action of the body when thrown back, or reflected*, by the surface, and the *normal* is called the *angle of reflection*.

From the similarity of the triangles *opm* and *rlm*, it is evident that the angles *pmo* and *lmr* are equal, hence their

complements, the *angles of incidence* and *of reflection*, are equal.

This is commonly stated as the *Law of Reflection: the angle of incidence equals the angle of reflection, and they lie in the same plane.*

If the body and the resisting surface are not perfectly elastic, the reaction *mn* is less than the normal component *op*, hence the angle of reflection will in this case be greater than the angle of incidence.

131. Work. — *Work is the production of motion against resistance.* In the *measurement* of work there are two factors involved: (1) the *effort* used, or the *resistance* overcome; (2) the *distance* through which the effort acts, or the *distance* through which the resistance is overcome in the direction in which it is acting. Work, being proportional both to the *distance* the body moves and to the *quantity of effort* exerted in that direction, is, therefore, measured by their product. Expressed in equational form, —

$$Work = FS = RS' \tag{11}$$

where S is the distance the effort F moves and S' the distance through which the resistance R is overcome.

132. Units of Work. — The C. G. S. unit of work, called an *erg*, is the quantity of work done by a unit force, 1 dyne, in moving against resistance through a unit distance, 1 cm.

The F. P. S. unit of work, called a *foot poundal*, is the work done by a unit force, 1 poundal, in moving against resistance through a unit distance, 1 ft.

The quantity of work represented by an erg is very

small, hence for practical purposes a unit equal to 10,000,-000 (10^7) ergs, called a *joule*, is used.

$$1 \textit{ joule} = 10{,}000{,}000 = 10^7 \textit{ ergs}.$$

In the F. P. S. system for practical purposes a unit is used which is g (32) times as large as a foot poundal, and is called a *foot pound.*

$$1 \textit{ foot pound} = g \textit{ foot poundals}.$$

Illustrations of the Measurement of Work. — 1. In placing a book, weighing 1 lb., resting on the table under one's arm, 2 ft. above the table, 2 ft. lb. of work are done.

2. In climbing the stairs from the first to the fourth floor, a height of 50 ft., if one's weight is 100 lb., $100 \times 50 = 5000$ ft. lb. of work are done.

3. In dragging a sled up a hill 400 ft. long, if the pull on the rope is 4 lb., $4 \times 400 = 1600$ ft. lb. of work are done.

4. In raising the dumb-waiter with a force of 20 lb., if 40 ft. of rope pass through one's hands, $20 \times 40 = 800$ ft. lb. of work are done.

133. Stability. — A body *in stable equilibrium* is said to have a greater or less *degree of stability* according as a greater or less quantity of work is required to overturn it. Let a, b, and c (Fig. 57) represent three positions of a body whose dimensions are, say, 10 cm. × 10 cm. × 20 cm., and suppose the body is to be turned over about the edge pq. Imagine a plane passed *through the center of mass*, m, and *perpendicular to the edge* pq, as $omdf$; then the degree of stability *with reference to this plane* is measured by the quantity of work required to turn the body over on the edge pq. With o as a center draw the arc md; through m draw the horizontal line mf; then the distance df ($= h$) is the vertical height through which the center of mass is raised by turning the body over, and this distance multiplied by the weight, W (which acts at the center of mass), equals the work done: $work = W \times h =$ measure of degree

of stability. In this case $h_1 : h_2 : h_3 = 3 : 15 : 5$ (approximately), therefore the several degrees of stability are to each other as,

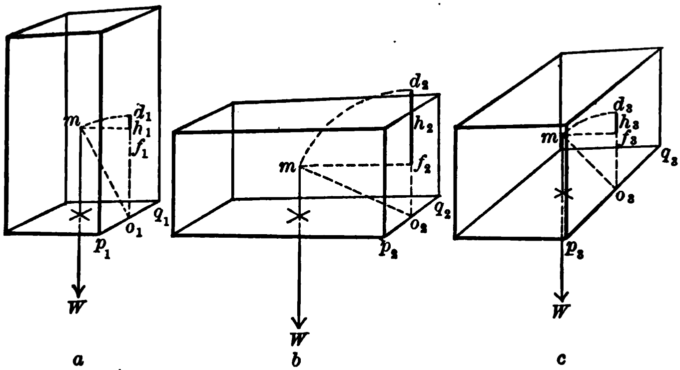


FIG. 57.

$W \times h_1 : W \times h_2 : W \times h_3 = h_1 : h_2 : h_3 = 3 : 15 : 5$. From this it follows:—

1st. That in considering any given case of stability of a body it is necessary to take into account, both the *center of mass* and *the edge* (or *axis*) about which the turning takes place; the forces involved act in a plane passed *through* the former and *perpendicular* to the latter.

2d. That with the same *horizontal* distance between the center of mass and the turning edge (*a* and *c*, Fig. 57), the degree of stability is *less* if the center of mass is *higher*.

3d. That with the same *vertical* distance between the center of mass and the turning edge (*b* and *c*, Fig. 57), the degree of stability is *greater* if the horizontal distance of the center of mass from the turning edge is greater. Neither of these relations, however, are proportional.

134. Energy. — *Energy may now be defined as the ability to do work.* According to the principle of the Conserva-

tion of Energy, no energy is ever created or destroyed; the sum total of the energy in the universe is a constant quantity. The only realities of the physical world are matter and energy, and energy is a possession of matter which it imparts or receives whenever it does work or has work done upon it. If, then, a body possesses energy, it obtained it from some other body because the second body performed a certain amount of work upon the first body, and in performing the work lost as much energy as the first body received. No body can exert force unless it possesses energy.

135. Potential Energy. — If a body possesses energy because of some *position* or *condition* it has acquired by virtue of work having been done upon it, this energy is called *Potential Energy, PE.*

For example, the energy of compressed air, of a wound clock spring, or of a raised pile driver is *potential energy.*

If a body of mass, m, is raised a vertical height, h, this body in its raised position possesses the ability to do work, *i.e. energy;* and the work it can do in returning to the place from which it was raised equals the work done upon it in raising it to that position. *Work* equals FS. To raise a body of mass, m, requires a force equal to its weight, mg. The distance, S, it is *raised* is denoted by the height, h. Therefore, the potential energy of a raised body is expressed by the equation, —

$$PE = mgh. \tag{12}$$

136. Unit of Energy. — The *unit of energy* is the unit of work. If m is expressed in grams, g in cm. per sec. per sec., h in cm., the quantity of energy is expressed in *ergs.*

137. Kinetic Energy. — The energy of a pile driver is *potential* before it is dropped, but during its fall all of its energy is being transformed into another kind, called *Kinetic Energy, KE*. *Kinetic energy* is the energy a body has because it is in motion, such as the energy of wind, of running water, of a striking hammer, or of a moving train.

The Kinetic energy which a moving body has equals the work done upon it to give it its motion.

Work $= FS$, but $F = ma$ and, by equation (4), $S = \frac{v^2}{2a}$; therefore $FS = ma\left(\frac{v^2}{2a}\right) = \frac{mv^2}{2}$.

The kinetic energy of a body of mass m moving with a velocity v is therefore expressed by the formula, —

$$KE = \tfrac{1}{2} mv^2. \qquad (13)$$

138. True Meaning of Mass. — In § 8 the *mass* of a body was provisionally defined as its quantity of matter. In § 119 attention was called to the conception of the *mass* of a body as that which determined the magnitude of the *effect of force* exerted by or upon the body. It is now possible to define the *mass* of a body as its *carrier of energy*, since the ability of a body to do work depends upon nothing appertaining directly to it other than its *mass*. The other factors involved in energy, g, h, or v^2, are conditions not inherent in the body. Energy is a possession of matter, and, these other conditions being equal, the greater the *mass* of a body, the greater is its *energy*.

139. Energy Changes in Doing Work. — *To do work* requires either the *transference of kinetic energy* of the body doing the work to the body upon which work is done, or the *transformation of kinetic energy* in the body

doing the work *into potential energy* in the body upon which work is done.

140. Power. — The quantity of work a motor does is independent of the time it takes to do the work. If one motor does more work *in unit time* than another, the first motor is said to have the greater *power.*

Power is the **rate** *of doing work*, or the quantity of work done per unit time.

141. Units of Power. — The C. G. S. unit of power is 1 *erg per sec.*, but this unit being too small for practical purposes, 1 *joule per sec.*, called a *watt*, is used.

$$1 \textit{ watt} = 1 \textit{ joule per sec.} = 10^7 \textit{ ergs per sec.}$$

The F. P. S. unit of power is 1 *foot poundal per sec.;* but for practical purposes a unit equal to 550 foot pounds per sec., called 1 *horse power*, is used.

$$1 \textit{ horse power} = 550 \text{ ft. lb. per sec.} = 33{,}000 \text{ ft. lb. per min.}$$

$$1 \textit{ horse power} = \frac{550 \times 30.48 \times 453.59 \times 980}{10{,}000{,}000} = 746 \textit{ watts.}$$

$$(1 \text{ ft.} = 30.48 \text{ cm., and } 1 \text{ lb.} = 453.59 \text{ gm.})$$

142. Problem in calculating the Power of a Motor. — To calculate the power of a steam engine: Suppose the piston is 50 sq. in. in area and that the *average* steam pressure during each stroke is 50 lb. per sq. in. Let the length of the stroke be 18 in. and the revolutions per minute of the fly wheel, 150.

Since the piston moves to and fro for each revolution of the fly wheel, the distance the piston ($50 \times 50 = 2500$ lb.) moves per revolution is 36 in., or 3 ft. The work done per revolution $= 2500 \times 3 = 7500$ ft. lb. The work done per minute $= 7500 \times 150 = 1{,}125{,}000$ ft. lb. The work done per second $= \frac{1{,}125{,}000}{60} = 18{,}750$ ft. lb. The power of the engine $= \frac{18{,}750}{550} = 34.05$ h.p. Expressed in watts, the power $= 34.05 \times 746 = 25{,}401.3$ watts $= 25.4$ kilowatts.

vl PROBLEMS

1. A force of 5000 dynes acts for 10 sec. upon a mass of 250 gm. which is free to move and starts from rest.

(*a*) What momentum is imparted to the body?

(*b*) What is its acceleration?

(*c*) What is its momentum after the force has been acting 3 sec.?

(*d*) How far will it move in the 10 sec.?

(*e*) If after the 10 sec. the force ceases to act upon the body and no other force acts upon it, with what velocity will it move and for how long a time?

Ans. (*a*) 50,000. (*b*) 20 cm. per sec. per sec. (*c*) 15,000. (*d*) 1000 cm. (*e*) 200 cm. per sec.; forever.

2. What force is required to bring a mass of 2000 T. moving with a velocity of 30 mi. per hr. (44 ft. per sec.) to rest in 22 sec.? *Ans.* 8,000,000 poundals (250,000 lb.).

3. A bullet is fired horizontally from the top of the mast of a ship 60 ft. above the sea with a velocity of 800 ft. per sec. The ship is moving with a velocity of 22 ft. per sec. How soon will the bullet strike the water? *Ans.* 1.93 sec.

4. A baseball whose mass is 250 gm., after falling from a height of 25 m. is caught by a boy, who in the act of bringing the ball to rest moves his hands with the ball a distance of 80 cm. downward. What is the average value of the force used? *Ans.* 7,656,250 dynes, or a force equal to the weight of 7.8125 kgm.

5. A boat whose mass is 200 T. is moving with a velocity of 3 ft. a sec.; what force will bring the boat to rest in 10 sec.? (Neglect friction of water.) *Ans.* A force equal to the weight of 3850 lb. (120,000 poundals).

6. A constant force of 500 lb. acts on a mass of 20 lb. (*a*) What velocity will be produced in 3 sec.? (*b*) How many foot pounds of work is done while the force moves the body 80 ft.? *Ans.* (*a*) 2400 ft. per sec. (*b*) 40,000 ft. lb.

7. An elastic body whose mass is 200 gm. while moving with a velocity of 25 cm. per sec. collides with a similar body at rest whose mass is 300 gms. After the collision the 200 gm. mass moves in the opposite direction with a velocity of 3 5 cm. per sec. (*a*) What is

the velocity of the 300 gm. mass after collision and in what direction? (*b*) What momentum did the first ball impart to the second? (*c*) What momentum did the second ball impart to the first?

Ans. (*a*) 19 cm. per sec. in the direction in which the 200 gm. mass was moving before collision. (*b*) + 5700. (*c*) − 5700.

8. The weight of a certain mass is 84 gm.; what is its weight, expressed in dynes? *Ans.* 82,320 dynes.

9. An elastic body moving northwest with a velocity of 20 ft. per sec. strikes an elastic surface extending due north and south; what is the velocity of the body after impact and in what direction? *Ans.* 20 ft. per sec. northeast.

10. How many joules of work is done in raising a mass of 5 kgm. a vertical height of 200 cm.? *Ans.* 98 joules.

11. How much work is done in pulling a sled whose mass is 12 lb. up a hill 200 ft. long having a grade of 30 degrees? Neglect friction. *Ans.* 1200 ft. lb.

12. A rectangular body, 20 cm. square at each end and 12 cm. long, whose mass is 40 kgm. rests on a side whose dimensions are 20 × 12 cm. If the density of the body is uniform, (*a*) how much work must be done to overturn the body about the 12 cm. edge? (*b*) about the 20 cm. edge? *Ans.* (*a*) 16.23 joules. (*b*) 6.5 joules.

13. A clock weight of 4 kgm. when wound up is 80 cm. higher than when completely run down. (*a*) How much work is the weight able to do when it is wound up? (*b*) If the clock runs for eight days, what is the power of the weight as a motor? *Ans.* (*a*) 31.36 joules. (*b*) .0000454 watt.

14. A shot traveling at the rate of 800 ft. per sec. is just able to pierce a 2-in. board; with what velocity must the shot move to pierce a 3-in. board? *Ans.* 980 ft. per sec.

15. If 2400 lb. of coal are burned in moving a train from New York to Philadelphia in 3 hr., how much coal must be burned to make the same trip in 2 hr., assuming that the energy of the coal is equally effective in producing motion? *Ans.* 5400 lb.

16. A motor raises an elevator weighing 2500 kgm. through a height of 50 m. in 15 sec. (*a*) How much work, expressed in joules, does the motor do? (*b*) What power, expressed in kilowatts, does the motor exhibit? (*c*) What is the average value of the force on the elevator? *Ans.* (*a*) 1,225,000 joules. (*b*) 75 k.w. (*c*) 2556.632 kgm.

17. (*a*) What is the power of a motor that can raise an elevator weighing 1500 kgm. through a height of 20 m. in 10 sec.? (*b*) If an additional load of 700 kgm. is put into the elevator, how long will it take this motor to raise the elevator 20 m.? *Ans.* (*a*) 29.4 k.w. (*b*) $14\frac{2}{3}$ sec.

CHAPTER VII

MACHINES

143. Mechanical Advantage. — A *machine* is a contrivance by means of which an effort can overcome a resistance to a greater advantage than if applied directly. *The advantage of a machine cannot be a gain in work*, because by the law of conservation of energy, energy is never created or destroyed. The energy put into the machine by the exertion of force upon it can never be less than the energy transferred to the resistance through the medium of the machine.

The advantage of a machine, called *mechanical advantage*, may be of two kinds: a gain in *force* or a gain in *speed*. There is a gain in *force* when the resistance R is greater than the effort F, and the value of this gain is the ratio $R : F$.

There is a gain in speed when the speed, Sp', of the resistance is greater than the speed, Sp, of the effort, and the value of this gain in speed equals the ratio $Sp' : Sp$.

144. The Efficiency of a Machine. — Since in every actual machine the parts have weight, and there is friction where the surfaces of the various parts rub over each other, a certain amount of work must be done by the effort to move the parts of the machine, even when the machine is applied to no external resistance. Hence the work, FS, done by

the effort is always greater than the work, RS', done upon the external resistance.

A machine becomes more *efficient* to the extent to which this element of wasted energy is eliminated. The *mechanical efficiency* of a machine is the *ratio* of its "output" of energy to its "intake" of energy, or

$$\textit{Mechanical efficiency} = RS' : FS.$$

For example, if with a given arrangement of pulleys, an effort of 250 lb., moving 40 ft., can raise a weight of 900 lb. a height of 10 ft., the work done by the effort (the "intake") = 250 × 40 = 10,000 ft. lb. The work done upon the resistance (the "output") = 900 × 10 = 9000 ft. lb. The efficiency of this machine is, therefore, 9000 : 10,000 = .90, or 90%. Again, if with a certain lever, an effort of 25 lb., moving ½ ft., raises a weight of 148 lb. a height of 1 in., the work done by the effort (the "intake") = $25 \times \frac{1}{2} = 12.5$ ft. lb.; the work done upon the resistance (the "output") = $148 \times \frac{1}{12} = 12.33$ ft. lb. The efficiency of this lever is, therefore, 12.33 : 12.5 = .9864, or 98.64%.

145. Three Principal Machines. — There are three fundamental machines: (1) lever, (2) pulley, (3) inclined plane. There are also three modifications of these: (*a*) wheel and axle, (*b*) screw, (*c*) wedge.

146. The Lever. — The *lever* has been previously considered as a problem in statics, in the discussion of the equilibrium of three parallel forces. It is now to be considered in motion, or as a problem in *kinetics*.

In the following discussions it is assumed that the *velocities* of the effort and of the resistance are *uniform*, so that after the machine has started the effort and resistance are in equilibrium (see First Law of Motion) and have therefore the same relation to each other as when the lever is at rest. It is also assumed that the parts of the machine are weightless and the bearings frictionless.

In the case of the lever, shown in Fig. 58, when F and R move, the paths, S and S', of their points of application are arcs of circles about f as center with a and b respectively as radii. The work done by $F = FS$, and that done upon $R = RS'$. Since the angles, 1 and 2, at the fulcrum are equal, being vertical angles, the lengths of the arcs, S and S', are proportional to the radii, a and b, or $S : S' = a : b$; but when the lever *is at rest*, $a : b = R : F$; $\therefore$ (since the velocities are uniform and R and F are in equilibrium) $S : S' = R : F$ or $FS = RS'$.

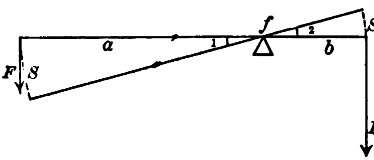


FIG. 58.

That is, if there is no work expended in the machine in moving its parts and in overcoming friction, the work done by the effort, FS, equals the work done upon the resistance, RS'.

The efficiency in this case, $RS' : FS = 1$, or 100%.

147. The Wheel and Axle. — The wheel and axle is a modified form of lever consisting in its simplest form of two concentric disks (Fig. 59) of radii, a and b, supported on a fulcrum, f, the axis at the center. To a rope wound about the larger disk, the effort F is applied. The resistance is applied to a rope wound in the opposite direction about the smaller disk.

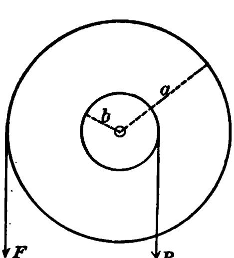


FIG. 59 *a*.

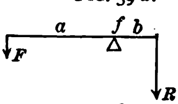


FIG. 59 *b*.

When the machine is turned through one revolution, the effort F moves a distance $2 \pi a$, equal to the

circumference of the larger disk; the resistance moves a distance, $2\pi b$, equal to the circumference of the inner disk. For each revolution the work done by the effort is $F \times 2\pi a$, and the work done upon the resistance is $R \times 2\pi b$.

But, considered as a lever (Fig. 59 *b*), $F:R=b:a$. Multiplying both terms of the second ratio by 2π will not change the value of the ratio; therefore, $F:R=2\pi b:2\pi a$ or $F\times 2\pi a=R\times 2\pi b$. Consequently, in this case also, if the machine is frictionless, the efficiency is 100%.

If a revolving arm is substituted for the larger disk, the machine is known as a windlass. A capstan used for hoisting anchors on ships and a hand coffee mill are familiar examples of the windlass.

148. The Pulley. — A pulley consists of one or more grooved disks, called *sheaves*, which can revolve within a supporting frame, called the *block*.

If while the sheave revolves, the block has a translatory motion, it is called a *movable* pulley. If the block remains stationary while the sheave revolves, it is a *fixed* pulley.

149. Single Fixed Pulley. — With a single fixed pulley (Fig. 60) the effort F equals the resistance R because, considered as a case of three parallel forces, the supporting force at the center of the sheave is midway between the two forces F and R; or, considering it as a lever, the arms r_1 and r_2 of F and R being equal, $F=R$. The distance S the effort moves down equals the distance S' the resistance is raised, the entire length of the rope being unchanged. Therefore $FS=RS'$, or the efficiency is 100% if the pulley is frictionless. There is no *mechanical advantage* in a single fixed pulley since $R:F=1$, and $S':S=1$. Its *use* is to *change the direction* in which a force must be applied to move a given resistance; for

example one can raise a body by pulling down or by pulling horizontally.

150. Single Movable Pulley. — This pulley shown in Fig. 61 is also a case of three parallel forces where the resistance R is the single opposing force midway between

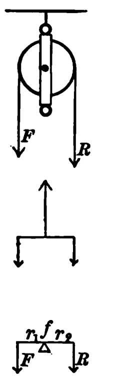


FIG. 60.

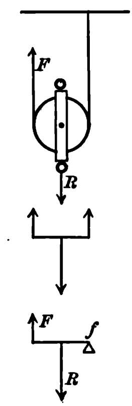


FIG. 61.

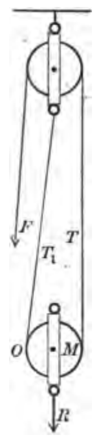


FIG. 62.

the effort F and the other supporting portion of the string whose tension is also F. Hence, $2F = R$, or $F = \frac{R}{2}$. The same result is obtained if a single movable pulley is considered as a second-class lever with the fulcrum at one end and the arm of R equal to $\frac{1}{2}$ the arm of F; hence $F = \frac{1}{2}R$. The mechanical advantage of the single movable pulley $R : F = 2$.

Figure 62 shows a combination of a single fixed and single movable pulley, the function of the fixed pulley being to change the direction of the effort from upward, as in

Fig. 61, to downward. By the single fixed pulley $F = T$; but $T = \frac{R}{2}$; hence $F = \frac{R}{2}$. The distance S' which the resistance R moves up is $\frac{1}{2}$ of the distance S which the effort F moves down, because each of the parts, T and T_1, of the cord must be shortened a length equal to S', and this total length of cord $2\ S'$ passes over the sheave of the fixed pulley allowing F to move that distance. Since $F = \frac{R}{2}$ and $S = 2\ S'$, $FS = \frac{1}{2}\ R \cdot 2\ S' = RS'$, or the efficiency $RS' : FS = 100\%$.

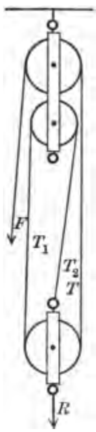


FIG. 63.

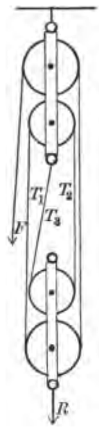


FIG. 64.

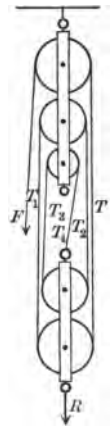


FIG. 65.

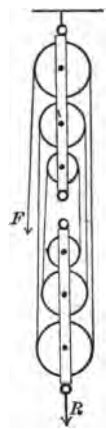


FIG. 66.

151. Double Fixed and Single Movable Pulley. — This arrangement is shown in Fig. 63, and since there are three portions, T, T_1, T_2, of cord supporting the movable block and since the tension of a cord is the same throughout its length, $R =$ tension $T +$ tension $T_1 +$ tension T_2. But $F =$ tension T, therefore $R = 3\ F$ or $F = \frac{R}{3}$. If the movable

block, instead of being assumed weightless, has a weight w, then $F = \frac{R + w}{3}$.

152. Double Fixed, Double Movable Pulley. — In this arrangement of pulleys (Fig. 64) it follows by the same reasoning as above that $R = T + T_1 + T_2 + T_3 = 4F$ or $F = \frac{R}{4}$. Assuming the weight of the movable pulley to be w, $F = \frac{R + w}{4}$.

153. Triple Fixed, Double Movable Pulley. — This arrangement shown in Fig. 65 gives a mechanical advantage, $R : F$, of 5, since there are five portions of cord supporting R, each under the same tension F, or $F = \frac{R + w}{5}$.

154. Triple Fixed, Triple Movable Pulley. — The mechanical advantage of this arrangement (Fig. 66) is a gain of force of 6, since R is supported by six portions of cord, each under the same tension F, or $F = \frac{R + w}{6}$.

155. General Formula for Any Number of Pulleys with a Continuous Cord. — From the examples given, it is evident that if, in general, n = the number of portions of the cord that *support the movable block*, $F = \frac{R}{n}$.

It should also be noted that the number of sheaves, counting those of the fixed block with those of the movable block, $= n$, and if n is an odd number, as 1, 3, 5, 7, the fixed block has one more sheave than the movable block and the fixed end of the cord must be attached to the movable

block; while if *n* is *even*, the fixed end of the cord must be attached to the fixed block.

156. The Inclined Plane. — If a body O (Fig. 67) is drawn up a frictionless inclined plane with uniform velocity, there is equilibrium among the three forces acting: (1) the weight, W, of the body O acting vertically downward, (2) the effort, F, pulling up the plane and parallel to the incline, (3) the reaction, P, of the plane equal to the pressure of the body upon the plane and acting normal to the plane.

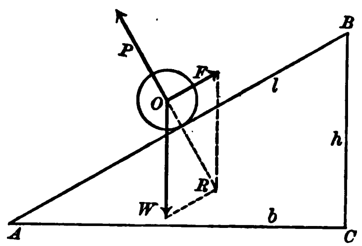


FIG. 67.

The resultant of any two of these forces, as F and W, is the diagonal of the parallelogram constructed upon these two forces as adjacent sides, and this diagonal, OR, is equal and opposite to force P.

Let l, b, and h denote the length AB, base AC, and height BC respectively of the incline. The $\triangle$ ABC and OWR are similar since

$$\measuredangle ORW = \measuredangle ACB \text{ and } \measuredangle WOR = \measuredangle BAC.$$

$$\therefore OW : WR = AB : BC.$$

But $$OW = W,\ WR = F,\ AB = l, \text{ and } BC = h.$$

$$\therefore W : F = l : h,$$

or the weight of a body moved up an incline is as many times the force required to move it as the length of the incline is times the height.

The pressure P of the body on the incline being equal to RO which is homologous to AC in the other triangle, it follows that $F : P = h : b$.

The mechanical advantage of an inclined plane $\frac{W}{F} = \frac{l}{h}$ or the gain of force equals the ratio of the length of the incline to its height.

The work done by the effort, F, in pulling the body up the entire length of the incline $= Fl$. The work done upon the weight, W, in raising it through the vertical distance, h, $= Wh$. By the above proportion $Fl = Wh$; therefore the efficiency of this machine is 100% provided that the plane is frictionless.

157. Friction. — Friction is the force acting between two bodies in contact, opposing the motion of the surface of one body over the surface of the other; the direction of this opposing force is tangent to the surfaces in contact.

One method of determining the quantity of friction between two surfaces makes use of the principle of the inclined plane.

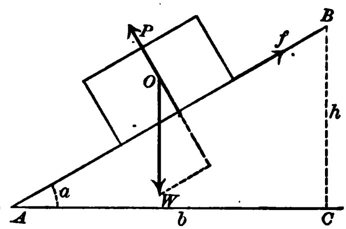


FIG. 68.

If a body O (Fig. 68) is placed upon the surface AB, and its inclination a is gradually increased until the body O just slides slowly down the plane with uniform velocity, the force of friction, f, the weight of the body, W, and the reaction, P, of the plane are in equilibrium.

The *coefficient of friction* for any two surfaces is the ratio of the force of friction f to the pressure P between the two surfaces, or the coefficient of friction $= \frac{f}{P}$.

But, by the law of the inclined plane, $\frac{f}{P} = \frac{h}{b}$, therefore

the coefficient of friction is the ratio $\frac{h}{b}$ when the plane is inclined, as described above.

158. The Screw. — If about a right cylinder M (Fig. 69 *a*) is wound a right triangle *abc*, whose base, *ac*, equals the circumference of the cylinder, the hypotenuse *ab* represents the *thread* of a screw, while the height *bc* represents the distance between two successive threads, or the *pitch* of the screw.

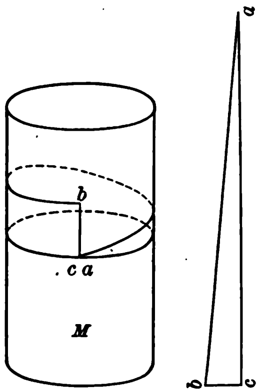


FIG. 69 *a*.

When a screw is turned through one revolution, its axis advances in a straight line a distance equal to the pitch of the screw. The effort F moves through a circular path, whose radius may be indicated by r. Then, by the law of the inclined plane, $F \times 2\pi r = R \times$ the pitch of the screw (the work done by the effort equals the work done upon the resistance).

The mechanical advantage of the screw is, therefore, $R : F = 2\pi r :$ pitch of screw.

For example, if the radius of the hand-wheel of a letter press (Fig. 69 *b*) is 8 in., and the screw has 6 threads to the inch, the mechanical advantage of the press equals $2 \times 3.1416 \times 8 : \frac{1}{6} = 301.6$; or a force of 1 lb. on the rim of the hand-wheel will produce a pressure of 301.6 lb. between the plates of the press, which are pushed together by the advancing motion of the screw.

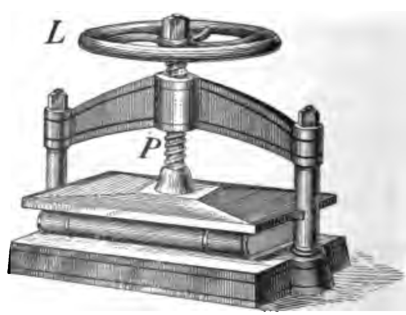


FIG. 69 *b*.

159. The Wedge. — If a wedge ABG (Fig. 70) is driven by a force F a distance equal to AK, the resistances R_1 and R_2, acting normally to each of the slant sides of the wedge, have each been moved a distance $DK = EK$; for the points D and E were originally together at point K.

Therefore, by the general law of machines, the work done by the force F, $F \times AK$, equals the work done upon the resistance,

$$R_1 \times DK + R_2 \times EK = 2R \times DK,$$

since $\quad R_1 = R_2 = R$ and $DK = EK$.

Therefore $\quad F : R = 2DK : AK.$

The $\triangle$ ADK and ACB are similar, therefore $DK : AK = BC : AB$. Hence, in a wedge, the mechanical advantage

$$\frac{R}{F} = \frac{AB}{2BC} = \frac{\text{length of wedge}}{\text{width of wedge}}.$$

In any actual case of the wedge, however, the frictional resistance to be overcome is so great that the above rate only approximately represents the advantage.

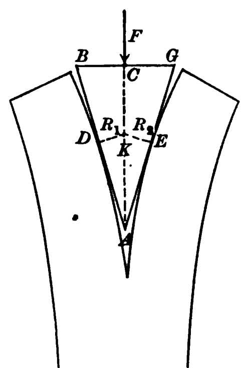


FIG. 70.

160. General Law of Machines. — From the consideration of these various machines it may be seen that, if the energy wasted in overcoming the friction in the machine is disregarded, there is a law common to all of them, viz. *the work done by the effort*, FS, *equals the work done upon the resistance*, RS'.

Also, since the forces, F and R, are inversely proportional to the distances, S and S', through which they move,

whenever the mechanical advantage $\frac{R}{F}$ is a *gain in force*, there is a *loss in speed*, S' being proportionally less than S.

161. Speed Gearing. — Unless a motor is directly connected with a machine, power is usually transmitted either by belt gearing or cogwheel gearing.

If the pulley A (Fig. 71) of a motor is belted with the pulley B of a given machine, for every revolution of the motor pulley there are as many revolutions of the other pulley as the circumference of pulley B is contained in the circumference of pulley A, or as the diameter of A is times the diameter of B.

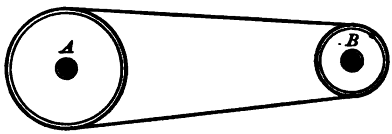


FIG. 71.

The diameter of a bicycle wheel is, as a rule, 28 in. If for one revolution of the pedals the bicycle moves forward a distance equal to the circumference of a wheel whose diameter is 80 in., the gear is 80. To produce this speed, the number of teeth in the driving sprocket, A : number of teeth in the rear sprocket, $B = 80 : 28$, or $= 20 : 7$. If this relation exists, for every revolution of the pedals, there are $\frac{20}{7} = 2\frac{6}{7}$ revolutions of the rear wheel.

To *increase* speed in a machine, the *driving pulley* must be *larger* than the driven; while to *reduce* speed the *driving pulley* must be the *smaller*.

PROBLEMS

1. What is the mechanical advantage of a lever whose resistance arm is 20 cm. long and effort arm 80 cm. long? *Ans.* Gain in force of 4.

2. By means of the lever in (1) an effort of 50,000 dynes moving 20 cm. lifts a weight of 175 gm. a height of 5 cm. What is the efficiency of the lever? *Ans.* 85.75%.

3. A uniform board 16 ft. long is balanced on a log. When a

girl weighing 110 lb. sits on one end of the board, where must a boy weighing 140 lb. sit to balance the seesaw? *Ans.* $1\frac{5}{7}$ ft. from other end of board.

4. (*a*) What force is required with a system of pulleys, consisting of a triple fixed and double movable block, to lift a weight of 540 lb., the weight of the movable block being 10 lb.? (*b*) What work is done to raise the weight 5 ft.? *Ans.* (*a*) 110 lb. (*b*) 2750 ft. lb.

5. If the force required in (4) were 120 lb., what would be the efficiency of the given system of pulleys? *Ans.* 91.66%.

6. If a block weighing 5 lb. rests upon a board and will just slide down the board if it is inclined at an angle of 30°, what is the force of friction between the block and board? *Ans.* 2.5 lb.

7. (*a*) What is the inclination of a plane when a force of 49,000 dynes can move a mass of 70.7 gm. up the plane with uniform velocity, the force being parallel to the incline and the plane assumed to be frictionless? (*b*) If the length of the plane is 100 cm., how much work is done in moving the mass the entire length? *Ans.* (*a*) 45°. (*b*) .49 joule.

8. A motor pulley 5 in. in diameter is belted with a 12 in. pulley on a counter shaft. An 8 in. pulley on the same counter shaft is belted with a 4 in. pulley on the head stock of a lathe. If the motor makes 1500 revolutions per min., what is the speed of the lathe? *Ans.* 1250 revolutions per min.

9. (*a*) The arms of a lever of the first class are 7 and 2 ft. respectively. What effort applied at the extremity of the long arm will raise 336 kgm. applied at the end of the short arm through a height of 10 cm.? (*b*) Through what distance must the effort move? *Ans.* (*a*) 96 kgm. (*b*) 35 cm.

10. (*a*) In a hydraulic press, if the piston is attached to a second-class lever 6 in. from the fulcrum, find the pressure on the piston produced by a force of 10 lb. applied at the end of the lever 27 in. long. (*b*) What is the pressure on the fulcrum? *Ans.* (*a*) 45 lb. (*b*) 35 lb.

11. (*a*) In a wheel and axle, if the diameter of the wheel is 2 m. and the diameter of the axle 12 cm., what effort is required at the circumference of the wheel to raise a weight of 500 kgm. attached to a rope wound about the axle? (*b*) What work is done by the effort to raise the weight 10 m.? *Ans.* (*a*) 30 kgm. (*b*) 49,000 joules.

12. A weight is raised by means of a rope passing around a horizontal cylinder 20 cm. in diameter which is turned by means of a crank 60 cm. long. What weight can be raised by a man using an effort of 35 kgm. on the handle of the crank? *Ans.* 210 kgm.

13. (*a*) A weight of 300 kgm. is raised by a cord passing around a drum 50 cm. in diameter, having on its shaft a toothed wheel also 50 cm. in diameter. Geared with this toothed wheel is a pinion 10 cm. in diameter, which is turned by a crank 30 cm. long. What effort is required at the crank handle? (*b*) How many revolutions of the crank are needed to raise the weight 5 m.? *Ans.* (*a*) 50 kgm. (*b*) 15.91 revolutions.

14. A weight of 30 lb. is to be raised by a single movable pulley. What effort is required (*a*) when the strings are parallel? (*b*) when each string is inclined 30° to the vertical? *Ans.* (*a*) 15 lb. (*b*) 17.32 lb.

15. A weight of 400 lb. is being raised by a pair of pulley blocks each having 2 sheaves. The fixed end of the rope is attached to the upper block. If each block weighs 10 lb., what is the pressure on the point from which the upper block hangs? (Neglect the weight of the rope.) *Ans.* 522.5 lb.

16. A weight of 200 lb. is being raised by a pair of pulley blocks, each weighing 15 lb., the movable block having 1 sheave, while the fixed block has 2 sheaves. The fixed end of the rope is attached to the movable block. What is the pressure on the point of support of the fixed block? *Ans.* $301\frac{2}{3}$ lb.

17. A man wishes to raise a barrel weighing 100 kgm. to a height of 150 cm. by rolling it along a plank 5 m. long. What effort must he use? *Ans.* 30 kgm.

18. The pitch of a screw is 4 mm. and the diameter of the screw is 17.5 mm. What effort applied at the circumference of the screw will produce a pressure of 30 kgm.? *Ans.* 2.18 kgm.

19. What is the pitch of a screw by which an effort of 36 lb. acting at the end of an arm 28 in. long produces a pressure of 2 T.? *Ans.* 1.584 in.

20. In a combination of pulleys the effort descends 96 ft. while the weight rises 3. What is the ratio of the effort to the weight? *Ans.* 1 : 32.

21. In a machine an effort of 35 lb. descends 17 ft. while raising a weight 6 in. Find the weight. *Ans.* 1190 lb.

22. What is the efficiency of a pulley by which an effort of 50 lb. moving 40 ft. can raise a weight of 270 lb. $6\frac{2}{3}$ ft.? *Ans.* 90%.

23. If the coefficient of friction between an iron weight and a certain board 40 in. long is $\frac{1}{4}$, at what angle can the board be inclined so that the iron weight will just slide slowly down the board? *Ans.* 14°, or one end of the board raised 9.7 in.

24. If a resistance of 250 lb. is overcome by an effort of 50 lb. and the resistance moves 10 ft., (*a*) how much work is done by the effort? (*b*) How much work is done upon the resistance? (*c*) If the work is done in 3 sec., what power was exhibited? *Ans.* (*a*) 2500 ft. lb. (*b*) 2500 ft. lb. (*c*) 1.515 h.p.

CHAPTER VIII

CURVILINEAR MOTION

162. The Value of Centripetal Force. — If two bodies, A and B (Fig. 72 *a*), of equal mass are joined by a cord and rotated in a horizontal plane about the vertical line YOY' as axis, and if the distances OA and OB are equal, the bodies will continue to rotate at the same distance from the axis; but if the distance OA is greater than OB, when the bodies are being rotated, the body A will gradually move farther from O, pulling body B toward O.

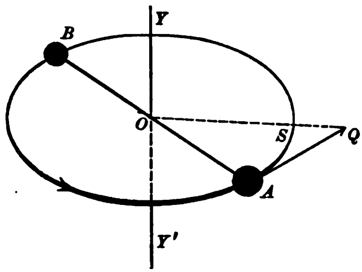


FIG. 72 *a*.

From the first law of motion it follows that if A is set in motion in the direction AQ, tangent to the circle at A, it *tends* to continue to move in that straight line, and to *compel* it to move in the circular path AS a force in the direction toward O must be continually acting upon it.

The tendency of A to move away from the center O, *i.e.* to be at Q instead of at S, is called its *centrifugal tendency*, and the force toward the center O compelling it to move in the circular path is called the *centripetal force*.

In the first case the *centripetal force* acting on A equals the *centrifugal tendency* of B and the *centripetal force* acting on B equals the *centrifugal tendency* of A.

In the second case, the centrifugal tendency of A is greater than the centrifugal tendency of B which is the centripetal force on A, hence this force is insufficient to balance the centrifugal tendency of A which increases as its distance from the axis increases.

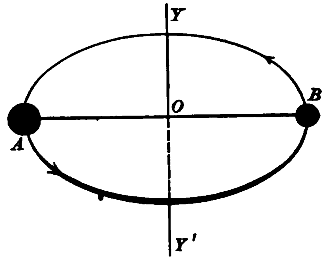


FIG. 72 *b*.

If two bodies, A and B (Fig. 72 *b*), of unequal mass, m_1 and m_2, are joined by a cord and rotated at equal distances from the axis YOY', the body A, of larger mass, will have the greater centrifugal tendency and will gradually move farther from O, pulling B toward O, showing that the centripetal force required to keep a body in a given circular path increases with an increase of mass.

The relation between the mass of a rotating body, its radius of rotation, and its centrifugal tendency is demonstrated by rotating the bodies, A and B, of mass m_1 and m_2, joined by a cord at such distances, r_1 and r_2, from the axis that $m_1 : m_2 = r_2 : r_1$; in which case the centrifugal tendencies balance (Fig. 73).

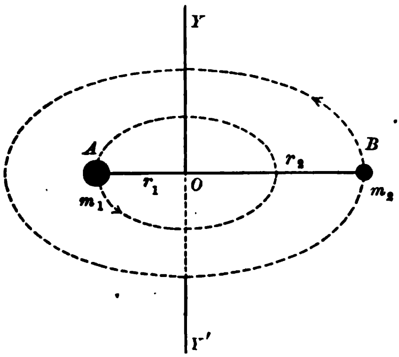


FIG. 73.

Since in the above proportion $m_1 r_1 = m_2 r_2$ and since the tension of the connecting cord which is the centripetal force for both masses keeps each in a fixed circular path, it

follows that the centripetal force is proportional to the product mr.

163. Illustrations of Centrifugal Tendency. — That centrifugal tendency is proportional to the rotating mass is usefully applied in the separation of liquids of different densities, *e.g.* cream from milk, by rotating them at high speed in a vessel with several concentric communicating compartments; the denser liquid moves into the compartment farthest from the axis, while the less dense liquid may be drawn off from the innermost compartment.

The principle of increase of centrifugal tendency with increase of distance from the axis of rotation explains the spheroidal shape of the earth. The equatorial portion of the earth's mass is, on the average, farthest from the axis of rotation and hence has the greatest centrifugal tendency. The same principle is used in potteries in the shaping of the plastic clay. The outer rail at a curve in a railroad track is raised in order to have a component of the weight of the train which will counterbalance its centrifugal tendency. The speed governor of some stationary steam engines utilizes the centrifugal tendency of the bobs of a conical pendulum to regulate the supply of steam.

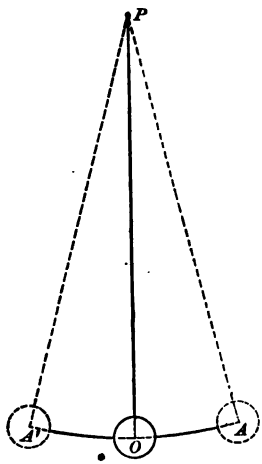


FIG. 74.

164. Pendulum. — A pendulum for ordinary experimenting (Fig. 74) consists of a heavy body, called the *bob*, suspended by a cord, the weight of which is negligible, from a fixed point P about which it can swing freely.

One half of the arc, AA', through which the pendulum swings, or OA, is its *amplitude* of vibration. By a vibration of a pendulum is meant a swing to *or* fro from one extreme of its path to the other. The *time of vibration* of a pendulum, or its *period* (t), is the time during which it makes one vibration. The *length of such a pendulum* for all practical purposes may be taken as the distance from the *point of suspension* P, to the center of gravity O of the bob.

165. Laws of the Pendulum. — By experimenting with such a pendulum the following laws may be demonstrated: —

1. The time of vibration is not affected by a change in the mass of the bob.
2. The time of vibration is not affected by a change in the amplitude of vibration, provided the amplitude is small.
3. The time of vibration is directly proportional to the square root of the length of the pendulum; $t \propto \sqrt{l}$
4. The time of vibration is inversely proportional to the square root of the acceleration due to gravity; $t \propto \frac{1}{\sqrt{g}}$.

These laws may be expressed in the equational form, —

$$t = \pi\sqrt{\frac{l}{g}}.$$

A pendulum which makes one vibration per second is called a *seconds pendulum*.

The length of a seconds pendulum in any place may be found by substituting for g in the above formula its value in that place. For example in New York $g = 980.26$ cm., therefore: —

$$1 \text{ sec.} = 3.1416\sqrt{\frac{l}{980.26}}.$$

$$l = \frac{980.26}{(3.1416)^2} = 99\ 32 \text{ cm.}$$

166. Use of the Pendulum in Clocks. — The value of the pendulum as a regulator of clocks lies in the fact that if its length is unchanged, it makes each vibration in exactly the same time. Its vibrations are therefore said to be *isochronous*, *i.e.* in equal times.

PROBLEMS

1. The length of a seconds pendulum in New York is 99.32 cm.; what is the time of vibration of a pendulum 50 cm. long in the same place? *Ans.* .71 sec.

2. A clock pendulum in New York is 24.83 cm. long, how many vibrations will it make in an hour? *Ans.* 7200.

3. Two pendulums, 81 cm. and 64 cm. long respectively, are set vibrating together; how soon afterward will they be vibrating together again? *Ans.* 14.45 sec.

4. What is the length of a seconds pendulum where the gravity acceleration is 978 cm. per sec. per sec.? *Ans.* 99.1 cm.

5. If a pendulum 1 m. long makes 60 vibrations per min., how many vibrations will a 2 m. pendulum make in the same time? *Ans.* 42.6.

6. If a pendulum which beats seconds at the equator gains 15 sec. in 2 hr. when carried to the north pole, compare the force of gravity at the two places. *Ans.* 1 : 1.00417.

7. A pendulum 1 m. long vibrates seconds; find the time of vibration of (*a*) $\frac{1}{2}$ m. pendulum; (*b*) $\frac{1}{3}$ m. pendulum; (*c*) $\frac{1}{4}$ m. pendulum; (*d*) $\frac{1}{5}$ m. pendulum.

Ans. (*a*) .707 sec. (*c*) .500 sec.
(*b*) .574 sec. (*d*) .442 sec.

CHAPTER IX

SOUND

167. Definition of Sound. — Sound is that kind of *wave motion* which is capable of causing the *sensation of hearing.* The term is used strictly in the objective sense. *To hear sound* is to become conscious, through the sensation of hearing, of certain external physical changes, the study of the nature of which constitutes the subject of *Sound* in Physics.

168. Characteristics of Sound Waves. — This particular kind of wave motion consists of the formation *in matter* of successive layers of *slightly greater* and of *slightly less than normal density* which follow each other rapidly. For convenience a layer of increased density is called a *compression*, and one of decreased density is called a *rarefaction.* A *wave* consists of the combination of one compression and one rarefaction. A *wave length*, in any given case, is equal to the combined thickness of a compression and a rarefaction; or, since these are of equal thickness, to double the thickness of a compression or of a rarefaction. The length of these waves ranges from a few centimeters to several meters according to circumstances. While these waves may be formed in all kinds of matter, the great majority and most important of them are formed *in the air.* The important characteristic of a sound wave is that the *vibration* is *longitudinal*, *i.e.* the particles of matter vibrate to and fro in the direction in which the wave advances.

169. Action of a Sounding Body. — In order to form these waves, *i.e.* produce *sound*, a body must impart energy to the surrounding contiguous matter which then becomes the *medium* in which the energy is propagated as sound. A body may thus impart energy either by *vibrating rapidly* in the medium, *e.g.* a tuning fork, violin string, bell, whistle, etc.; or by making a single, very sudden push or impact against the medium, *e.g.* the crack of a whip, snap of an electric spark, report caused by a firecracker, gun or cannon. Bodies while acting in this way are called *sounding bodies*. By the first method a long series of successive sound waves is formed which, if heard, causes a sustained sensation lasting as long as the sounding body vibrates; while by the second method there is likely to be but one well-defined wave which would cause a single momentary sensation.

The vibrations of a sounding tuning fork, violin string, or bell are visible or may be made so by holding a pith-ball pendulum against the body. The vibrations of the air column of a sounding organ pipe may be seen by lowering into the pipe a membrane upon which some sand has been sprinkled. Sound waves formed in air by the snapping of an electric spark may be photographed with suitable arrangements. Figure 75 shows a number of these photographs which were made by Professor R. W. Wood of Johns Hopkins University. The dark spot at the centers represents two small brass knobs, slightly separated, one being behind and therefore hidden by the other. The black lines extending outward from these spots are the rods of an electrical machine producing the charge which passes between the knobs as a spark, producing a slight sound wave which may be heard as a sharp snap. Three successive stages of progress of one of these

waves are shown in *a*, *b*, and *c* (Fig. 75), while in *d* and *e* are shown two stages of such a wave after reflection from a plane surface, the horizontal black line being the edge of a plate of glass against which the wave has struck.

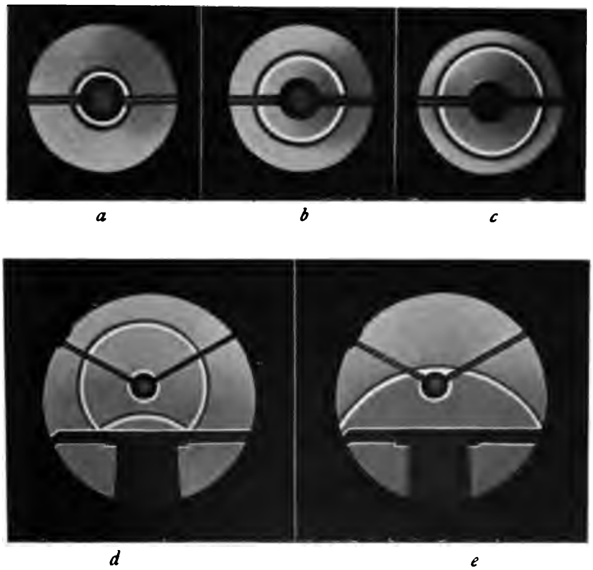


FIG. 75.

170. A Material Medium Essential for Propagation. — That a material medium is essential for the propagation of sound may be demonstrated by putting in action an electric bell supported by the least possible material connection within a bell jar from which the air is exhausted. If the exhaustion is very complete, nothing will be heard, showing that no sound is produced, because, although a vibrating body is ready to impart energy, there is no material medium in touch with it into which the energy may be received as sound.

171. Formation of Sound Waves. — To illustrate the character of the sound wave produced by a sounding body which vibrates many hundreds of times before coming to rest, suppose Fig. 76 to represent a tuning fork in action.

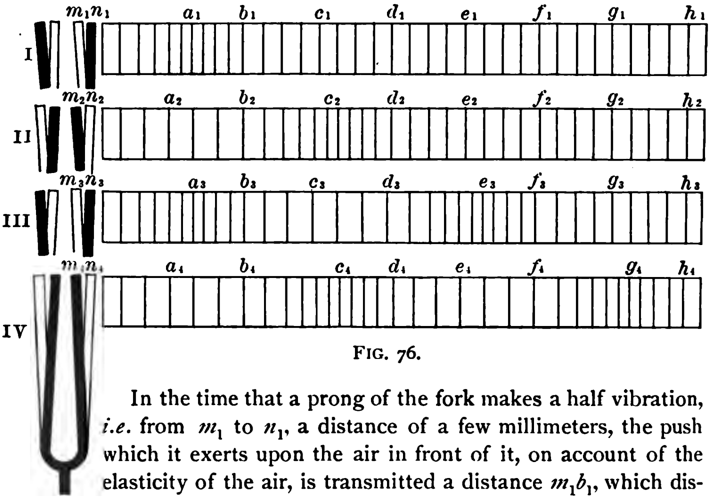


FIG. 76.

In the time that a prong of the fork makes a half vibration, *i.e.* from m_1 to n_1, a distance of a few millimeters, the push which it exerts upon the air in front of it, on account of the elasticity of the air, is transmitted a distance $m_1 b_1$, which distance would be as much as 65 cm. if the fork were making 256 vibrations per sec.

Within this distance the particles of the air are crowded together more closely than they are normally, hence the density of the air within this space is above normal and is called a *compression*. During this *first* half vibration of the fork this space $m_1 b_1$, as shown in the diagram, is the only portion to which the vibratory motion of the fork has been transmitted, the space beyond from b_1 to h_1 being as yet unaffected.

While the prong of the fork is moving from n_2 to m_2, thus completing its vibration, the particles of air in the portion of space $m_2 b_2$ become more separated than normal, as shown in II of the diagram, producing a *rarefaction*; while, by the transmission of the compression, the portion $b_2 d_2$ has the condition which the part $m_1 b_1$ had in the first half of the vibration.

By the end then of the first complete vibration of the fork from *m* to *n* and back again to *m*, the motion of the fork has been transmitted to

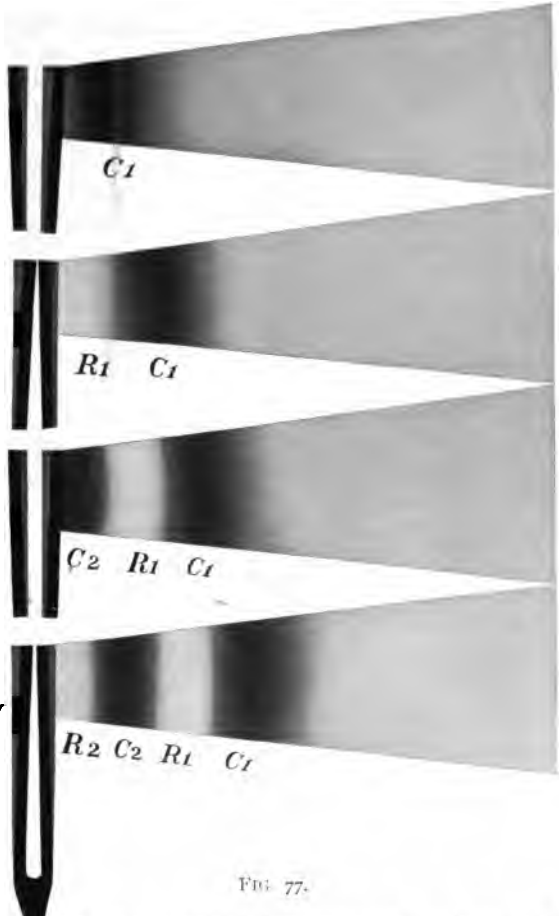


FIG. 77.

the air in which it has progressed as far as m_2d_2, in the first half of which space is formed a rarefaction m_2b_2, and in the second half a compression b_2d_2, constituting *one complete wave* which, it should be observed, has been formed during one *complete vibration* of the fork;

and further, m_2d_2 is the *length of one wave*, and is equal to the distance the disturbance is propagated during one complete vibration of the fork.

During a second vibration of the fork the disturbance will have progressed so far that by the end of the third half vibration it has the appearance of III in the diagram, and by the end of the second complete vibration it will have progressed a distance m_4h_4, in which space are two rarefactions and two compressions constituting two complete waves.

Figure 77 also represents the initial formation of sound waves by a tuning fork. The uniform gray indicates a normal state of the air; the dark zones, regions of compression; and the light zones, regions of rarefaction. In I, the fork is shown at the extreme position of the *first outward* spring, having produced one complete layer of *compression*, C_1 (a half wave). In II is shown the extreme position of the *first inward* spring with the production of one complete layer of *rarefaction*, R_1 (another half wave), the compression layer, C_1, having moved forward a distance equal to its own thickness. These two layers constitute one complete sound wave which, it will be observed, was formed by and during one complete vibration of the fork. In III and IV the *second* outward and inward spring of the fork is shown producing a new compression, C_2, and a new rarefaction, R_2, which follow in the wake of the first. This process continues as long as the fork vibrates.

172. Sound Wave from Single Impulse. — Consider next the character of a sound wave produced by a sudden momentary impact, as in the case of an exploding firecracker. The large quantity of gas produced by the rapid combustion of the powder in the cracker, by its sudden pressure or impact, compresses the air on all sides of it. There is thus formed a layer of compression $C_1C_2C_3C_4$, in the shape of a spherical shell, as shown in Fig. 78, about the center of disturbance F, which rapidly spreads farther and farther from this center. This layer of compression is succeeded by a layer of rarefaction $R_1R_2R_3R_4$, formed in the same way as with the tuning fork, which follows in the wake of the shell of compression.

This succession of slightly increased and decreased den-

sities of the medium caused by the particles of the medium *vibrating to and fro in the direction of propagation* constitutes a *sound wave.* The effect upon the tympanum, or

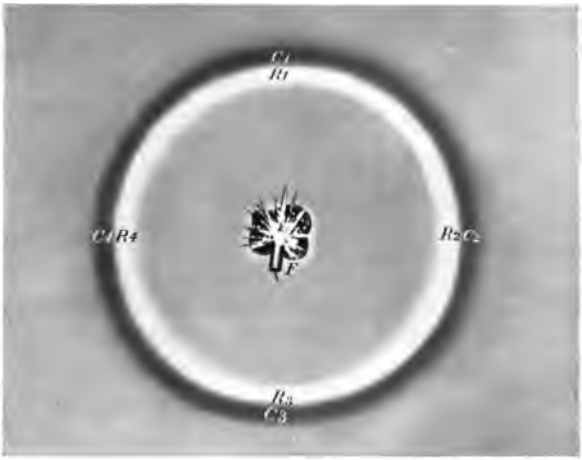


FIG. 78.

ear drum, of such a rapid, though slight, change in the density of the air in the external ear, when communicated by the auditory nerve to the brain, is the production of *the sensation of hearing.*

173. Direction of Propagation of a Sound Wave. — These waves exist not only in the direction of the vibration of the fork, as shown in the diagram, but, because an elastic fluid transmits a pressure equally in all directions, they exist to the same extent in all directions, being in the form of concentric spherical shells about the prong as a center, and having a thickness equal to *md,* Fig. 76.

The particles of the air themselves move but a small distance, while the wave moves relatively a large distance

One half of a to *or* fro movement of any particle is the *amplitude of vibration* just as in the case of the pendulum.

At the instant the prong has reached the point n_1, Fig. 76, the particle at b_1 has just begun to feel the transmitted pressure, and is about to begin its vibration, while the particle at a_1 is moving with the greatest velocity it ever has in the direction m_1b_1 making a_1 a point of maximum density. By the time the prong has returned to m_2, the particle at d_2 has just begun to feel the transmitted pressure, and the particle at c_2 has its maximum velocity in the direction m_2d_2 (toward the right), while the particle at a_2 has its maximum velocity in the direction d_2m_2 (toward the left). By the end of the first complete vibration of the fork there is produced at c_2 a condition of *maximum* density, and at a_2 a condition of *minimum* density.

174. Velocity of Sound in Terms of Wave Length and Frequency. — If the Greek letter λ (pronounced *lambda*) represents a wave length, or the distance the wave travels during *one* complete vibration of the sounding body, and if the latter makes n vibrations per second, the total distance the wave travels in one second, or the velocity of sound, v, = $n\lambda$. The number of vibrations per second represented by n is called the *frequency of vibration*, or simply the *frequency*.

Since $v = n\lambda$, and in a given medium v is constant, it follows that $n \propto \frac{1}{\lambda}$, *i.e.* the frequency varies inversely as the wave length.

It will be shown later that it is possible to determine experimentally both the value of n and of λ (in air); having done which, it is a simple matter to calculate the velocity of sound in air by this formula. (First Method.)

175. Velocity of Sound in Terms of Pressure and Density. — The velocity of sound in any gas may be determined from the formula $v = \sqrt{\frac{P}{\rho}}$, where P is the pressure of the

gas on unit area expressed in absolute units of force, and ρ is its density.

The pressure of air per unit area, under normal conditions, is 1033.6 gm. per sq. cm. = 1033.6 × 980 = 1,012,928 *dynes* per sq. cm.

When a gas is compressed suddenly, its temperature rises, and with an increase of temperature the pressure is increased more than if the compression was more slowly produced, allowing the heat of compression to escape by radiation, and the temperature, consequently, to remain constant during the compression.

It has been experimentally determined that when the compression is sudden the pressure is 1.41 times as great as when the temperature remains constant. Since the compressions in a sound wave are produced rapidly, the value of the pressure of the air in the wave is 1.41 × 1,012,928 dynes = 1,428,228.48 dynes per sq. cm. The density of air at 0° C. and 760 mm. pressure is .001293 gm. per cc.

Therefore the velocity of sound in air at 0° C.,

$$v = \sqrt{\frac{1428228.48}{.001293}} = \sqrt{1104585058} = 33235 \text{ cm.} = 332.35 \text{ meters}$$

per sec. (Second method.)

176. Velocity of Sound in Air in Terms of Distance and Time. (Third Method.) — By noting the interval of time, t, between seeing the flash of a gun fired at a distance, D, and hearing the report, the velocity of sound in air, $v = \frac{D}{t}$, has been determined to be 332.4 m. or 1090 ft. per sec., at a temperature of 0° C. The calculated value from the formula of the second method closely agrees with this experimental result.

177. Velocity of Sound in a Gas is Proportional to its Absolute Temperature. — When a gas is heated, it expands, and in consequence its density grows less. From the Law of Charles (see expansion of gases due to heat), it follows that the density of a gas is inversely proportional to its absolute temperature, or $\rho_1 : \rho_2 = T_2 : T_1$.

By the formula given in the second method for velocity of sound, the velocity is inversely proportional to the square root of the density, the value of P being considered unchanged, *i.e.* $V_1 : V_2 = \sqrt{\rho_2} : \sqrt{\rho_1}$. Taking the square root of each term of the first proportion and inverting each ratio, $\sqrt{\rho_2} : \sqrt{\rho_1} = \sqrt{T_1} : \sqrt{T_2}$. Substituting, $V_1 : V_2 = \sqrt{T_1} : \sqrt{T_2}$, *i.e.* the velocity of sound in a gas is directly proportional to the square root of its absolute temperature.

Let $V_0 =$ the velocity of sound in air at 0° C. (332 m. per sec.), and let $V_t =$ the velocity at a temperature t° C. The absolute temperature of 0° C. $= 273 + 0 = 273°$; the absolute temperature of t° C. $= 273 + t°$.

Substituting in the above direct proportion between the velocity of sound and the square root of the absolute temperature,

$$V_o : V_t = \sqrt{273°} : \sqrt{273 + t°}.$$

178. Temperature Coefficient for Velocity of Sound.—To find the velocity of sound in air at a temperature of 10° C. Substitute for V_0, 332 m. per sec., and for t° C., 10° C.,

$$332 : V_{10°} = \sqrt{273} : \sqrt{273 + 10}.$$

$$V_{10°} = \frac{332\sqrt{283}}{\sqrt{273}} = \frac{332(16.8226)}{16.5227} = 338.03 \text{ m. per sec.}$$

Similarly, for the velocity of sound in air at a temperature of 20° C.,

$$V_{20°} = \frac{332\sqrt{293}}{\sqrt{273}} = \frac{332(17.1172)}{16.5227} = 343.94 \text{ m. per sec.}$$

The *increase* in the velocity for a rise of 10° above 0° C. $= 338.03 - 332.00 = 6.03$ m. per sec., or .603 m. per sec. for an average increase per 1° rise in temperature. The increase in velocity for 20° rise above 0° $= 343.94 - 332 = 11.94$ m. per sec., or .597 m. per sec. for an average increase per 1° rise in temperature.

The mean of these two averages is .6 m. (= 2 ft.) per sec., which can be used as a very approximate value for the increase in the velocity of sound in air for each degree of rise in temperature above 0° C. It is called the *temperature coefficient* of the velocity of sound in air.

179. Velocity Independent of Atmospheric Pressure. — Since in any gas $v=\sqrt{\frac{P}{\rho}}$, if the values of P and ρ change the same relatively, the value of v is constant, for the value of the fraction $\frac{P}{\rho}$ is unchanged. Any change in the pressure of the atmosphere changes the elastic force of the air, which always equals the pressure upon it, and its density in the same ratio; therefore the velocity of sound in air is not affected by variations in the atmospheric pressure. At the same temperature the velocity of sound at the top of a mountain is the same as it is in the valley below.

180. Velocity of Sound in Liquids and Solids. — The velocity of sound in liquids is greater than in gases, the velocity in water at 4° C. being 1400 m. per sec., or about 4 times the velocity in air. The velocity of sound in most solids is still greater, the value of v in iron being 5093 m. per sec., or approximately 15 times the velocity in air.

181. Reflected Sound; Echo. — Sound is reflected from surfaces just as light is reflected. When a reflected sound is heard and is recognized as such, it is called an *echo*. If a person produces a sound, *e.g.* shouts, or claps his hands, or fires a pistol, and hears the echo, the distance, l, of the reflecting surface from him may be calculated by noting the time, t, between the production of the sound and the hearing of the echo; for in this case the sound has traveled the distance l in going to the reflecting surface, and the same distance again in returning, making a total distance, $2\,l$; then,

$$v=\frac{2\,l}{t}, \text{ or } l=\frac{vt}{2}.$$

In order that an echo may be distinctly heard, the distance l must be great enough to give a perceptible inter-

val between the time of hearing the original sound, which is practically instantaneous if the experimenter produces the sound, and the time of hearing the echo.

182. Tone and Noise. — Naturally many of the characteristics of sound are best studied through and by the aid of the sensation of hearing which it causes. Outside of the mind there is no such thing as *noise*, or *tone*, or *music* considered as a pleasing sequence and combination of tones; but certain external physical changes occur which give rise to these sensations, and these terms are therefore used in the subjective sense.

If the frequency of a sounding body (and consequently the frequency of the sound waves produced by it) is *uniform*, and not less than about 16 nor more than 40,000, the sensation produced is a *musical tone*. If, on the contrary, the disturbances are non-periodic or isolated, *e.g.* footsteps on a bare floor, or an explosion, the sensation is discontinuous or momentary, and is called a *noise*.

183. Characteristics of Tones. — There are three characteristics by which tones are distinguished from one another, viz.: **loudness**, **pitch**, and **quality** (*timbre*).

Loudness refers to the *intensity* of the sensation. Thus a tone is said to be faint or loud according to the degree to which the sensation is excited.

Pitch refers to the *graveness or acuteness* of the sensation. A grave tone is said to have a low pitch while an acute tone is one of high pitch.

Quality refers to the degree of *complexity* of the sensation and, generally, it is this characteristic of a tone which betrays the nature of the sounding body.

184. Characteristics Due to Physical Conditions. — *Loudness* depends upon the rate at which the kinetic energy

of sound is received into the ear; and this in turn depends upon two conditions: (1) the *rate* at which such energy is imparted to the medium by the sounding body, and (2) the *distance* of that body from the ear.

(1) If a certain quantity of kinetic energy, KE, is given to a sounding body whose mass, m, is fixed and whose mean velocity of vibration is v, its kinetic energy, $KE = \frac{1}{2}mv^2$. As v dies down to zero, the KE of the body becomes zero, *i.e.* disappears from the body and at the same time reappears in part as kinetic energy of sound in the surrounding medium. If, now, a greater quantity of energy is put into the body, the value of v is greater since m remains unchanged, and this means that the amplitude (not the frequency) of vibration of the sounding body is increased. This produces greater amplitude in the vibrating medium which in turn causes a more intense sensation. This may be illustrated by touching a key of a piano, first gently, then vigorously, and listening to the effect resulting from giving to the sounding body a greater quantity of kinetic energy.

Again, if the mass m of the sounding body is increased, *e.g.* when the stem of a vibrating tuning fork is held against a box which then, to some extent, vibrates with it, v will die down more rapidly because of the greater area of the sounding body in contact with the medium into which the energy is received at a correspondingly higher rate; hence a louder tone. The sounding board or the body of a musical instrument is constructed on this principle.

(2) A spherical sound wave moving rapidly outward in all directions from a sounding body possesses a certain definite quantity of energy which the sounding body imparted to it and which remains practically unchanged for a considerable distance. Since the area of a spherical surface is

$4\pi r^2$, the area of the wave increases as the square of the radius or distance from the source. From this it follows that *the quantity of sound energy per unit area, which determines the degree of loudness, varies inversely as the square of the distance from the sounding body.*

This law applies in general to energy radiating from a point source and is called *the law of inverse squares.*

185. Pitch. — *Pitch* depends upon the *frequency* of the sound wave in the medium in contact with the ear, and this, in turn, generally depends upon the frequency of the sounding body. The pitch of a tone becomes more acute as the frequency of vibration becomes greater and, conversely, it grows more grave as the frequency decreases.

186. Experimental Determination of Frequency. — The frequency of a tuning fork may be found by attaching a

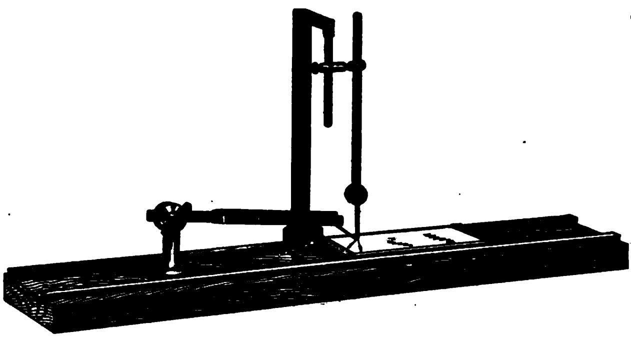


FIG. 79.

stylus to one of its prongs and, while the fork is vibrating, drawing it over a smoked glass plate upon which is being made at the same time a trace by a second stylus attached to a vibrating pendulum whose period is known; the number of fork vibrations to each pendulum vibration is shown

on the glass, and this number divided by the period of the pendulum equals the frequency of the fork (Fig. 79).

187. Another method of finding the frequency of any sounding body is to foree a current of air through regularly spaced holes in a disk which is revolved at such speed that the tone given by this instrument, called a *siren* (Figs. 80 and 81), is in unison with, *i.e.* has the same pitch as, the tone given by the sounding body. The product of the number of holes in the disk and the number of revolutions it makes per second equals the number of separate puffs of air (each producing a sound wave) made per second, and therefore equals the frequency of the sound waves formed by either sounding body.

FIG. 80.

188. Range of Hearing. — Investigation shows that the sensation will not be a smooth, continuous tone unless the frequency is higher than about 16, and that if the frequency is gradually increased, the pitch of the tone rises higher and higher, becoming exceedingly acute and finally inaudible when the frequency has reached about 40,000.

189. Intervals. — Between two tones differing in pitch there is said to be an *interval*, and the value of this interval is the ratio of the frequencies of the two tones.

If the attention is fixed on a tone of any pitch and the frequency is then increased, there will be recognized, occurring periodically as the pitch rises, other tones the pitch of which seems to bear a simple, harmonious relation to the first. The interval which separates the most conspic-

uous of these closely related tones is called the interval of the *octave*. If the frequencies of the tones giving that interval are determined, which is easily done by means of the siren, they are found to be in the simple ratio of 1 : 2 : 4 : 8 : 16, etc., *i.e.* each of a series of tones separated by the interval of the octave is produced by double the frequency of the preceding lower tone, or by half the frequency of the next higher tone.

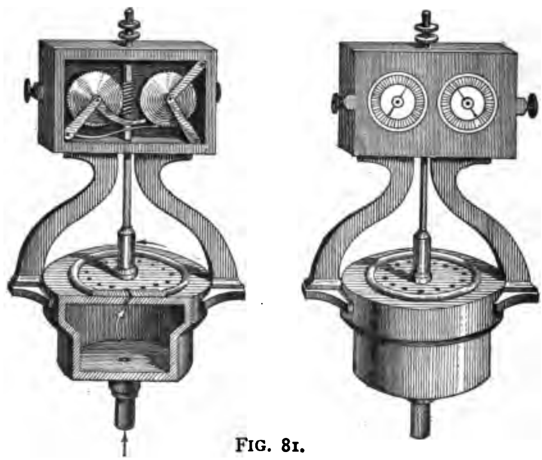


FIG. 81.

Other intervals besides that of the octave are also easily recognized and whenever this is the case, the frequencies giving such intervals are found to bear simple ratios to each other: *the smaller the terms of the ratio the simpler the interval.* Thus 3 : 2 is the ratio of the frequencies giving the *interval of the fifth*; 4 : 3, the *interval of the fourth*; 5 : 4, the *interval of the third*; 5 : 3 the *interval of the sixth*; etc.

Furthermore, if the tones defining such intervals are heard together, the effect of the combination has a pleas-

ing simplicity corresponding to the smallness of the terms of the frequency ratios.

The musical intervals arranged in the order of their simplicity are as follows: —

1. Interval of the Octave, frequency ratio 2 : 1.
2. Interval of the Fifth, frequency ratio 3 : 2.
3. Interval of the Fourth, frequency ratio 4 : 3.
4. Interval of the Major Third, frequency ratio 5 : 4.
5. Interval of the Major Sixth, frequency ratio 5 : 3.
6. Interval of the Minor Third, frequency ratio 6 : 5,
7. Interval of the Minor Sixth, frequency ratio 8 : 5.
8. Interval of the Major Second, frequency ratio 9 : 8.
9. Interval of the Major Seventh, frequency ratio 15 : 8.

etc., etc.

It has been shown by Von Helmholtz, a noted German scientist, that the simplicity of any combination of tones depends upon the absence of *beats* accompanying the tones or their harmonics. This is explained later under the subject of *interference.*

190. Chords: Major and Minor. — A combination of two or more tones whose frequency ratios are small, is a *chord.* If the frequency ratios of three tones are 4 : 5 : 6, they constitute a *major chord;* if 10 : 12 : 15, they constitute a *minor chord.*

191. The Musical Scale. — The *major diatonic,* or *natural scale,* is a series of eight tones within the interval of the octave as follows: —

Tone	C_3	D_3	E_3	F_3	G_3	A_3	B_3	C_4
Frequency	256	288	320	$341\frac{1}{3}$	384	$426\frac{2}{3}$	480	512
Frequency Ratios	1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2
or,	24 :	27 :	30 :	32 :	36 :	40 :	45 :	48

The frequency ratios of these eight tones are the same for each octave above or below the octave C_3–C_4. The

groups of tones, C_3–E_3–G_3, G_3–B_3–D_4, F_3–A_3–C_4, are major chords since their frequencies reduce to the ratios $4:5:6$ in each case; consequently the major diatonic scale may be regarded as built up of three major chords interrelated as shown above.

The ratios of the frequencies defining each of the successive intervals in the scale are:—

C–D,	D–E,	E–F,	F–G,	G–A,	A–B,	B–C.
$\frac{9}{8}$	$\frac{10}{9}$	$\frac{16}{15}$	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{9}{8}$	$\frac{16}{15}$

It is apparent that these intervals are unequal. The two, E–F and B–C, being much smaller than the others, are called *half intervals* or "*semitones*"; the others are called *whole intervals* or "*whole tones*."

192. The Chromatic Scale.— Such musical instruments as a piano or organ have a "fixed" keyboard and in order to produce the scale just described, *beginning with any other tone than* C, it would be necessary to insert several extra tones in each interval. That tone having a frequency $\frac{25}{24}$ of the tone C is called the "sharp" of that tone, and is written $C^\sharp$; another tone having a frequency $\frac{24}{25}$ of the next tone D is called the "flat" of that tone, written $D^\flat$. The frequency of $C_3^\sharp$ is $\frac{25}{24}$ of $256 = 266.6$; the frequency of D_3 is $\frac{24}{25}$ of $288 = 276.5$.

To insert all the required tones would increase the original eight tones to such a number that the keyboard would be very awkward; hence a compromise is effected by inserting only *one* additional tone in each *whole* interval, making the number of tones in the scale thirteen in all; and each half interval is made to have the same value. Since in the scale of thirteen tones there are twelve half intervals and the upper tone of this scale has double the fre-

quency of the first tone, each half interval has the ratio $1 : \sqrt[12]{2} = 1 : 1.059$.

To equalize the intervals of a scale in this manner is called *tempering*, and the scale is then known as the *equally tempered scale.*

The following is a comparison of the equally tempered scale with the major diatonic scale, beginning with $C_3 = 256$.

Equally Tempered Scale		Major Diatonic Scale	
Tone	Frequency	Tone	Frequency
C_3	$256 \times \overline{1.059}^{0} = 256$	C_3	$256 \times 1 = 256$
$C_3\sharp$ or $D_3\flat$	$256 \times \overline{1.059}^{1} = 271.2$	$C_3\sharp$ $D_3\flat$	$256 \times \frac{25}{24} = 266.6$ $288 \times \frac{24}{25} = 276.5$
D_3	$256 \times \overline{1.059}^{2} = 287.4$	D_3	$256 \times \frac{9}{8} = 288$
$D_3\sharp$ or $E_3\flat$	$256 \times \overline{1.059}^{3} = 304.4$	$D_3\sharp$ $E_3\flat$	$288 \times \frac{25}{24} = 300$ $320 \times \frac{24}{25} = 307.7$
E_3	$256 \times \overline{1.059}^{4} = 322.5$	E_3	$256 \times \frac{5}{4} = 320$
F_3	$256 \times \overline{1.059}^{5} = 341.7$	F_3	$256 \times \frac{4}{3} = 341.3$
$F_3\sharp$ or $G_3\flat$	$256 \times \overline{1.059}^{6} = 362.0$	$F_3\sharp$ $G_3\flat$	$341.3 \times \frac{25}{24} = 355.5$ $384 \times \frac{24}{25} = 368.6$
G_3	$256 \times \overline{1.059}^{7} = 383.6$	G_3	$256 \times \frac{3}{2} = 384$
$G_3\sharp$ or $A_3\flat$	$256 \times \overline{1.059}^{8} = 406.4$	$G_3\sharp$ $A_3\flat$	$384 \times \frac{25}{24} = 400$ $426.6 \times \frac{24}{25} = 409.5$
A_3	$256 \times \overline{1.059}^{9} = 430.6$	A_3	$256 \times \frac{5}{3} = 426.6$
$A_3\sharp$ or $B_3\flat$	$256 \times \overline{1.059}^{10} = 456.1$	$A_3\sharp$ $B_3\flat$	$426.6 \times \frac{25}{24} = 444.4$ $480 \times \frac{24}{25} = 460.8$
B_3	$256 \times \overline{1.059}^{11} = 483.3$	B_3	$256 \times \frac{15}{8} = 480$
C_4	$256 \times \overline{1.059}^{12} = 512$	C_4	$256 \times 2 = 512$

From the mathematical nature of the musical scale it will readily be seen that if the pitch of a tone is not indicated in terms of its *frequency*, it may be designated by

reference to its *position* in the scale. To the musician who has the sense of "absolute pitch," F_8 means a certain tone the pitch of which has a fixed value and bears definite relations to that of other tones; while to the physicist it means a tone caused by sound the frequency of which is 341.3.

The equally tempered scale of modern musical instruments is based upon the international pitch of A_8 as 435. This gives C_8 a frequency of 258.6. The frequency 256 ($= 2^8$) is taken for C_8 in the physical study of sound.

193. Quality. — *Quality* depends upon the complexity of the wave in the transmitting medium, and this in turn upon the manner in which the sounding body vibrates. When a tuning fork is properly handled, each prong has a simple to and fro motion *as a whole*, like the pendulum, and the sound waves formed by it are in consequence perfectly simple and well defined, giving a tone of the simplest quality that can be produced. Most sounding bodies, however, vibrate not only *as a whole*, but also *in parts* which are the simplest subdivisions of the whole. Each of these vibrating parts produces its own set of waves, all of which coexist in the medium as a complex wave whose form is the resultant of the several separate vibrations. Such a sound wave causes a complex sensation, consisting of what is called the *fundamental tone*, due to the vibration of the body as a whole, and of what are known as the *harmonics*, due to the vibrating parts. These parts, being $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, etc., of the whole vibrating body, have frequencies inversely proportional to their lengths, viz.: 2, 3, 4, 5, 6, etc., times that of the fundamental tone. The harmonics are, therefore, related to the fundamental by certain definite intervals as follows: —

	Frequency Ratio with Fundamental Tone	Interval
1st harmonic	2 : 1	1 octave
2d harmonic	3 : 1	1 octave and a fifth
3d harmonic	4 : 1	2 octaves
4th harmonic	5 : 1	2 octaves and a third
5th harmonic	6 : 1	2 octaves and a fifth
etc.,	etc.,	etc.

No two sounding bodies vibrate *in parts* in exactly the same way, owing to difference in *size*, *shape* or *material;* hence the number and relative intensity of the resulting harmonics are never quite duplicated. Therefore the complexity of the tone produced, *i.e.* its *quality*, is characteristic of each sounding body.

The *quality* of tone of two musical instruments may, however, be made practically identical if great care is taken to construct them alike as to size, shape, and material. It is this characteristic of quality which enables one to recognize the many different instruments in a full orchestra, even should the tones be of the same pitch and loudness; or by which one distinguishes the voices of friends, or in which lies the many-fold value of one violin over another.

194. Resonance. — If sound waves from two independent sources in passing simultaneously through the same medium coincide in phase, *i.e.* each compression of one set of waves joins with each compression of the other set, the amplitude of vibration of the parts of the medium is the *sum* of the amplitudes which each wave has separately and consequently the sound is *strengthened or reinforced.* This phenomenon is called *resonance.*

195. Interference. — If, on the contrary, the two sets of waves are opposite in phase, *i.e.* the compressions of one set join with the rarefactions of the other set, the resultant

amplitude is the difference of the amplitudes of the two waves separately and the sound is consequently *weakened.* This phenomenon is called *interference.*

196. Length of Resonating Air Column. — If a vibrating tuning-fork is held over the open end of a tube filled with air and closed at the other end in such a manner that the length of the air column in the tube may be adjusted, it is found that there is a certain length of air column which, by its vibration, reënforces the sound of the fork. This length of the air column is equal to *one fourth* of the length of the sound wave produced by the fork for the following reason: —

Referring back to Fig. 76, it will be seen that while the prong of the fork is moving from m_3 to n_3 the air in the space m_3b_3 or d_3f_3 is moving in the same direction, and while the prong is moving from n_4 to m_4 the air in the space m_4b_4 or d_4f_4 is moving in the direction n_4m_4. Now m_3b_3, d_3f_3, m_4b_4, d_4f_4, are each equal to one half a wave length; therefore, while the fork is vibrating over a tube containing a column of air, from position m_3 to n_3, the wave travels through the air in the tube to the closed end and *back* again, a distance of one half a wave length. Hence the length of an air column which is vibrating with a fork so as to produce a wave in the same phase is *one fourth* of the length of the sound wave generated by the fork.

197. Relation of Frequency to Length of Air Column. — Since the length of the vibrating air column in a tube closed at one end is one fourth that of the corresponding sound wave, no matter what that length may be, it follows that *the wave length varies directly as the length of the column.* But $\lambda \propto \frac{1}{n}$; therefore *the frequency varies inversely as the length of the column,* which means that the shorter the column the higher the pitch of the tone produced by it. A column one half the length of another will give a tone an

octave higher, while one one third as long will give a tone of an interval of an octave and a fifth, and so on. On account of the *inertia* of the air in the column vibrating with the fork, the actual length of the column extends somewhat beyond the open end of the tube, the distance increasing as the diameter of the tube increases and amounting to about .4 of the diameter. Therefore to obtain the actual length of the column, add .4 of the diameter of the tube to the length of the tube. If L is the length and D the diameter of a tube, then the wave length, $\lambda = 4\,(L + .4\,D)$.

198. Laws of Vibrating Air Columns. — A tube open at one end and closed at the other is called a "closed pipe"; one open at both ends, an "open pipe," Fig. 82.

FIG. 82.

Experiment gives the following results: —

1. The frequency of the tone of a closed pipe is *one half* that of an open pipe of the same length. Consequently the interval of the tones produced by two such pipes is an octave, the lower tone being given by the closed pipe.

2. The frequency of the tone emitted by either open or closed pipes is inversely proportional to the length of the pipe.

As a corollary from (1) it follows that, since a closed pipe is one fourth of the length of the sound wave it produces, an open pipe is one half as long as the wave it produces.

EXAMPLE. — If a closed pipe 65 cm. long gives the tone C_2 128, what tone will be produced by an open pipe 130 cm. long? By the 2d

law it follows that a closed pipe 130 cm. long would produce the tone C_1 64 and since an open pipe of the same length gives a tone an octave higher, an open pipe 130 cm. long will produce the tone C_2 128.

199. Laws of Vibrating Strings. — By a *vibrating string* is meant any string or wire stretched between two points or supports so that it may be made to vibrate freely, *e.g.*

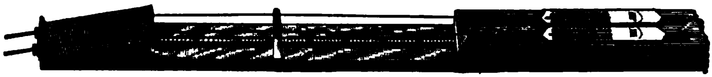


FIG. 83.

a violin or guitar string, a piano or sonometer wire, as shown in Fig. 83.

The following results are determined by experiment with vibrating strings: —

Other conditions being the same; —

1. The frequency is *inversely* proportional to the *length* (l) of the string: $n \propto \frac{1}{l}$.

2. The frequency is *inversely* proportional to the *diameter* (d) of the string: $n \propto \frac{1}{d}$.

3. The frequency is *directly* proportional to the *square root* of the *tension*, τ (pronounced *tau*), of the string: $n \propto \sqrt{\tau}$.

4. The frequency is *inversely* proportional to the *square root* of the *density* (ρ) of the string: $n \propto \frac{1}{\sqrt{\rho}}$.

These laws may be condensed into one statement as follows: The frequency is *directly* proportional to the square root of the tension and inversely proportional to the continued product of the length, the diameter, and the square root of the density: $n \propto \frac{\sqrt{\tau}}{l \cdot d \cdot \sqrt{\rho}}$.

EXAMPLE. — If a steel wire 60 cm. long, .25 mm. diameter under a tension of 4 kgm., will give the tone G_2 192, what tone will be produced by a steel wire 40.9 cm. long, .33 mm. diameter when under a tension of 9 kgm.?

$$n_1 : n_2 = \frac{\sqrt{\tau_1}}{l_1 \cdot d_1 \cdot \sqrt{\rho_1}} : \frac{\sqrt{\tau_2}}{l_2 \cdot d_2 \cdot \sqrt{\rho_2}}.$$

$$192 : n_1 = \frac{\sqrt{4}}{60 \times .25 \times \sqrt{\rho}} : \frac{\sqrt{9}}{40.9 \times .33 \times \sqrt{\rho}}.$$

$$192 : n_2 = \frac{2}{15\sqrt{\rho}} : \frac{3}{13.497\sqrt{\rho}}.$$

$$\frac{2\,n_2}{15\sqrt{\rho}} = \frac{576}{13.497\sqrt{\rho}}.$$

$$26.994\, n_2 \sqrt{\rho} = 8640 \sqrt{\rho}.$$

$$26.994\, n_2 = 8640.$$

$$n_2 = 320, \text{ which is } E_3.$$

200. Beats. — Two sounding bodies whose frequencies differ but little will form two sets of waves of slightly different length which will periodically coincide in phase, alternately reënforcing and interfering with each other. An analogous illustration is seen in the steps of two persons walking *together* if one takes shorter and more rapid steps than the other. Periodically the right feet (or the left feet) of both persons will move together: this corresponds to coincidence in phase of the sound waves; but at the middle of these periods the right foot of one and the left foot of the other person will move together, and this is analogous to interference in the sound waves. When such a wave combination is heard, the loudness varies periodically, corresponding to the reënforcements and interferences, giving to the sensation a throbbing or beating effect. Such variations in loudness are called *beats*.

It is evident in the case of two persons walking together

that if one takes 50 double steps per minute and the other takes 49 or 51 such steps in the same time, they will be perfectly "in step" just once each minute; and again, if one takes 50 and the other either 48 or 52 steps per minute, they will be "in step" twice each minute and so on; *i.e.* the difference in the number of steps per minute is the number of times they will be "in step" each minute. In the case of sound waves alternately reënforcing and interfering with each other *the number of beats per second is equal to the difference of the frequencies.*

The harmony or discord of tones heard together is due to the absence or presence of beats between the tones or their harmonics.

PROBLEMS

1. A cannon ball is fired against a target 2 mi. distant with an average velocity of 1200 ft. per sec. (*a*) Which reaches the target first, the ball or the sound of the firing? (*b*) What is the interval of time between the two arrivals if the temperature of the air is 16° C.? *Ans.* (*a*) The ball. (*b*) .61 sec.

2. A flash of lightning is seen and 6 sec. later the thunder is heard. How far away was the lightning, the temperature being 24° C.? *Ans.* 2080.8 m.

3. A tuning fork vibrates 384 times per sec. Neglecting the diameter, what is the length of the tube which will produce maximum resonance with this fork at a temperature of 0° C.? *Ans.* 21.64 cm.

4. What is the length of the sound wave in air produced by a body whose frequency is 384, the temperature being 20° C.? *Ans.* 89.7 cm.

5. What is the length of the air column in a closed pipe 6 cm. in diameter at a temperature of 25° C. which will reënforce the tone E_3 320? *Ans.* 24.74 cm.

6. Write the major chord beginning with the tone B_3 480, and show that the frequencies give the required ratios. *Ans.* B_3–$D\sharp_4$–$F\sharp_4$.

7. Write the major diatonic scale in the key of G and show that the intervals are correctly given. *Ans.* G_3–A_3–B_3–C_4–D_4–E_4–$F\sharp_4$–G_4.

8. Write the minor chord beginning with the tone A_3 426.6, and prove your answer. *Ans.* A_3–C_4–E_4.

9. If two fifths of a second is required to say "hello" and if *immediately after* shouting it the echo of these two syllables is distinctly heard, what is the distance of the reflecting surface from you, the temperature of the air being 12° C.? *Ans.* 67.92 m.

10. What are the frequencies of the sounds each of which will produce 4 beats per sec. with the tone C_4 512? *Ans.* 508 and 516.

11. If a closed pipe 4 ft. long gives the tone D♯ 72, what tone will an open pipe 3 ft. long give? *Ans.* G_2 192.

12. If a steel wire 50 cm. long, .25 mm. in diameter under a tension of 4 kgm., will produce the tone E_3 320, what tension on a steel wire 30 cm. long, .40 mm. in diameter, will produce the tone C_8 256?

Ans. 2.36 kgm.

13. The second harmonics of two tones have the ratio $\frac{15}{12}$. What is the ratio of their fundamentals? *Ans.* Same ratio.

14. What is the frequency of the tone which is a fifth above A_3 426.6? *Ans.* E_4 640.

15. What is the frequency ratio of the tones C_3 and A♭$_3$? *Ans.* $\frac{5}{8}$.

16. (*a*) What tone is a major third above B♭$_3$? (*b*) What is the frequency ratio of the interval B_3–D_4? *Ans.* (*a*) D_4. (*b*) $\frac{5}{6}$.

17. When the tones C_2 128 and B_2 240 are sounded together, how many beats per second are produced between B_2 and the first harmonic of C_2? *Ans.* 16.

18. Write the minor chord beginning with E♭$_3$.

Ans. E♭$_3$–G♭$_3$–B♭$_3$.

19. If two tuning forks, vibrating respectively 256 and 252 times per second, are simultaneously sounded near each other, what phenomena will follow? *Ans.* 4 beats per sec.

20. A musical string, known to vibrate 300 times a second, gives a certain tone. A second string, sounded a moment later, seems to give the same tone. When sounded together, two beats per second are noticeable. Are the strings in unison? If not, what is the rate of vibration of the second string? *Ans.* 298 or 302.

21. A tuning fork produces a strong resonance when held over a jar 14 in. long and 4 in. in diameter. (*a*) Find the wave-length. (*b*) Find the wave period. *Ans.* (*a*) 62.4 in. (*b*) .057 sec. at 0° C.

22. A certain string vibrates 200 times a second. (*a*) Find the vibration number of a similar string, twice as long, stretched by the same weight. (*b*) Of one that is half as long.

Ans. (*a*) 100. (*b*) 400.

23. A string sounding C_3 is 54 cm. long. Must it be lengthened or shortened and how much to give the tone D_3? *Ans.* 6 cm. shorter.

24. A certain string vibrates 200 times per sec. Find the vibration number of another string that is twice as long, and weighs four times as much per foot, and is stretched by the same weight. *Ans.* 50.

25. A certain string sounds the tone C_2 when it is stretched by a weight of 4 kgm. What weight must it carry to give the tone F_2? *Ans.* $7\frac{1}{9}$ kgm.

26. A musical string 5 ft. long emits a tone in unison with that of a fork that is known to vibrate 128 times a second. What will its vibration number be when it has been shortened 2 ft.? *Ans.* $213\frac{1}{3}$.

27. A sonometer string is stretched by a load of 9 lb. What load must be given to it so that it may sound a tone an octave lower? *Ans.* $2\frac{1}{4}$ lb.

28. (*a*) Find the length of an organ pipe which produces waves 2 ft. long, the pipe being open at both ends. (*b*) Find the length, the pipe being closed at one end. *Ans.* (*a*) 1 ft. (*b*) 6 in.

29. If a musical sound is due to 288 vibrations, to how many vibrations will its major third, fifth, and octave, respectively, be due? *Ans.* 360, 432, 576.

30. If a tone is produced by 132 vibrations per sec., what number will represent the vibrations of the tone a fifth above its octave? *Ans.* 396.

31. A given tone is found to be in unison with the tone emitted by the inner row of 24 holes of a siren when the disk is turned at the uniform rate of 640 times in 30 sec. Assigning 256 vibrations for middle C, name the given tone. *Ans.* C_4.

32. Determine the vibration number for each tone of a scale the keynote of which has 522 vibrations.

33. Is there any difference in the pitch of a locomotive whistle when the locomotive is standing still, when it is rapidly approaching the observer, and when it is rapidly moving from him? If so, describe and explain it.

34. Why does the sound of a circular saw cutting through a board fall in pitch as the saw enters the board?

35. If 10 sec. intervene between the flash and report of a cannon, what is its distance, the temperature being 0° C.? *Ans.* 10,900 ft.

36. Steam was seen to escape from the whistle of a ferryboat, and the sound was heard 4 sec. later. The temperature being 20° C., how far was the boat from the observer? *Ans.* 4520 ft.

37. What is the length of sound waves propagated through air at a temperature of 20° C. by a tuning fork that vibrates 113 times per sec.? *Ans.* 10 ft.

38. A shot is fired before a cliff and the echo heard 5 sec. later. The temperature being 10° C., determine the distance of the cliff. *Ans.* 2770 ft.

39. How many vibrations per second are necessary for the formation of sound waves 2 ft. long, the velocity of sound being 1100 ft.? Determine the temperature at the time of the experiment. *Ans.* 550. 5° C.

CHAPTER X

HEAT

201. Definition of Heat. — *Heat is molecular kinetic energy, i.e.* the energy of the vibrating molecules of matter. When a bullet strikes an iron target its motion is suddenly stopped and it becomes hot. The kinetic energy of the mass disappears, being changed into kinetic energy of the molecules. The kinetic theory of matter affirms that the molecules of a body are in motion. A body possesses energy owing to this molecular motion; the more rapidly the molecules move, the greater is their energy and the hotter is the body.

202. Temperature. — The distinction between hot and cold bodies is familiar to all, and is associated with the difference of the sensations experienced in touching various bodies, according as they are hot, warm, cool, or cold. These words refer to the state of a body which is designated by the term *temperature.* Hot bodies are said to have a high temperature, and cold bodies a low temperature.

If the temperature of a body is higher than that of the surrounding bodies, it loses heat to them. If the temperature is lower than that of the surrounding bodies, it gains heat from them. If the body neither gains nor loses heat, it is at the same temperature as the surrounding bodies.

203. Definition of Temperature. — *The temperature of a body is its thermal state considered with reference to its ability to give heat to or to receive heat from other bodies.*

Since kinetic energy equals $\frac{1}{2}mv^2$, the temperature of a given mass is determined by the mean square of the velocities of its molecules, and a change of temperature is proportional to the change of the mean square of the velocities of the molecules.

204. Temperature not a Measure of the Quantity of Heat. — Bodies at equal temperatures do not necessarily possess equal quantities of heat; nor does a body of higher temperature necessarily possess more heat than another body of lower temperature. A cupful of water dipped from a pail of water will have the same temperature as the water in the pail, but, owing to its smaller mass, will not possess the same quantity of heat. Temperature is only one of several factors determining the *quantity* of heat contained in any body.

205. Measurement of Temperature. — The thermal state of a body may vary consecutively from very hot to very cold, and some means must be devised to measure and express these temperatures. Temperature cannot be measured directly, for there is no way of measuring the velocity of the molecule. The sensations cannot be relied upon, for if one hand be placed in hot water and the other in cold water, and then both are placed together in warm water, the warm water will seem hot to the hand that was in the cold water and cold to the hand that was in the hot water. Use must be made, then, of some property of a substance which alters continuously with the temperature.

The volume of most substances increases continuously as the temperature rises, and this property is usually employed to measure the temperature of a body. Mercury and glass both expand when heated, but the expansion of mercury is greater than the expansion of glass. If a bulb

of glass, with a fine tube attached, is filled with mercury and then heated, both glass and mercury will expand; the capacity of the bulb increases, but the volume of the mercury increases still more, so that not only the bulb, but part of the tube also, will be filled. As the temperature is increased, more and more of the tube becomes filled with the mercury.

FIG. 84.

A *thermometer* is an instrument used for measuring temperature, its action being based on the change of volume which accompanies a change of temperature.

206. Construction of a Mercury Thermometer.— A glass tube with heavy walls and having a very fine bore has a bulb blown at one end and a small cup at the other (Fig. 84). The tube is held upright and a little mercury is poured into the cup; but owing to the fineness of the bore, the mercury will not run into the tube, and so the following method is used to fill the thermometer. The bulb is heated so as to expand the air in it and cause some to escape through the mercury in the cup; as the air in the bulb cools some mercury is forced by the atmospheric pressure into the tube and bulb. To expel the remainder of the air, the mercury now in the bulb is heated until it boils, when the air, along with the vapor of mercury, escapes. After the boiling ceases and the mercury cools slightly, more of the mercury from the cup runs into the tube and completely fills both bulb and tube; the tube is then sealed at the top (Fig. 85).

FIG. 85.

When the mercury contracts by further cooling, the space above it is free from air.

207. Graduation of the Thermometer. — Experiments have proved that under constant pressure the temperatures of melting ice and of steam are constant. These temperatures are known as the *fixed points*, and are used in comparing and graduating thermometers. A thermometer after being filled and sealed is laid aside for some months. It is then placed in finely crushed melting ice so that the mercury is entirely surrounded by the ice. The position of the top of the mercury column is then marked on the glass.

The thermometer is next placed in steam at a pressure of 760 mm., and the new position of the top of the mercury column is again marked. The distance between these two marks is graduated into a number of equal divisions called degrees. The graduations are extended above the boiling and below the melting points.

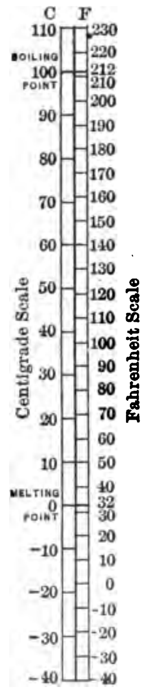


Fig. 86.

208. Thermometer Scales. — Two scales of temperature are in general use, the Centigrade and the Fahrenheit. In the Centigrade scale the temperature of melting ice is marked zero (0°) and that of steam 100°. The distance between them is divided into one hundred equal parts called Centigrade degrees. In the Fahrenheit scale the temperature of melting ice is marked 32° and that of steam 212°. The distance between these two points is divided into 180 equal divisions known as Fahrenheit degrees. Temperatures below 0° in both scales are negative (Fig. 86).

209. Comparison of Temperature Scales. — Since 180 Fahrenheit degrees represent the same temperature change (*i.e.* from melting to boiling point) as 100 Centigrade degrees, each Fahrenheit degree represents $\frac{5}{9}$ of the temperature change that each Centigrade degree represents; and conversely each Centigrade degree represents $\frac{9}{5}$ of the temperature change of a Fahrenheit degree.

A temperature of 50° C. means 50 Centigrade degrees above the melting point, and this equals $\frac{9}{5}$ of 50, or 90 Fahrenheit degrees above the melting point. As the melting point of the Fahrenheit scale is marked 32°, this must be added to 90 to give the reading in the Fahrenheit scale of the same temperature as 50° C.

$$50^\circ \text{ C.} = \tfrac{9}{5} \times 50 + 32 = 122^\circ \text{ F.}$$

To change from the Centigrade scale to the Fahrenheit scale take $\frac{9}{5}$ of the Centigrade reading and then add 32.

A temperature of 50° F. means 50 − 32, or 18 Fahrenheit degrees above the melting point, and as each Fahrenheit degree equals $\frac{5}{9}$ of a Centigrade degree, 18 Fahrenheit degrees = 10 Centigrade degrees above the melting point.

$$50^\circ \text{ F.} = \tfrac{5}{9}(50 - 32) = 10^\circ \text{ C.}$$

To change from the Fahrenheit scale to the Centigrade scale subtract 32 *from the Fahrenheit reading and take $\frac{5}{9}$ of the remainder.*

Letting F. and C. represent the *readings* in the two scales respectively, these rules may be written: —

$$\text{F.} = \tfrac{9}{5}\text{ C.} + 32,$$

$$\text{C.} = \tfrac{5}{9}(\text{F.} - 32).$$

210. Limitations of the Mercury Thermometer. — Mercury solidifies at about −38° C. and boils at 360° C. A mercury thermometer is not accurate as the temperature approaches these limits and cannot be used at all for temperatures higher or lower than these limits. To measure tempera-

tures between $-38°$ C. and about $-100°$ C. alcohol thermometers are used. For still lower temperatures electrical methods may be employed which depend on the change of the current flowing in a circuit, caused by a change of the temperature. To measure temperatures above 350° C. metallic thermometers are used, the action of which depends on the expansion of solids with the rise in the temperature, *e.g.* an expanding wire (Fig. 87), or a compound bar of two different metals. Electrical methods may also be used for very high temperatures.

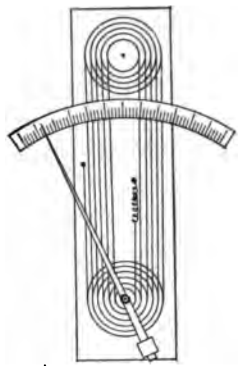


FIG. 87.

211. The Air Thermometer. — This instrument in its simplest form consists of a glass tube of small bore with a bulb filled with air at the upper end. The lower open end of the tube dips into a liquid and a scale is mounted back of the tube (Fig. 88). To construct an air thermometer the air in the bulb is slightly heated, thus expelling some of it; as the air cools again, the liquid will be forced part way up the tube. On heating the air in the bulb the liquid in the tube is depressed and on cooling it the liquid will rise. In this simple form an air thermometer is *not used to measure* temperature, but only *to indicate a change* of temperature.

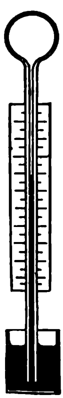


FIG. 88.

212. Sources of Heat. — Most of the heat energy on the surface of the earth is received from the *sun.* This energy is transmitted as radiant energy from the sun by wave motion in the ether.

The *interior of the earth* is at a higher temperature than the surface and heat is transmitted thence

to the surface. Volcanoes and hot springs are evidence of this higher temperature; deep mines also are noticeably warmer than the surface.

Chemical combination is generally accompanied by an evolution of heat. When this action is very rapid it is accompanied by the production of light and is then called *combustion* or *burning.* In the burning of coal the oxygen of the air combines with the carbon of the coal; most artificial heat is produced in this way.

When an *electric current* flows through a conductor it generates heat, as seen in electric heaters and in the incandescent lamp.

When *mechanical energy* disappears on account of *friction* it is changed into heat, as seen in the heating of the bearings of engines, vehicles, and machines.

In the *impact* of two bodies the kinetic energy of the mass may be largely changed into heat. A lead bullet striking an iron plate may be melted by the heat produced.

In the *compression of a gas* its temperature is raised, the work done in compressing it being partly changed into heat. A gas on suddenly expanding has its temperature noticeably lowered. Use is made of this fact in ice machines where ammonia, which has been liquefied by cooling and compression, is allowed to evaporate and expand rapidly, causing a fall of temperature by the absorption of heat from surrounding water.

213. Transmission or Diffusion of Heat. — Heat is transmitted from one point to another in three ways: by *conduction*, by *convection*, and by *radiation.*

214. Conduction is the process by which heat is transmitted from one molecule to the next, and from that to the next, and so on. If one end of a solid is heated, the mole-

cules at this end are made to move faster, and they communicate this increased motion to their neighbors, which in turn pass it on. Those substances which transmit heat readily in this way are called *good conductors*, and those which do not transmit it readily are called *poor conductors* or *nonconductors*. Metals are the best conductors, and among them silver and copper rank first. Wool, asbestos, and wood are nonconductors. Liquids and gases are very poor conductors.

215. Convection. — By this method, which is possible only in fluids, the heated portion of the substance, by expanding and becoming less dense, is forced upward by the surrounding colder and denser parts. In rising, this portion carries its heat with it, and the colder parts, taking the place of the portion first heated, are in turn heated and forced upward, thus setting up currents through the fluid — ascending currents of the heated part and descending currents of the colder part. Solids cannot transmit heat by convection because their parts cannot change their relative positions. Houses are ventilated and heated by making use of these convection currents.

216. Heating of Houses. — In the hot-air furnace the fire box is surrounded by an air drum having pipes leading out of its *top* to the registers in the several rooms. The supply pipe for the cold fresh air leads into the drum near the *bottom*. The air having become heated in the drum is forced upward through the pipes by the cold air outside, and flows into the rooms. This method provides a continuous supply of fresh warm air to the house.

In the hot-water system the water is heated in tubes or drums above the fire box. The main supply pipe leads from the *top* of the heater, with branches to the radiators

in the various rooms. A pipe leads from the bottom of each radiator to the return main, which enters the bottom of the heater. The whole system is kept filled with water, which upon being heated rises, is cooled in the radiators and then returns to the heater, a constant circulation being thus maintained.

217. Radiation. — Energy is transmitted through the *ether* by means of a wave motion (see § 262). This process of transmitting energy is called *radiation*, because the waves spread out in all directions from the source, and the energy so transmitted is called *radiant energy*. When this energy is absorbed by matter, the matter becomes hotter; hence radiation is sometimes spoken of as a method of heat transmission. The rapidly vibrating molecules of a hot body set the ether within and around the body into a similar vibration, thus starting the waves which travel in all directions through the ether. When these ether waves strike a body of matter they may be reflected, transmitted, or absorbed. When *absorbed* they set the molecules of the body into vibration and again appear as heat. In radiation *only the matter which absorbs the energy is heated*, the intervening ether medium remaining unaffected in this respect.

As gases are very poor conductors of heat, practically all heat transmitted *horizontally* or *downward* through a gas is by *radiation*.

EFFECTS OF HEAT

218. *A change in the quantity of heat in a body causes either a change in temperature or a change of state, both of which are usually accompanied by a change in volume.*

219. Expansion. — In general, when the temperature of a body is raised there is an accompanying increase in its

volume, called *expansion*. A part of the energy absorbed increases *the kinetic energy* of the body by increasing the velocities of the molecules, while the remaining portion increases the *potential energy* of the body by doing work in producing this expansion. The amount of expansion produced in a given body is generally proportional to the rise of temperature, but different substances have different rates of expansion. Solids in general have a smaller rate of expansion than liquids have; but solids differ among themselves in this respect.

220. Coefficient of Linear Expansion. — *The coefficient of linear expansion of a solid is the increase in length of each unit of length of the solid at 0° C., for a rise of temperature of one degree.*

If L_0, L_1, and L_2 are the lengths of a solid at 0°, t_1°, and t_2° respectively, and if α is the coefficient of linear expansion, then $L_0\alpha$ is the total increase in the length for a rise of 1° above 0° C.; $L_0\alpha t_1$ is the total increase in the length for a rise of t_1° above 0° C., and $L_0\alpha t_2$ is the total increase in the length for a rise of t_2° above 0° C.

Hence,
$$L_1 = L_0 + L_0\alpha t_1 = L_0(1 + \alpha t_1),$$
$$L_2 = L_0 + L_0\alpha t_2 = L_0(1 + \alpha t_2),$$
$$L_0 = \frac{L_1}{1 + \alpha t_1} = \frac{L_2}{1 + \alpha t_2},$$
$$L_2 = L_1 \frac{1 + \alpha t_2}{1 + \alpha t_1} = L_1[1 + \alpha(t_2 - t_1)],$$

or, the length, L_2, of a solid at any temperature t_2 equals the length, L_1, at some lower temperature t_1 times the quantity, 1 plus the coefficient of expansion times the difference of these temperatures.

The coefficient of cubical expansion of a substance is the increase in volume of each unit of volume of that substance at 0° C. for a rise of temperature of 1°. The coefficient of cubical expansion practically equals *three times* the coefficient of linear expansion (proof in Appendix).

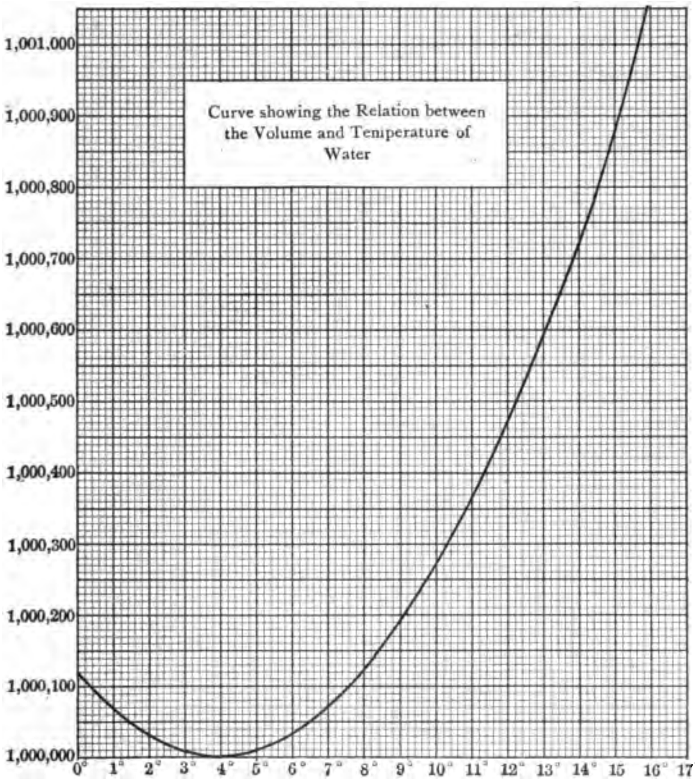
The cubical expansion of a liquid is approximately proportional to the change of temperature, but different liquids have different coefficients of expansion.

221. Table of Coefficients of Expansion. — In the following table are given the coefficients of expansion of a number of common substances. When not otherwise indicated, the value given is the mean coefficient between 0° and 100° C.

Substance	Linear Coefficient	Cubical Coefficient
Glass	.00000837	.0000254
Aluminum	.000023	.000069
Copper	.0000168	.0000531
Lead	.00002882	.000088
Tin	.00001959	.000059
Zinc	.00002976	.000089
Iron	.00001204	.000035
Platinum	.00000857	.0000266
Brass	.000019	.000057
Alcohol	(between 0° and 80°)	.00104
Ether	(between −15° and 38°)	.00215
Glycerine		.000534
Mercury		.000182
Turpentine	(between −9° and 106°)	.00105
Water	(between 10° and 100°)	.00043

222. Anomalous Expansion of Water. — When water at 0° C. is heated it *contracts* instead of expanding, and this contraction continues until the temperature of 4° C. is reached, then *expansion* begins, and at 8° the water has practically the same volume as at 0° C. (Fig. 89). The temperature of 4° C., therefore, is the temperature of *minimum volume* and consequently that of *maximum density of water.*

But for this peculiarity of water the formation of ice on the surface of ponds and rivers would be delayed for some time, and when once started the freezing would progress



Temperature C.

FIG. 89.

somewhat more rapidly than it. does. The cooling takes place at the surface and this cooled water sinks, the process continuing until the entire body of water has reached a

temperature of 4°; then, if there is further cooling, the colder surface water, being less dense than the water beneath which is at 4°, remains at the surface and finally drops to 0°, when it begins to freeze.

223. Expansion of Gases. — The effect of heat upon a gas may be measured by determining the change in its volume while the pressure is kept constant or by determining the change in its pressure while the volume is kept constant. Experiment shows these two coefficients are the same for all gases. The value obtained is .003665 or $\frac{1}{273}$ of the volume, or pressure, at 0° C. for a change of 1° C.

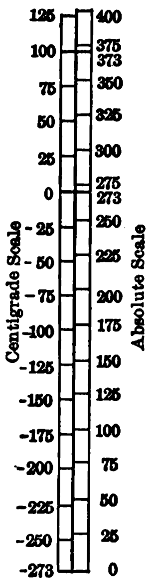


FIG. 90.

Let V_0 = volume at 0° C., V_t = volume at t° C., then if the pressure is kept constant

$$V_t = V_0(1 + \tfrac{1}{273}t),$$

$$V_t = \frac{V_0(273 + t)}{273};$$

at any other temperature t', this becomes

$$V_{t'} = \frac{V_0(273 + t')}{273}$$

or

$$\frac{V_{t'}}{V_t} = \frac{273 + t'}{273 + t}.$$

224. Absolute Temperature Scale. — If another temperature scale is used in which the degree is equal to the centigrade degree but having the zero at − 273° C. and temperatures are measured in this scale, the above becomes, —

$$\frac{V_{t'}}{V_t} = \frac{273 + t'}{273 + t} = \frac{T'}{T},$$

in which T and T' express the temperature in the new scale. This scale of temperature is called the ***absolute temperature scale*** (Fig. 90).

It is believed that the zero of this scale is the lowest possible temperature; that at this temperature bodies would possess no heat energy, and that the molecules would be at rest. This zero of temperature has never been obtained, but by liquefying hydrogen gas a temperature of about $-260°$ C., or $15°$ absolute temperature, has been reached.

From the above it is evident that *the volume of a gas at constant pressure is directly proportional to its absolute temperature.* This is known as *Charles' Law.*

225. Combination of Boyle's and Charles' Laws. — From experiments with gases at constant temperature it is found, as previously shown, that —

$$V : V' = P' : P.$$

From the preceding paragraph

$$V : V' = T : T', \text{ under constant pressure ;}$$

therefore, if both the pressure and the temperature vary,

$$V : V' = P'T : PT';$$

whence,

$$VPT' = V'P'T,$$

$$\frac{VP}{T} = \frac{V'P'}{T'} = \text{a constant.}$$

226. Calorimetry. — The process of measuring heat is called *calorimetry.*

227. Units of Heat. — The quantity of heat required to raise the temperature of 1 gram of water 1° C. is the C. G. S. unit of heat and is called the *calorie.* The F. P. S. unit of heat is the quantity of heat required to raise the temperature of 1 pound of water 1° F. and is known as the *British Thermal Unit* (B. T. U.).

228. Heat Absorbed or Liberated by Water. — To calculate the quantity of heat, expressed in calories, required to

raise the temperature of any mass of *water* any number of degrees it is only necessary to multiply the mass in grams by the rise of temperature in degrees centigrade.

If M represents the mass of water, t_1 its initial temperature, and t_2 its final temperature, the number of calories absorbed in the rise of temperature from t_1 to t_2 is expressed thus: calories $= M(t_2 - t_1)$. If the final temperature t_2 is lower than the initial temperature t_1 of the water, then the number of calories *taken from* it is expressed thus: calories $= M(t_1 - t_2)$.

229. Specific Heat. — It is found by experiment that most substances require less heat to raise their temperature a given number of degrees than is required to produce the same rise of temperature in an equal mass of water. *The specific heat of a substance is the quantity of heat necessary to raise the temperature of unit mass of the substance one degree.*

The quantity of heat absorbed in raising the temperature, or liberated in lowering the temperature, of a given mass of any substance is found by multiplying its *specific heat* by its *mass* and by the *change of temperature.*

If M is the mass of a substance, s its specific heat, t_1 the initial temperature, and t_2 the final temperature, then

$$\text{calories absorbed} = Ms(t_2 - t_1),$$

or

$$\text{calories liberated} = Ms(t_1 - t_2).$$

230. Method of Mixtures. — To measure the quantity of heat given to or taken from a substance the *method of mixtures* may be used. In this method a known mass of the given substance at a known temperature is mixed with a known mass of water at a known temperature and the temperature of the resulting mixture is noted. Knowing the mass of the water and its change of temperature the quantity of heat it has received (or liberated) can be

directly calculated in calories, and this heat must have come from (or been given to) the substance mixed with the water.

Hence, $$M_s s_s(t_s - t_r) = M_w s_w(t_r - t_w),$$
in which M_s and M_w are the two masses; s_s and s_w, the two specific heats; t_s and t_w, the two initial temperatures; and t_r the resulting temperature of the substance and of the water respectively. If all but one of the above quantities are known, that one may be calculated.

EXAMPLE. — If 500 gm. of lead at a temperature of 96° C. are immersed in a calorimeter containing 1000 gm. of water at 20° C. and the resulting temperature is found to be 21.2° C., the specific heat of lead can be found as follows: —

$$500 \times s_{L.} \times (96 - 21.2) = 1000 \times 1 \times (21.2 - 20),$$

from which $$s_L = \frac{1000(21.2 - 20)}{500(96 - 21.2)} = \frac{1000 \times 1.2}{500 \times 74.8} = .032 \text{ cal.}$$

231. Table of Heat Constants.

	Specific Heat	Melting Point	Boiling Point	Heat of Fusion	Heat of Vaporization
Alcohol	0.65	−130° C.	78.2° C.	——	206
Aluminum	0.212	700° C.	——	——	——
Brass	0.094	912° C.	——	——	——
Copper	0.093	1100° C.	——	——	——
Glass	0.19	——	——	——	——
Gold	0.032	1100° C.	——	——	——
Iron	0.11	1635° C.	——	28.	——
Lead	0.032	330° C.	——	5.86	——
Mercury	0.033	−39° C.	357° C.	2.8	——
Silver	0.056	1000° C.	——	21.	——
Sulphur	0.17	114° C.	448° C.	9.4	362
Tin	0.055	228° C.	——	14.	——
Turpentine	0.5	−10° C.	160° C.	——	70
Zinc	0.093	420° C.	——	28.	——
Ice	0.5	0° C.	——	80.	——
Water	1.0	——	100° C.	——	536
Steam	0.48	——	——	——	——

232. Change of State. — *Change of state* signifies the passing from one of the three states in which matter may exist, *viz.*, solid, liquid, or gaseous, into either of the other states.

233. Transformation of Energy. — If heat is applied to a solid, its temperature will rise until a certain point is reached, when the temperature remains constant while the body melts or fuses. After the solid is entirely changed to liquid, the temperature will again rise until a second point is reached, when it again remains constant while the liquid changes into a vapor. After all the liquid is vaporized the temperature will once more begin to rise. The heat, which is absorbed while the change of state is taking place, is transformed into potential energy, work having been done by it in overcoming the molecular forces. There is usually a marked change of volume accompanying the change of state. The heat required to produce a change of state is sometimes called latent heat.

234. Fusion. — The changing of a substance from the solid to the liquid state is known as *fusion* or *melting*, and the temperature at which this change takes place is the *melting point* of that substance. *Solidification* is the opposite of fusion, and the temperature at which a liquid solidifies, under ordinary conditions, is the same as the melting point for the same substance.

235. Laws of Fusion. — 1. Every crystalline substance begins to melt at a definite temperature, which is invariable for each substance at the same pressure.

2. The temperature of a solid while melting remains constant until the entire body is melted.

Some substances, as wax, glass, wrought iron, and plat-

inum, are exceptions to this law. These substances pass through a plastic state, the viscosity decreasing as the temperature rises. It is owing to this fact that these substances can be welded.

236. Effects of Pressure. — Substances which *expand* on solidifying have their melting points (or freezing points) *lowered* by an increase of pressure, while the opposite is true of substances which *contract* on solidifying. Ordinary atmospheric changes of pressure, however, produce so small an effect on the melting point of ice as not to be measurable. Water, bismuth, and cast iron expand on solidifying, but most other substances contract. In the casting of iron and type this property is of great utility, because these substances on solidifying expand slightly and completely fill the mold, thus producing a perfect cast. On the other hand gold and silver coins must be stamped by dies, because they contract on solidifying and would give an imperfect cast.

237. Regelation. — When two pieces of ice at 0° C. are pressed together they unite about the points of contact. This phenomenon, to which the name *regelation* is given, is a result of pressure causing a lowering of the melting point. The pressure liquefies the ice at the points of contact, but the water thus formed is below its normal freezing point, and as soon as it escapes from the pressure by flowing away from the points of contact it again freezes, joining the two pieces of ice together. This is the explanation of the making of snowballs and also of the flow of glaciers.

238. Heat of Fusion. — The quantity of heat required to change *unit mass* of a substance from the solid to the

liquid state, without change of temperature, is the *heat of fusion* of that substance. The number of calories required to *melt* 1 gm. of ice at 0° C. is 80, which then is the value of the heat of fusion of ice.

239. Measurement of the Heat of Fusion. — The heat of fusion of ice may be measured as follows: —

A known mass, M_i, of ice at 0° C. is placed in a known mass, M_w, of water at a known temperature, t_w, and at the moment the ice is completely melted, the resulting temperature, t_r, of the mixture is taken. The heat liberated by the water is $M_w(t_w-t_r)$. This is the heat which has melted the ice and then raised the temperature of the water (melted ice) from zero to the temperature of the mixture. The heat required to melt the ice is M_iF, F being the heat of fusion of ice. The heat required to raise the temperature of the water formed from the melted ice is $M_i(t_r-0)$.

Equating the heat *liberated* and the heat *absorbed*, —

$$M_w(t_w-t_r)=M_iF+M_i(t_r-0).$$

Hence,

$$F=\frac{M_w(t_w-t_r)-M_i(t_r-0)}{M_i}.$$

240. Solution. — Heat is absorbed when a solid changes to the liquid state by dissolving in a liquid, as is illustrated in the cooling of water when sugar is added, or, in a more marked way, when ammonium nitrate is added to water.

Energy is required to change a solid into a liquid, and in the process of solution this energy must come from the molecular kinetic energy (heat) of both the liquid and the solid, and is transformed into the potential energy of the liquid state. A decrease in the molecular kinetic energy means a lowering of the temperature of the solution.

241. Freezing Mixtures. — Use is made of this principle in "freezing mixtures," as ice and salt in the freezing of

ice cream. The ice alone would reduce the temperature of the cream to 0° C., but no lower, because the cream and ice being then at the same temperature, there is no further flow of heat from the cream to the ice. To freeze the cream an additional quantity of heat must be withdrawn from it equal to its heat of fusion, and to do this it is necessary to lower the temperature of the ice below zero. This is accomplished by mixing salt with the ice, which in the process of dissolving may lower the temperature 10 or 15 degrees below 0° C.

242. Crystallization. — When a solid crystallizes rapidly from a solution the temperature usually rises, because in the change from the liquid to the solid state the potential energy of the liquid state becomes molecular kinetic energy, or heat, and so the temperature is raised. This is shown very strikingly by crystallization from a solution of sodium sulphate or of sodium thiosulphate.

243. Vaporization. — Vaporization is the changing of a substance from the liquid to the vapor form. This may take place in two ways: (1) *evaporation*, (2) *ebullition* or *boiling*.

244. Evaporation. — If the change from liquid to vapor takes place slowly *from the surface* of the liquid, the process is called *evaporation*.

245. Theory of Evaporation. — According to the kinetic theory of matter all the particles of matter are in continual vibratory motion, and in liquids there is perfect freedom of motion of the molecules among themselves. The molecules, however, have very different velocities, and some of those having the greatest velocity, when moving

toward the free surface of the liquid, break through the surface into the space above. The number of molecules thus leaving the liquid depends upon the *temperature* of the liquid, the *extent of the free surface*, and the *condition of the space above the liquid.* If a liquid is evaporating into an inclosed space, evaporation is said to cease when the number of particles leaving the liquid equals the number reëntering it from the space above. When this condition is reached the space above is said to be saturated with the vapor of the given liquid.

Since in the process of evaporation the liquid loses those molecules having the greatest velocities, the mean square of the velocities of the remaining liquid molecules decreases, which means that the temperature of the liquid falls.

246. Laws of Evaporation. — 1. The rate of evaporation increases with the temperature.

2. The rate of evaporation increases with the extent of the free surface of the liquid.

3. If the air in contact with the liquid is continually replaced by other air, the rate of evaporation is greater than if the air is not so changed; for in the latter case, as the air becomes more and more saturated with the vapor, the rate of evaporation decreases.

These laws of evaporation are well illustrated in the drying of clothes. The clothes are spread out on a line in order to give a large surface in contact with the air, thus securing more rapid evaporation than if placed in a pile. If a wind is blowing, they dry more quickly because of the continual change of the air in contact with them. Other conditions being equal, the clothes dry less quickly on cold days than on warm days because the rate of evaporation is slower at a lower temperature. On very damp or muggy

days, when the humidity is high, clothes do not dry rapidly because the air is already nearly saturated with water vapor.

247. Volatile Liquids. — Liquids which evaporate readily, such as alcohol, gasolene, ether, etc., are called *volatile*.

248. Cooling by Evaporation. — Since it is the most rapidly moving molecules that leave the liquid in evaporation, the temperature of the liquid is noticeably lowered if the evaporation is rapid. Use is made of this fact in cooling water for drinking purposes in hot climates. The water is placed in a porous vessel, and the evaporation from the large surface cools the remaining liquid.

249. Dew-point. — The *dew-point* is the temperature at which the water vapor in the atmosphere begins to condense. The quantity of water vapor which can be held by the atmosphere increases with the temperature of the atmosphere. When the quantity of water vapor in the air at any given temperature equals its vapor capacity at that temperature, the air is said to be *saturated*. If air containing moisture is cooled, a temperature will be reached at which the air is saturated, and then any further cooling will cause some of the water vapor to condense. The condensed vapor appears in the form of clouds, fog, dew, rain, etc.

This phenomenon is analogous to the squeezing of a damp sponge. The more it is squeezed the less is its capacity for holding water, and when squeezed until its capacity for holding water equals the quantity of water present in it the sponge is saturated; any further squeezing causes water to drip from the sponge. Cooling the air "squeezes it," or decreases its vapor capacity.

250. Relative Humidity. — The ratio of the quantity of water vapor in the air to the quantity required to saturate it *at the given temperature* is called the *relative humidity.* Saturation is represented by 100 and relative humidity by a percentage less than 100.

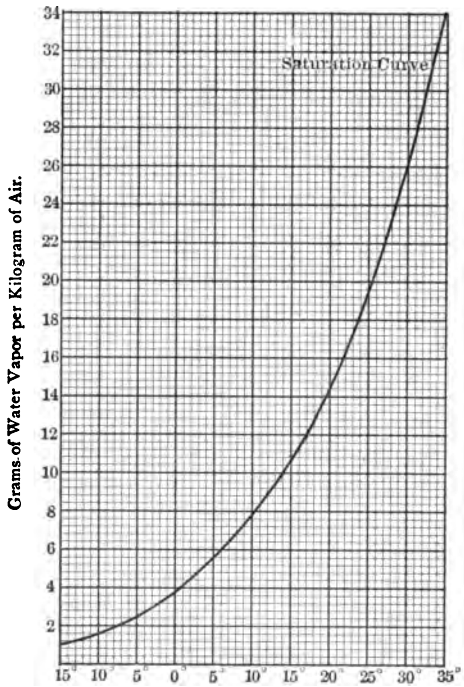


FIG. 91.

The curve (Fig. 91) shows the actual quantity of water vapor required to saturate unit mass of air at the different temperatures.

If air at 20° C. must be cooled to 8° C. to bring it to the dew-point it is found from the curve that the actual quantity of water vapor present in the air is 6.8 gm. per kilogram of air, while to saturate air at 20° C. requires 14.2 gm. of water vapor per kilogram of air. The relative humidity is therefore —

$$\frac{6.8}{14.2} \times 100 \doteq 47.9\%.$$

251. Boiling or Ebullition. — If the change from liquid to vapor takes place rapidly and violently within the body of the liquid, the process is called *boiling* or *ebullition.* The bubbles of vapor form *in* the liquids at the hottest points, rise to the surface, and break through.

252. Theory of Boiling. — Whenever the temperature of a portion of a liquid is such that the molecular velocities are sufficient to overcome both the cohesion between the molecules and the restraint due to the pressure of the liquid and the atmosphere above it, these molecules push away the surrounding liquid and occupy a space enormously large compared with that originally occupied by them, forming in the liquid what is called a bubble, which rises to the surface and emerges into the space above. The molecules in this bubble are in the gaseous state. The liquid is then said to be boiling. This phenomenon occurs at a certain definite temperature for a given liquid under a given pressure. As the heat is usually applied to the under surface of vessels containing liquid, the bubbles are formed in the lower portion, where the liquid first reaches the boiling temperature. Obviously, if the pressure on the liquid is varied, the temperature at which this change takes place varies correspondingly, *i.e.* the boiling point is lowered by a decrease of the pressure and raised by an increase of the pressure on the liquid.

253. Laws of Boiling. — 1. Each liquid has a certain boiling point which is invariable for that liquid under the same conditions.

2. When the liquid begins to boil the temperature remains constant until the entire body of liquid is changed to vapor, if the pressure and other conditions remain constant.

3. The boiling point is dependent on the character of the inner surface of the containing vessel.

4. The boiling point is raised by salts, and lowered by gases, dissolved in the liquid.

5. The boiling point rises with an increase of pressure and drops with a decrease of pressure (see curve, Fig. 92).

A change of pressure of 2.7 mm. of mercury when the pressure is near 760 mm. changes the boiling point of water .1° C. At high altitudes water boils at much lower temperatures than at sea level. The temperature of the water in a boiler giving steam at a pressure of 760 cm. of mercury (10 atmospheres) is 180° C.

254. Evaporation and Boiling Contrasted. — From the foregoing it is evident that the following differences exist between the phenomena of evaporation and of boiling.

1. Evaporation is a phenomenon limited to the surface of a liquid and is affected by a change in the extent of the free surface; boiling takes place within the body of the liquid and is not affected by changes in the extent of the free surface.

2. Evaporation takes place at all temperatures; boiling takes place at one temperature only for a given pressure on a given liquid.

3. Evaporation is but slightly affected by changes in the pressure upon the liquid surface, being mainly dependent upon the degree of saturation of the space above the liquid with the vapor of the given liquid; the presence of other vapors in this space increases the pressure, but the rate of evaporation of the given liquid is only slightly decreased thereby; the boiling point, however, changes with each change in the pressure upon the liquid.

4. Evaporation is a phenomenon of individual molecules; boiling, of groups of molecules. Hence, evaporation is an invisible process, while boiling is a visible process.

5. Evaporation is accompanied by a fall of temperature; in boiling the temperature remains constant throughout the process.

255. Heat of Vaporization. — The *heat of vaporization* of a given liquid is the number of calories required to change

unit mass of that liquid into vapor without change of temperature.

The heat of vaporization of water at 100° C. is 536 calories, *i.e.* 536 calories of heat are absorbed in changing 1 gm. of water at 100° C. into steam at 100° C., and 536 calories of heat are liberated in condensing 1 gm. of steam at 100° C. into water at 100° C.

MECHANICAL EQUIVALENT OF HEAT

256. Heat and Work. — About 1840 Joule investigated the relation between the heat unit and the unit of mechanical energy. By revolving a paddle in water, and measuring the mass and the rise of temperature of the water, the power being supplied by falling weights, he determined the quantity of mechanical work done and the quantity of heat developed by it, and from these data he calculated the ratio of the two units. According to Joule, 772 ft. lb. of work would raise the temperature of one pound of water 1° F., or 1390 ft. lb. would raise the temperature of one pound of water 1° C.

Later investigations place the values as follows: —

778 ft. lb. = 1 B.T.U.

.427 kgm. m. = 1 cal.

4.19×10^7 ergs = 4.19 joules = 1 cal.

This numerical relation is known as the *mechanical equivalent of heat.*

257. Conversion of Heat into Work. — Other forms of energy are readily converted into heat, but it is more difficult to change heat into other forms of energy, especially into mechanical energy. This change can only be made by employing some kind of heat engine, and then only a small fraction of the heat energy is converted into useful

mechanical work, most of the heat energy escaping to the surrounding substances.

258. The Steam Engine is one means of converting heat into work. In this operation water is placed in a boiler over a furnace and changed into steam at high pressure

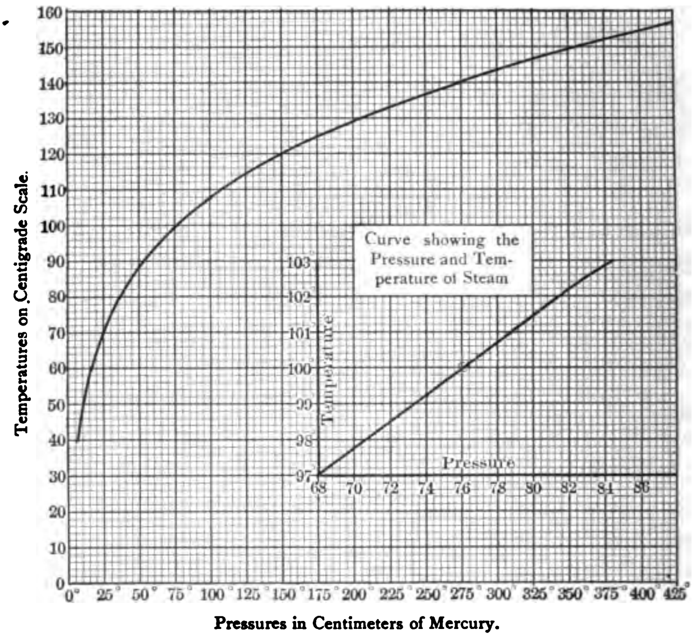


FIG. 92.

and temperature. This steam is then passed into a cylinder with a movable but tight-fitting piston, and the pressure of the steam against this piston pushes it forward. The piston is connected by a piston rod and a connecting rod to a revolving fly wheel. In this way the reciprocating motion of the piston is changed to rotary motion.

259. Details of the Steam Engine. — Figure 93 shows the essential parts of an automatic steam engine. *C* is the cylinder; *P*, the piston; *V*, the valve; *B*, the steam pipe from the boiler; *E*, the exhaust pipe; *X*, the cross head; *F*, the fly wheel; *R*, the connecting rod; *K*, the crank pin; *T*, the eccentric by which the valve *V* is moved.

The diagram shows the piston at the beginning of the forward stroke, steam being admitted through the port back of the piston. The pressure of the steam forces the piston forward, turning the fly wheel, and the eccentric

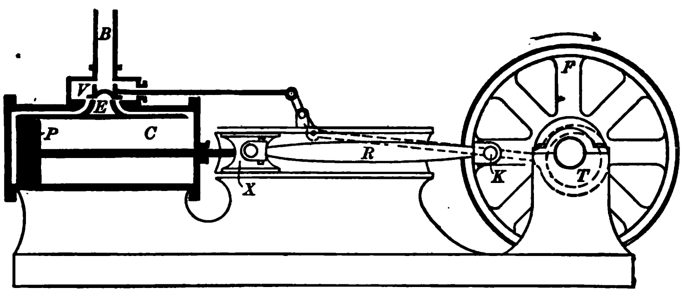


FIG. 93.

T, connected with the valve rod, moves the valve *V*. When the piston reaches the front end of the cylinder, steam is admitted through the port at that end, the port at the rear end then being open to the exhaust.

It should be noted that the pressure at the exhaust is atmospheric or about 15 lb. to the square inch, while the pressure of the live steam may be from 50 to 200 lb. per square inch, according to the character of the engine used and the power required.

Figure 94 gives five different positions of the piston and valve, showing the various events during one stroke of the piston: —

1. The admission of the steam through one port, the other port being open to the exhaust, thus beginning the stroke.

2. The steam port open wide at about one fifth of the forward stroke.

3. The cut-off, *i.e.* the steam port, closed when expansion of the steam in the cylinder begins, the other port still being open to the exhaust.

4. The steam port still closed and the other port closed to the exhaust, the remaining steam in the front end being compressed, thus cushioning the stroke of the piston, bringing it gradually to rest and at the same time rapidly increasing the pressure in this end of the cylinder to a value approaching that of the live steam.

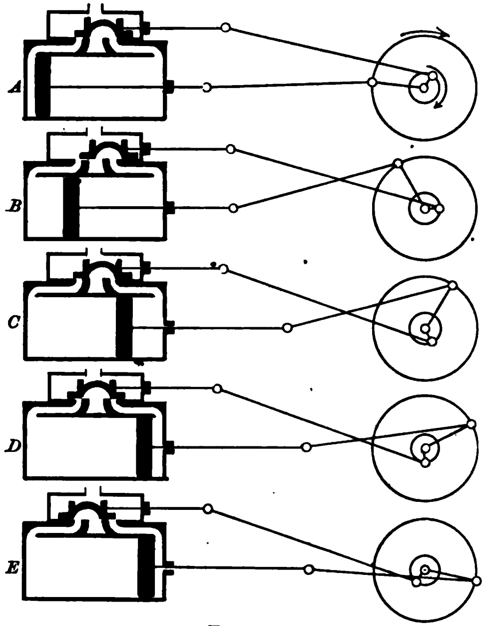


FIG. 94.

5. The beginning of the back stroke, live steam having been admitted through the other port at the end of the forward stroke.

260. Graphical Representation of the Steam Pressure during the Stroke. — The value of the steam pressure during these successive stages of the stroke may be represented by an *indicator diagram* in which the distance along the horizontal axis represents the length of the stroke of the piston, and the vertical distance above the horizontal axis (which represents atmospheric pressure) shows the pressure of the steam behind the piston.

The straight line AB (Fig. 95) indicates full pressure of steam during the stages 1 and 2, the hyperbola BC represents the decreasing pressure as the steam works expansively in stage 3; CD represents the further decrease in pressure due to the gradual exhaust and the decrease to nearly atmospheric pressure; the straight line DE indicates practically atmospheric pressure during the first part of the back stroke, the exhaust being closed at E; EF shows the rapid rise in pressure during compression; the vertical line FA indicates the admission of live steam and a return to boiler pressure, the admission of steam having begun at a point between E and F, but near F.

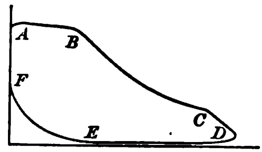


FIG. 95.

261. Gas Engine. — In this form of engine a mixture of air and an inflammable gas, such as gasoline vapor, or illuminating gas, is received into the cylinder of the engine

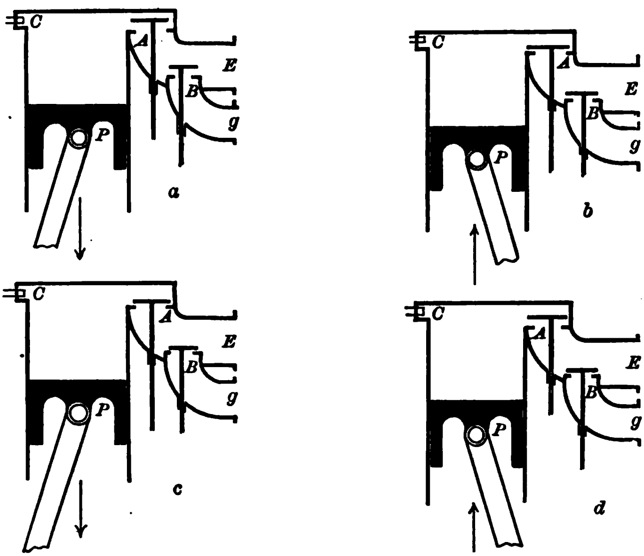


FIG. 96.

where, upon being ignited by an electric spark, hot tube, or an open flame, it is exploded, producing gases at high temperature and consequently at high pressure which move the piston forward. It differs from a steam engine in that the gases at high pressure are *produced within the cylinder* and that the gas pushes upon but one side of the piston, being admitted into one end of the cylinder only.

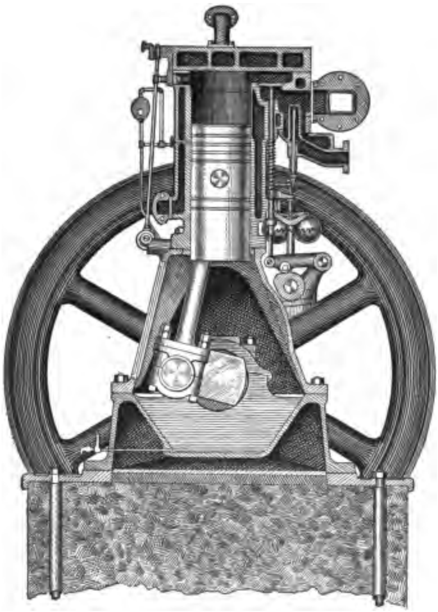


FIG. 97.

Figure 96 shows (in a simplified form) the cylinder of a "four-cycle" gas engine.

The gas issues from the port guarded by the valve *B*, and moves with a current of air into the cylinder through the port guarded by valve *A*. This port also serves as the exhaust during the last half of each second double stroke.

Figure 96 *a* shows the piston moving downward with ports *A* and *B* open, through which the explosive mixture enters the cylinder during this stroke. During the second stroke shown in Fig. 96 *b*, the piston is moving upward; ports *A* and *B* are both closed, and the explosive mixture is being compressed. At the beginning of the third stroke shown in Fig. 96 *c* this mixture is exploded by the igniter *C* and the high pressure gases thus produced force the piston downward, ports *A* and *B* both remaining closed. During the fourth stroke shown in Fig. 96 *d*, port *B* is closed, but *A* is open and the piston moving upward forces the expanded gases out through port *A* as the exhaust.

The momentum of the heavy fly wheel shown in Fig. 97 carries the piston through the remaining three strokes after each explosion within the cylinder.

The four cycles may be briefly described thus:—

First, Admission;
Second, Compression;
Third, Explosion (Power Stroke);
Fourth, Expulsion (Exhaust).

PROBLEMS

1. Reduce to F. reading: (*a*) 20° C., (*b*) −40° C., (*c*) 100° C., (*d*) 36° C.

Ans. { (*a*) 68° F. (*c*) 212° F.
(*b*) −40° F. (*d*) 96.8° F.

2. Find the difference in temperature between 10° C. and 10° F.: (*a*) in C.°, (*b*) in F.°

Ans. (*a*) $22\frac{2}{9}$ C. degrees. (*b*) 40 F. degrees.

3. Reduce to C. reading: (*a*) 62° F., (*b*) 98° F., (*c*) 32° F., (*d*) 0° F.

Ans. { (*a*) $16\frac{2}{3}$° C. (*c*) 0° C.
(*b*) $36\frac{2}{3}$° C. (*d*) $-17\frac{7}{9}$° C.

4. Reduce to absolute temperature reading: (*a*) 0° C., (*b*) −20° C., (*c*) 40° C., (*d*) 100° C.

Ans. { (*a*) 273° A. (*c*) 313° A.
(*b*) 253° A. (*d*) 373° A.

5. A brass rod is 59.8 cm. long at 20° C. and 59.886 cm. long at 98° C. Find the coefficient of linear expansion of brass. *Ans.* .0000184.

6. A certain mass of gas occupies a volume of 78 cc. when the temperature is 25° C. and the pressure is 752 mm. of mercury. Find its volume under standard conditions, *viz.*, 0° C. and 760 mm. pressure. *Ans.* 66.3 cc.

7. The volume of a certain mass of air at 60° C. was found to be 94.1 cc. Its volume at 10° C. is 80 cc. Find the coefficient of expansion of air. *Ans.* .00365.

8. When 100 gms. of aluminum (sp. ht. = .21) at a temperature of 80° C. is placed into 150 gms. of water at a temperature of 10° C., what will be the temperature of the mixture? *Ans.* 18.6° C.

9. How many units of heat are needed to raise the temperature of 120 gm. of water from 18° C. to 42° C.? *Ans.* 2880 cal.

10. The mass of a calorimeter is 110 gm. and its sp. ht. = .1. The mass of water in the calorimeter is 150 gm. and its temperature is 10°. A coil of wire is placed in the calorimeter whose mass is 2 gm. and sp. ht. = .09. An electric current is sent through the coil of wire for half an hour, when it is found that the temperature of the calorimeter and contents is 28° C. (*a*) How much heat was produced by the electric current per second? (*b*) What power is expended in the wire by the electric current? *Ans.* (*a*) 1.61 calories. (*b*) 6.762 watts.

11. Find the heat of fusion of ice from the following:—

Mass of calorimeter	110 gm.
Sp. ht. of calorimeter	.1
Mass of water in calorimeter	150 gm.
Temp. of water just before putting in ice .	80° C.
Temp. of water when ice is melted . .	36° C.
Mass of ice put into the water . . .	61 gm.

Ans. 80.13 cal.

12. With what velocity must a lead bullet (sp. ht. = .033), whose mass is 20 gm., strike a target so that its temperature is raised 100° C., assuming that all of the energy of the bullet is transformed into heat and that it is all absorbed by the bullet? *Ans.* 166.5 meters per sec.

13. What is the specific heat of a substance whose temperature falls 60° in raising the temperature of the same mass of water 12°?

Ans. .2 cal.

14. If 150 gm. of water at 50° C. are poured upon 60 gm. of ice at 0° C., what will be the resulting temperature? *Ans.* 12.9° C.

15. Find the heat of vaporization of water from the following data:

Mass of calorimeter	110 gm.
Sp. ht. of calorimeter	.1
Mass of calorimeter and cool water . . .	260.9 gm.
Temperature of cool water	18.5° C.
Temp. after passing steam into the cool water	40.1° C.
Mass of calorimeter and water . . .	267 gm.

Ans. 513.4 cal.

16. What is the relative humidity when, the temperature of the air being 22° C., the dew-point is 4° C.? *Ans.* .325.

17. If, when the temperature of the air is 85° F., the relative humidity is .70, what is the dew-point? *Ans.* 74.5° F.

18. A copper ball whose mass is 3 kgm., taken from a furnace and plunged into 8 kgm. of water at 10° C., heated the water to 25° C. If

the sp. ht. of copper is .09, what is the temperature of the furnace? *Ans.* 469.4° C.

19. A calorimeter containing 500 gm. of water and 250 gm. of ice, all at 0° C., has steam at 100° C. introduced till half the ice is melted. (*a*) What will be the weight of the water and ice then? (*b*) Would more or less steam have been required if 1000 gm. of water had been in the calorimeter? *Ans.* (*a*) 765.7 gm. (*b*) Neither more nor less.

20. For the purpose of keeping one's feet warm on a sleigh ride, which is preferable, a 10-lb. hot-water bottle or a 10-lb. plate of iron, both at the same temperature, say 100° C. at the start? Why?

21. Two basins are placed side by side on a hot stove, and in one is placed an 8-pound flatiron, in the other 8 lb. of water, both the iron and water being at the temperature of the room. Having placed one hand on the iron and the other on the water, which hand would you be compelled by the heat to remove first? Why?

22. If some way were discovered of making infusible ice, how would it affect its value for cooling purposes? Why?

23. (*a*) If, on a cold night, a complete, heavy, woolen suit were put on a marble statue out of doors, how would the temperature of the statue be affected in the course of a few hours, the temperature of the air remaining constant, say 0° C.? Why? (*b*) What effect would the suit have on the temperature of the statue if the temperature of the air changed? (*c*) What, then, is the function of "warm" clothing?

CHAPTER XI

LIGHT

262. Definition of Light. — *Light is that portion of radiant energy which is capable of causing the sensation of sight. Radiant energy* is the term applied to kinetic energy acting outside of ordinary matter in what is called the *ether.* This *ether* fills all space, both intermolar and intermolecular, not occupied by ordinary matter; is exceedingly elastic; is incompressible, and apparently offers no resistance to the passage of ordinary matter through it. Energy also exists in the ether in the potential form, as, for example, in the magnetic field, in which case the energy is embodied in a strained condition of the ether.

263. Propagation of Radiant Energy. — Radiant energy is transferred through the ether by a progressive vibratory, or wave, motion of the ether. These waves differ greatly as to amplitude and frequency, but they all travel with the same velocity in the ether, *viz.*, 3×10^{10} cm., or about 186,000 mi. per sec.

264. Production and Effects of Ether Waves. — The ether waves are produced and communicated to the ether by disturbances occurring in ordinary matter in touch with it. Once formed, they travel outward in every direction, *i.e. radiate*, until perchance they are intercepted by a body of ordinary matter which may absorb their energy, whereupon there is produced in the matter a disturbance similar to that by which the waves were formed and sent out on their

journey. The character of the wave will depend on the nature of the disturbance in the sending matter; and the nature of the disturbance produced in the receiving matter will, in turn, depend on the kind of ether wave that reaches it, and on the degree to which the matter may respond thereto. The length of the ether wave has much to do in determining the kind of effect produced when it impinges on ordinary matter. Many of the waves are exceedingly short and it is convenient to measure them by a unit called the *micron*, which is the one millionth of a millimeter.

265. Heat, Light, and Other Ether Waves Contrasted. — If ether waves are not longer than 720 microns, or shorter than 380 microns, they produce the sensation of sight when they strike upon the retina of a normal eye. Other effects caused by radiant energy are *chemical*, *thermal* (radiant heat), and *electrical*.

Waves that are too short to cause the sensation of sight, as well as the shorter ones that do cause this sensation, are very *active chemically*, *i.e.* are readily transformed into *atomic* disturbances, the motion of the atom, the chemical unit of matter, being closely attuned to these short waves. Thus it is that a photograph may be made by means of radiant energy which would not affect the eye.

Again, the longer waves that cause the sensation of sight, together with those that are still somewhat longer than these, readily *produce heat*, *i.e.* cause molecular agitation when received into ordinary matter, the motion of the molecule responding sympathetically to these waves.

Electrical effects are produced by radiant energy in general and those portions which do not affect the *eye* or the *atom* or the *molecule* directly must be studied through the electrical phenomena which they may be made to produce.

266. Character of Ether Waves. — In general the wave motion of the ether is *transverse*, *i.e.* the vibratory motion is at right angles to the line of propagation. A *wave front* is the locus, within a given wave, of particles *in the same phase* of vibration. The *advance wave front* may be considered as the locus of particles just on the point of being disturbed and the term *wave front* generally refers to this locus.

A *wave length* (as in sound) is the distance between successive wave fronts of like phase.

When the source of light is a *point* within a medium of uniform density, the wave front is a spherical surface surrounding the source as a center, and the wave in this case is called a *spherical wave.* If the source is at a great distance, a small portion of the spherical surface does not differ essentially from a plane and the wave front may be considered a plane. This form of wave is called a *plane wave.*

267. Ray, Pencil, and Beams of Light. — A *ray* is a line normal to a wave front. A bundle of rays belonging to a spherical wave front is conical in shape and is called a *pencil* of light; while in a bundle belonging to a plane wave front the rays are parallel and this is called a *beam* of light.

268. Luminous and Illuminated Bodies.—A body emitting light generated *within itself* is called a *luminous* body. All bodies which are made visible by light received from an outside source are called *illuminated* bodies.

269. Apparent Position of a Body. — Objects are generally seen by means of light coming from them through the air, and since experience shows that light as a rule

travels in straight lines, the mind persists in assigning the object's direction from the observer in the line along which the light immediately enters the eye. And although the mind may come to understand perfectly well that the light does not, in many cases, travel the entire distance from the body in a straight line, it nevertheless fully accepts the deception without which there would be no appreciation of the picture seen in the looking-glass, or of the apparent bending of a spoon in a glass of water.

270. Sighting and Sight Line. — Sighting is determining the line of direction, or *sight line*, from the observer's eye to an object point. Since two points determine a straight line, to establish a particular sight line it is only necessary to fix, at convenient places, two points in the line of sight to the object point; then a line drawn through these points will if prolonged pass through the object point.

If a second sight line to the same object point is established in like manner, it is evident that the intersection of the two sight lines is the exact location of the object point.

Binocular vision, or seeing with both eyes, makes use of this principle in locating near-by objects.

271. Visual Angle — Apparent Size of a Body. — The visual angle of a (linear) object is the angle formed by two sight lines, one to each extremity of the object, as shown in Fig. 98.

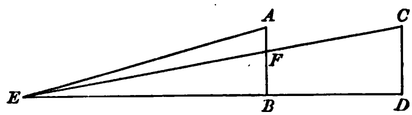


FIG. 98.

$\measuredangle AEB$ is the visual angle of the object AB, and $\measuredangle CED$ of the object CD or of object FB. From the similarity of the $\triangle FEB$ and CED,

$$FB : CD = EB : ED. \tag{1}$$

That is, for two objects having the same visual angle their lengths are directly proportional to their distances from the observer. Hence, if the relative distances are known the relative lengths may be easily estimated, and *vice versa*. Again, let $AB = CD$, then

$$FB : AB = EB : ED, \tag{2}$$

where FB is the apparent length of CD relative to AB.

Hence, for two objects of the same length their apparent lengths are inversely proportional to their distances from the observer.

These simple principles greatly aid the eye in determining the size and distance of objects, and are fundamental in the rules of perspective. The eye is quite helpless in dealing with such a problem if both size and distance are unknown, as is so strikingly illustrated in attempting to estimate the size and distance of the moon.

272. Parallax. — Of two objects, the one nearer a moving observer changes its direction from him faster than the more remote one which, therefore, seems to move *with* the observer while the nearer appears to move in the *opposite* direction. *Parallax* is the apparent relative displacement of two points unequally distant from an observer, due to his motion across the line passing through the points. It is measured by the angle between this line and the line passing through the observer and the nearer point.

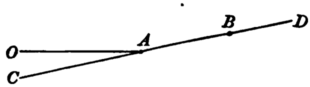


FIG. 99.

In Fig. 99, CD is the line through the points A and B unequally distant from the observer O; OA is the line through the observer and the nearer point A; and $\measuredangle\, OAC$ measures the apparent relative displacement, or parallax, given to the points A and B by the observer having changed his position from C to O.

It is evident that: —

1st. When the observer is in line with the two points the parallax is zero, for the measuring angle is then zero.

2d. The parallax increases as the observer moves away from this line and around the nearer point as a center.

3d. In order that any other point shall have the same parallax as the nearer point it must coincide with the nearer point.

273. Use of the Principles of Parallax.—These principles of parallax may be used to determine the distance of a point from an observer when direct measurement is impossible, or to locate the image formed by a lens or mirror. Since an image does not consist of matter it is more difficult to fix its exact location than in the case of a material body. If, however, a small object such as a pin, is so placed as to have the same parallax as the image has, then by the third principle, the position of the pin coincides with that of the image and may be taken as the location of the image for any measurements.

Error due to parallax must be avoided in reading the position of a pointer or index on a fixed scale with which it is not quite in contact. Since the true reading is a point on the scale *perpendicularly behind* the index, the eye must be placed in the line through this point and the index; then there will be zero parallax. This rule must be observed in order to make a correct reading of any instrument of measurement as the ruler, clock, thermometer, ammeter, etc.

274. The Pin-hole Camera. — A pin-hole camera is a box having a small hole in the center of one side. If this side is turned toward a brightly illuminated object a picture of the object may be seen on the inner surface of the opposite side. This picture will be a faithful reproduction of the object as to form and color, but since the rays of light entering the box through the hole travel in

straight lines, they cross at the hole, as shown in Fig. 100, and consequently the picture is inverted and reversed. Further, from the similar ⧍ AOB and $A'OB'$, $AB : A'B' = M : N$, *i.e.* the size of the object and that of the picture are proportional to their respective distances from the pin hole.

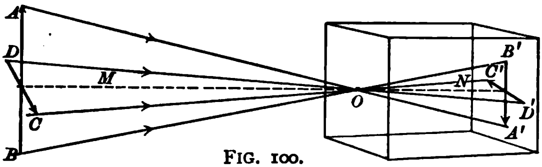


FIG. 100.

275. Transparent, Translucent, and Opaque Bodies. — According to the degree to which they transmit light, substances are classified as *transparent*, *translucent*, or *opaque*: transparent, if they permit of distinct vision through them; translucent, if they permit of indistinct vision; opaque, if no light is transmitted through them.

276. Shadows. — A shadow is a portion of space darkened by the interposition of an opaque body between this space and a source of light. What is usually spoken of as a "shadow cast" on the wall, or on the ground, is simply a *section* of the true shadow. When one stands in the shade of a tree or sits on the shady side of a building, he is then within the shadow proper.

If the source of light were a geometric point, the shadow would be uniformly dark, but as all luminous bodies have dimensions, only a part of the space is completely cut off from the light, while the remainder receives light from a portion of the luminous body. Hence shadows have two parts: (1) the *umbra*, or the part from which all light is cut off; (2) the *penumbra*, the part from which only a portion of the light has been excluded.

277. Size and Shape of a Shadow. — The size and shape of a shadow are determined by three conditions: (1) the size and shape of the opaque body, (2) its distance from the source of light, and (3) the shape of the cross section of the opaque body, as made by a plane cutting it perpendicular to the line joining the luminous and the opaque bodies.

278. Shadows and Eclipses. — Since the planets and satellites are spherical, opaque, nonluminous bodies at different distances from the sun, which is a larger spherical luminous body, it follows that each planet or satellite has a shadow extending from it in a direction away from the sun. The umbra and penumbra of such a shadow are respectively defined by the externally and the internally tangent conical surfaces, a longitudinal section of which is shown in Fig. 101. If one of these bodies passes into the shadow of another it suffers an eclipse, either total or partial: total, if within the umbra, and partial, if within the penumbra.

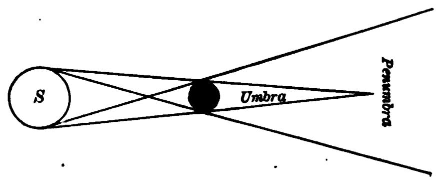


FIG. 101.

279. A Method of Determining the Velocity of Light. — The velocity of light in the ether was first measured by Römer, a Danish astronomer, at Paris in 1676, by means of observations of the time interval of successive eclipses of the largest of Jupiter's moons. The orbits of the Earth and of Jupiter are concentric, Jupiter being the more distant planet. The distance from the Sun to the Earth is 93,000,000 mi., to Jupiter, 480,000,000 mi. The Earth takes 1 year to make one revolution about the Sun; Jupiter

makes one revolution in 12 years. Hence, if at a given time the Earth and Jupiter are in conjunction, *i.e.* are on the same side of the Sun and in the same straight line with it, about $6\frac{1}{2}$ months later they will be in opposition, *i.e.* on opposite sides of the Sun and in the same straight line with it. This moon of Jupiter revolves about it and passes into its shadow at regularly recurring intervals, thus suffering eclipse. The interval between two successive eclipses was observed when the Earth and Jupiter were in conjunction, and from this observation the time of each successive eclipse was calculated. It was observed, however, that the eclipses occurred a little later than the time calculated, the delay gradually increasing until the Earth and Jupiter were in opposition, when the difference in time was $16\frac{2}{3}$ min. From that time on this delay decreased until they were again in conjunction, when the observed and the calculated times again coincided.

It thus became evident that the time at which the eclipse takes place is earlier than the time at which it is *seen* to occur by an observer on the Earth, by the length of time it takes light to travel from Jupiter's moon to the observer on the Earth. Since this distance is 186,000,000 mi. (the diameter of the Earth's orbit) *greater* when the two planets are in opposition than when in conjunction, the maximum delay of $16\frac{2}{3}$ min. must be the time required for light to travel 186,000,000 mi. $16\frac{2}{3}$ min. = 1000 sec.; therefore the velocity of light in ether is 186,000,000 ÷ 1000 = 186,000 mi. per sec. Stated in C. G. S. units, this velocity is 3×10^{10} cm. per sec.

280. Intensity of Illumination and Intensity of Light. — The *intensity of illumination of a surface* must be carefully distinguished from the *intensity of light of an illuminant.*

The *intensity of illumination* (I) is the quantity of light received per unit area. The *intensity of light* (L) of a given illuminant is the ratio of the rate of emission of light by the illuminant to the rate of emission by a standard candle, and is expressed as so many *candle power.* The *intensity of illumination* (I) may be found by dividing the total quantity of light (L), received on a spherical surface about the illuminant as a center, by the area, a, of the spherical surface, *i.e.* $I = \frac{L}{a}$.

281. Relation of Intensity of Illumination to Intensity of Light and to Distance. — Light waves emanating from a point source are, like sound waves, spherical shells, and the quantity of light received by each expanding shell is the same as that of the preceding shell. The area (a) of a shell increases as the square of the radius (r), or distance (d), from the point source increases, —

i.e. $$a = 4\pi r^2 = 4\pi d^2; \text{ therefore } I = \frac{L}{a} = \frac{L}{4\pi d^2}.$$

This means that *the intensity of illumination varies directly as the intensity of the light emitted by, and inversely as the square of the distance from, the source:* $I \propto \frac{L}{d^2}$. This relation is another instance of the *law of inverse squares.*

282. The Principle of Photometry. — If a surface is *equally* illuminated by light from two illuminants, L_1 and L_2, whose respective distances from the surface are d_1 and d_2, then —

$$I = \frac{L_1}{4\pi d_1^2} = \frac{L_2}{4\pi d_2^2}, \text{ whence } L_1 : L_2 = d_1^2 : d_2^2,$$

i.e. the intensities of the illuminants are *directly* proportional to the *squares* of their respective distances from the

equally illuminated surface. This principle is made use of in the *photometer*, which is an instrument for measuring the intensity or candle power of any illuminant.

There are two principal types of photometers, the Bunsen and the Rumford. In the Bunsen photometer (Fig. 102) a

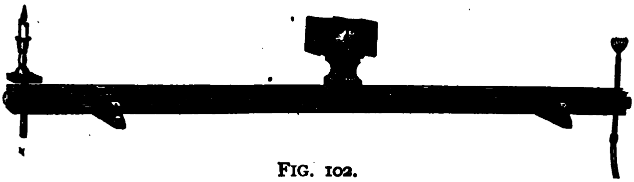


FIG. 102.

translucent screen is moved to such a point between the given illuminant and the standard illuminant that its two surfaces are equally bright; then, if L_1 is a standard candle, $L_2 = \frac{d_2^2}{d_1^2}$ *candle power.* In the Rumford photometer (Fig. 103) the two illuminants are placed at such distances from

FIG. 103.

a white surface that the two shadow sections cast by an object on that surface have equal intensities of illumination; then, as with the other instrument, $L_2 = \frac{d_2^2}{d_1^2}$ *candle power,* if L_1 is a standard candle.

PROBLEMS

1. At what distance from the eye would a quarter of a dollar subtend the same visual angle as a nickel held 35 cm. from the eye, the diameters of the two coins being 25 and 19 mm., respectively? *Ans.* 46 cm.

2. A street lamp is 8 ft. above the pavement. Find the length of shadow cast by a man 6 ft. tall when 6 ft. from the lamp. *Ans.* 18 ft.

3. How far from the moon does the umbra of its shadow extend if the distance from the sun to the moon is $93\frac{1}{4}$ million miles, the diameters of the sun and moon being 800 thousand and 2 thousand miles, respectively? *Ans.* 237,468 mi.

4. Compare the quantities of light received by a photographic negative held successively 25 and 40 cm. distant from an electric arc.

Ans. 2.56 to 1.

5. A standard candle is 300 cm. from a 16 c.p. electric lamp; where should a translucent screen be placed between them to receive equal illumination from the two lights? *Ans.* 60 cm. from the candle.

6. If a kerosene lamp is 65 cm. and a standard candle 35 cm. from a translucent screen when it is equally illuminated by the two lights, what is the intensity of the light from the lamp? *Ans.* 3.44 c.p.

CHAPTER XII

REFLECTION OF LIGHT

283. Light Incident on a Surface. — When light strikes the surface of matter it is said to be *incident* on that surface. A part of the incident light is turned back or *reflected* from the surface, while the remainder passes into the matter and is either transmitted through it or else transformed by it into some other form of energy, as heat, or chemical action.

284. Reflection. — When light is incident on a smooth polished surface, reflection is *regular* (Fig. 104), *i.e.* the reflected portion of a beam or pencil of light is not scattered promiscuously as it is when the reflecting surface is rough or unpolished (Fig. 105), in which case the reflected light is said to be *diffused*.

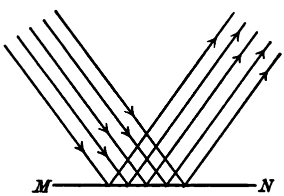


FIG. 104.

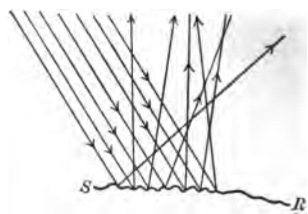


FIG. 105.

The visibility of nonluminous objects depends upon the diffusion of light incident on their surfaces. A surface from

which the reflection of light is perfectly regular is itself quite invisible.

285. Effect of Reflection of a Light Wave. — Let MN, Fig. 106, be a plane polished surface of an opaque body. Let O be a source of light, and let F_1F_1, F_2F_2, F_3F_3, etc., be successive wave fronts. When the front a_1 of wave front F_6F_6 strikes the mirror MN, it is bent back, traveling

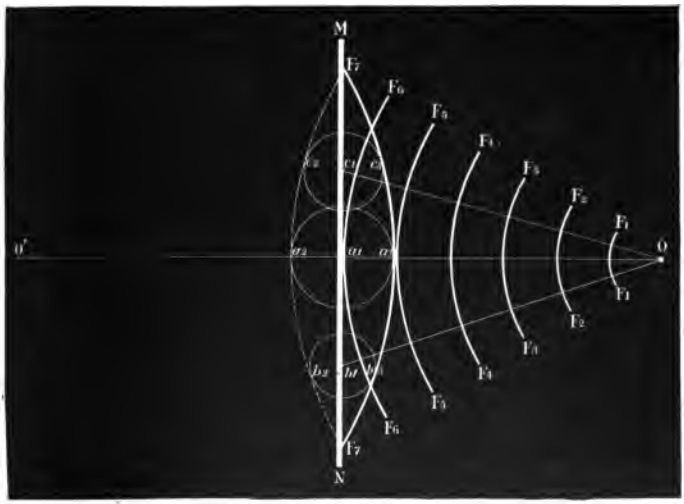


FIG. 106.

the distance a_1a_3 equal to the distance a_1a_2 it would have traveled in the same time if the surface had not turned it back. Similarly the front b_1 travels the distance b_1b_3 equal to b_1b_2 and c_1 travels the distance c_1c_3 equal to c_1c_2. If about a_1 as a center a circle of radius a_1a_2 is drawn, and if about points b_1 and c_1 also circles are drawn of radii b_1b_2 and c_1c_2 respectively, it is evident from the symmetry of the figure that the reflected wave front $F_7c_3a_3b_3F_7$, drawn

tangent to these circles, is a portion of another spherical surface similar to the front $F_7c_2a_2b_2F_7$, which would have existed had the wave not been reflected, and having O' as its center.

It follows then that the point O' from which the reflected wave seems to come is situated as far behind the mirror as is the original source O in front of it.

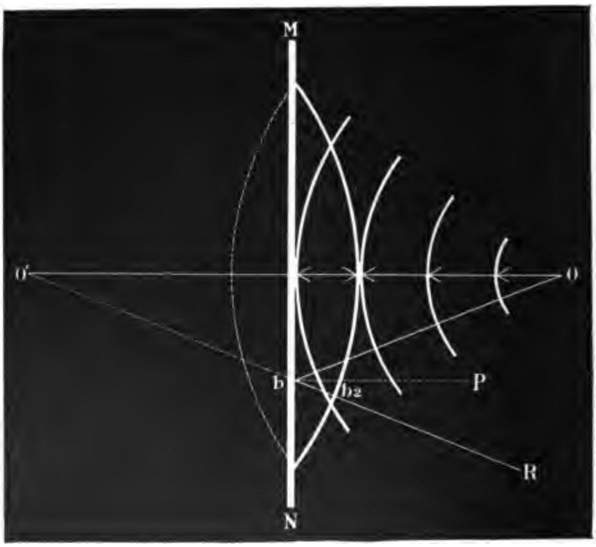


FIG. 107.

286. Proof of the Law of Reflection. — Light from a point b_1, Fig. 107, in a wave front after reflection travels in the direction b_1b_2 as if it came from the point O'. The line OO' joining the centers of the two equal intersecting spheres is perpendicular to the mirror line MN, which is their common chord. Draw a perpendicular Pb_1 to the mirror at the point of incidence. The triangle $OO'b_1$ is

isosceles since its base OO' is bisected at right angles by the line MN Pb_1 is parallel to OO', being perpendiculars to the same line MN. Therefore $\measuredangle Pb_1O = \measuredangle O$ (alt. int. $\measuredangle$s) and $\measuredangle Pb_1b_2 = \measuredangle O'$ (corresponding $\measuredangle$s). But $\measuredangle O = \measuredangle O'$ (base $\measuredangle$s of isos. $\triangle$). Therefore $\measuredangle Pb_1O = \measuredangle Pb_1'R$.

The ray Ob_1 is the *incident* ray and b_1R is the corresponding reflected ray. The angle Ob_1P formed by the incident ray and the normal at the point of incidence is the *angle of incident;* while the angle Rb_1P formed by the reflected ray and the normal is the *angle of reflection.*

Therefore, when light is reflected, each incident ray makes the same angle with the perpendicular to the surface at the point of incidence that its corresponding reflected ray makes with the same perpendicular, and the angles are situated on opposite sides of the perpendicular. This relation is commonly stated as the *Law of Reflection:* —

The angle of incidence equals the angle of reflection and the incident ray, the reflected ray and the normal to the mirror at the point of incidence all lie in the same plane.

287. A Second Proof of the Law of Reflection. — Let EFG, Fig. 108, be a portion of the front of a plane light wave which is incident on the reflecting surface MN. At A, where the wave front AB first strikes the surface, the wave is reflected and the light from A travels a distance equal to BD during the time the light from B travels to D. With A as a center and a radius equal to BD, draw an arc. The reflected light from A will travel to some point of this arc during this time. In the same way, from Q draw an arch with a radius equal to SD, SQ being perpendicular to FQ; the reflected light from Q will reach some point of this arc at the instant the light from S reaches D. In the same manner with other rays draw arcs with radii as is

indicated in the figure. The reflected light will reach each of these arcs at the instant that the light from B reaches D. The reflected wave front is therefore the line CRD drawn from D tangent to each of these arcs.

Draw the perpendiculars AP' and DP'' at the two points of incidence, A and D.

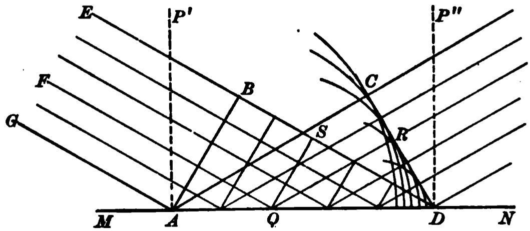


FIG. 108.

The rt. $\triangle$ ABD and ACD are equal, having a common hypotenuse AD and the arm AC equal to the arm BD.

$$\measuredangle BDA = \measuredangle CAD \text{ (hom. pts. of = } \triangle\text{)}.$$

$$\therefore \measuredangle BDP'' = \measuredangle CAP' \text{ (Ax. 3)}.$$

But $\measuredangle BDP''$ is the angle which the incident rays make with the perpendicular to the surface at the point of incidence, and $\measuredangle CAP'$ is the angle which the reflected rays make with this perpendicular; therefore the $\measuredangle$s of incidence and reflection are equal and lie in the same plane with the normal to the surface at the point of incidence.

288. Definition of Focus, Real and Virtual. — A *focus* is the apex of a pencil of light the rays of which have been made to pass through that point by a change in their direction. It is a *real focus* if the pencil of light thus formed is complete to the apex, and a *virtual focus* if the pencil is cut off short of the apex.

289. Definition of an Image. — *The image of a point object is the focus of light from that point. The image of*

a body object is the locus of the foci of light from all the points of the body.

The point O', Fig. 109, is the *image* of O, because it is the *focus* of light from O after being reflected by the mirror MN, and it is *virtual* because the reflected pencil of light $TLSK$ is cut off short of its apex O' by the plane of the mirror.

290. Geometrical Proof of the Location of the Image of a Point formed by a Plane Mirror. — Draw from the point object O, Fig. 109, any two incident rays, such as OT and OS. Erect the perpendiculars PT' and SP to the mirror at the points of incidence. Draw the corresponding reflected rays TL and SK making $\measuredangle 1 = \measuredangle 2$ and $\measuredangle 6 = \measuredangle 7$ (law of reflection). These rays *appear* to come from the point O', which is therefore the image of O.

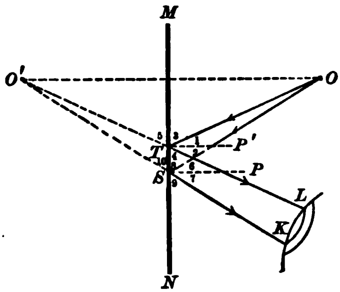


FIG. 109.

$\measuredangle 3 = \measuredangle 4$ (Ax. 3).

$\measuredangle 4 = \measuredangle 5$ (vertical $\measuredangle$s).

$\measuredangle 3 = \measuredangle 5$ (Ax. 1).

$\therefore \measuredangle O'TS = \measuredangle OTS$ (Ax. 3).

In a similar manner it may be proved that $\measuredangle 8 = \measuredangle 10$.

In the $\triangle$s OTS and $O'TS$

$$TS = TS \text{ (identity).}$$

$$\measuredangle OTS = \measuredangle O'TS.$$

$$\measuredangle 8 = \measuredangle 10.$$

$$\therefore \triangle OTS = \triangle O'TS.$$

$$\therefore OT = O'T \text{ (hom. sides} = \triangle\text{s).}$$

$$\therefore \triangle OTO' \text{ is isosceles.}$$

$\therefore$ Since $\measuredangle 3 = \measuredangle 5$, the mirror line MN bisects the line OO' at right angles. (The bisector of the vertical angle of an isosceles triangle bisects the base at right angles.)

Therefore the image O' of a point O, as seen in a plane mirror, is so situated that a line joining the image and object is *bisected at right angles* by the mirror.

291. Characteristics of an Image. — In order to fully describe an image it is necessary to determine its *characteristics* with reference to the object of which it may be called the optical counterpart. These characteristics are six in number: —

1. *Location:* referring to the respective positions of the object and image, whether on the same or opposite side of the mirror or lens.
2. *Kind:* real or virtual.
3. *Distance:* from the mirror or lens stated either in terms of the distance of the object or in terms of some length which is a property of the mirror or lens, such as its *focal length* or its *radius of curvature.* (§§ 295 and 300.)
4. *Size:* relative to that of the object, and which is generally dependent on (3).
5. *Arrangement vertically:* whether erect or inverted.
6. *Arrangement laterally:* whether exchanged right and left, or reversed.

292. Size of a Plane Mirror Image. — *A plane* mirror image is the same size as the object.

Let AB (Fig. 110) be the object and $A'B'$ its image.

Then $Ax = A'x$ and $By = B'y$ as previously shown.

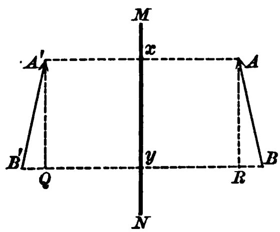


FIG. 110.

Draw AR and $A'Q \perp$ to BB'.

In $\triangle$s ARB and $A'QB'$

$AR = A'Q$ (opp. sides of $\square$).

$RB = QB'$ (Ax. 3)

(since $Qy = Ry = A'x = Ax$).

$\measuredangle R = \measuredangle Q$ (rt $\measuredangle$s).

$\therefore \triangle ARB = \triangle A'QB'$.

$\therefore AB = A'B'$ (hom. sides $= \triangle$s).

293. Arrangement of Image, Vertically and Laterally. — If one looks into a vertical plane mirror, he sees —

(1) that his image is vertically erect;

(2) that his right hand appears to be the left hand of his image and *vice versa*. This is expressed by saying that a plane mirror image is reversed.

The image of a tree on the bank of a quiet pool of water appears upside down, *i.e.* the image of a vertical object formed by a horizontal plane mirror is inverted.

294. Characteristics of a Vertical Plane Mirror Image. — A vertical plane mirror image has then the following characteristics: —

1. *Location:* on the side of the mirror opposite to the object.
2. *Kind:* virtual, following from (1), since the rays do not actually pass through the mirror.
3. *Distance:* equal to the distance of the object from the mirror, the plane of which bisects at right angles the lines joining each point of the object with its image.
4. *Size:* equal to the size of the object, following from (3), since all lines joining object points with their respective images are parallel.
5. *Arrangement vertically:* erect.
6. *Arrangement laterally:* reversed.

295. Reflection by a Concave Mirror. — A concave mirror is the polished *concave* surface of a small portion of a spherical shell.

Let MN (Fig. 111) be a section of a concave mirror whose center of curvature is at C. Suppose a light wave emanates from O as a source of light. When the point a_1 in wave front F_4 strikes the mirror, it is reflected back toward O since it strikes it perpendicularly. When the point F_4 reaches F_5, point a_1, which would have been at a_2 but for the mirror, is at a_3, which is such a point that $a_1a_2 = a_1a_3$. The reflected wave front $F_5a_3F_5$ is a *flatter* curve than Ma_1N; hence as the wave recedes from the mirror, it contracts toward a point I as a center which is *farther* from the mirror *than either O or C*. Beyond the point I the

reflected wave diverges. OF_5 is an incident ray, CF_5 is a radius of the mirror called a *radius of curvature*, and hence is perpendicular to it at F_5. Therefore OF_5C is the angle of incidence of the ray OF_5. F_5I is the direction this ray takes after reflection; hence it is the reflected ray from F_5, and CF_5I is the angle of reflection for this ray. By the law of reflection the angle OF_5C equals the angle CF_5I.

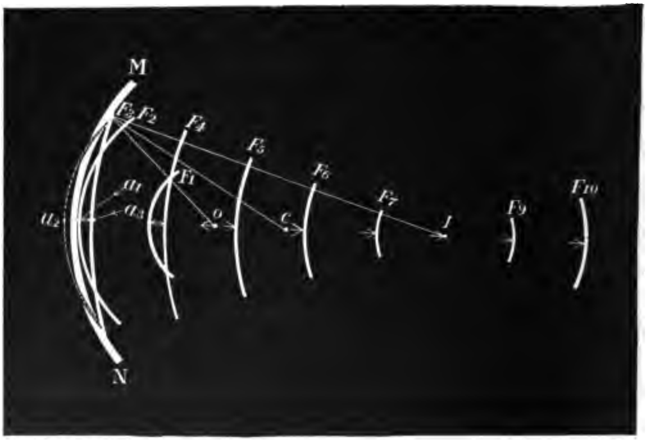


FIG. 111.

Since a focus is the apex of a pencil of light after reflection (or refraction), point I is the *focus* of rays from point O. If point I is the source of light, the wave front strikes the mirror first at F_5; then while retreating toward F_4 the point a_8 would advance to position a_1, and then be reflected, retreating toward a_8. Hence the reflected wave front would be F_4a_1, and the wave would converge to point O, which is therefore the focus of light from point I.

296. Conjugate Foci. — Two points, so related that if the object is placed at either of them the image is formed

at the other, are called *conjugate foci.* O and I are conjugate foci.

297. Principal Focus and Principal Axis. — The *principal focus* of a mirror is the point of convergence of rays which were parallel to the principal axis when incident. The *principal axis* is the line which passes through the *middle of the mirror* V and the *center of curvature* C, Fig. 112. Any other line passing through the center of curvature C, and any point of the mirror, is called a *secondary axis.*

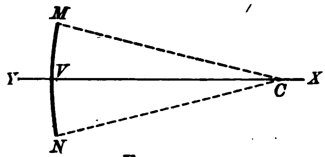


FIG. 112.

298. Aperture of a Mirror. — The *aperture* of a mirror is the angle formed at the center of curvature by two lines to opposite points on the margin of the mirror.

In Fig. 112 the angle MCN is the aperture, C being the center of curvature and XY the principal axis passing through the center of curvature C and the middle point of the mirror, V, called the vertex.

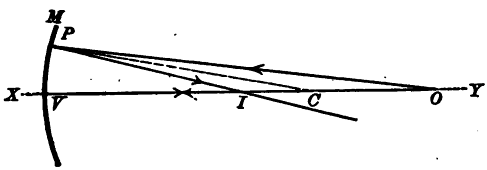


FIG. 112 *a*.

299. Relation between the Conjugate Focal Distances and the Radius of Curvature of a Concave Mirror. — In Fig. 112 *a* let MN represent a concave mirror, C its center of curvature, and XY its principal axis. If a source of light is placed at O, a ray from it incident on the mirror at P will be reflected, making the angle IPC equal to the angle

OPC. A second ray incident at V will be reflected back on itself since the angle of incidence is zero; the point of intersection I, of these two rays, is the position of the image of O; for all rays from O after reflection from the mirror intersect in one point.

In the triangle OPI the line CP bisects the angle OPI, therefore

$$OP : IP = OC : CI.$$

If the aperture of the mirror is small so that P is near V, OP will be practically equal to OV and IP to IM, so the proportion becomes

$$OV : IV = OC : CI.$$

Representing the distance from the mirror to the object by D_o, to the image by D_i, and to the center of curvature by R, the proportion may be written

$$D_o : D_i = D_o - R : R - D_i,$$

$$D_oR - D_oD_i = D_oD_i - D_iR,$$

$$D_iR + D_oR = 2\,D_oD_i.$$

Dividing by D_oD_iR,

$$\frac{1}{D_o} + \frac{1}{D_i} = \frac{2}{R}.$$

That is, *the sum of the reciprocals of the conjugate focal distances equals twice the reciprocal of the radius of curvature of the mirror.*

300. Relation between the Conjugate Focal Distances and the Principal Focal Distance (*focal length*). — If the source of light is very distant, so that the rays falling on the mirror are practically parallel, D_o being very large, $\frac{1}{D_o} = 0$, and the preceding equation then becomes,

$$\frac{1}{D_i} = \frac{2}{R}, \text{ whence, } D_i = \frac{R}{2}.$$

The point at which incident parallel rays are brought to a focus after reflection is by definition the *principal focus*, and the distance to this point from the mirror the *focal length* (f) of the mirror. The value of D_i just found is this distance, and is seen to be equal to one half the radius.

The focal length of a concave mirror is one half the radius of curvature, or the principal focus is midway between the vertex of the mirror and the center of curvature.

Since $\frac{1}{f} = \frac{2}{R}$, therefore $\frac{1}{D_o} + \frac{1}{D_i} = \frac{1}{f}$.

The sum of the reciprocals of the conjugate focal distances equals the reciprocal of the focal length of the mirror.

301. Foci of Concave Mirrors. — The relation of conjugate foci of concave mirrors may be summarized as follows: —

1. When $D_o = \infty$, $D_i = f$.
2. When $D_o < \infty > R$, $D_i > f < R$.
3. When $D_o = R$, $D_i = R$.
4. When $D_o < R > f$, $D_i > R < \infty$.
5. When $D_o = f$, $D_i = \infty$.
6. When $D_o < f$, D_i is negative and the image is virtual.

302. Principles of Geometrical Construction of Images formed by Concave (or Convex) Mirrors. — The image of a given object as formed by reflection from a concave (or convex) mirror may be determined by making use of the following principles of construction: (1) An axial ray, *i.e.* one passing along an axis, is reflected through the center of curvature. (2) A ray parallel to the principal axis is reflected through the principal focus, or conversely, a focal ray, *i.e.* one passing through the principal focus, is reflected in a direction parallel to the principal axis.

303. Application of the Principles. — It is only necessary to draw two such rays from an object point and find their intersection after reflection in order to locate the image; for all rays emanating from a given object point and reflected by the mirror are brought to the same focus (if the aperture is small).

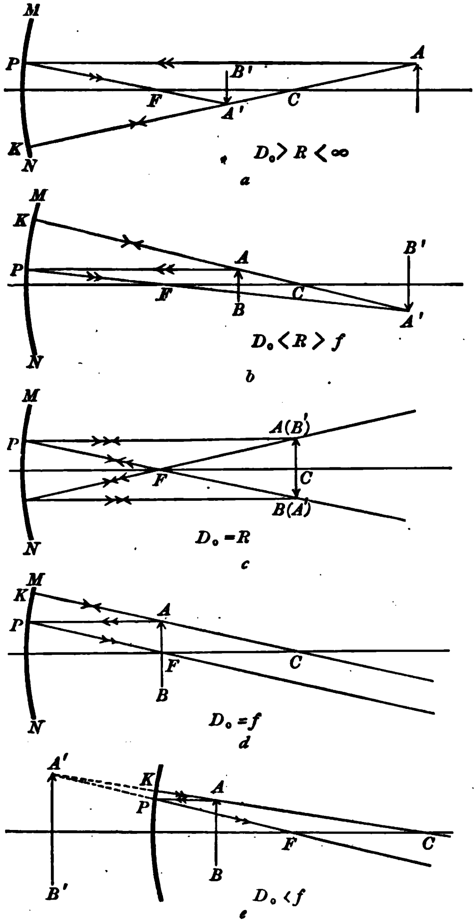


FIG. 113.

In the accompanying diagrams (Fig. 113 *a*, *b*, *c*, *d*, and *e*) let *AB* be the object, *AK* an axial ray which is reflected back on itself, *AP* a ray parallel to the principal axis, which after reflection becomes a focal ray, *i.e.* passes through *F*. The point of intersection, *A′*, of these rays, is the image of *A*. Similarly *B′*, the image of *B*, is found.

304. Characteristics of Images formed by a Concave Mirror. — 1. When the object is beyond the center of curvature (Fig. 113 *a*): —

(1) *Location:* on the same side of the mirror as the object.
(2) *Kind:* real.
(3) *Distance:* nearer the mirror than the object.
(4) *Size:* smaller than the object.
(5) *Arrangement vertically:* inverted.
(6) *Arrangement laterally:* reversed.

2. When the object is between F and C (Fig. 113 *b*):—

The characteristics (1), (2), (5), and (6) are the same as in 1.
(3) *Distance:* farther from the mirror than the object.
(4) *Size:* larger than the object.

3. When the object is at C the image is formed at the same place, is real, inverted, and of the same size (Fig. 113 *c*).

4. When the object is at F there is no image formed, since all the reflected rays are parallel (Fig. 113 *d*).

5. When the object is at infinity the image is a point and is located at F (Fig. 113 *d*).

6. When the object is between F and the mirror (Fig. 113 *e*):—

(1) *Location:* on the opposite side of the mirror to the object.
(2) *Kind:* virtual.
(3) *Distance:* farther behind the mirror than the object is in front of it.
(4) *Size:* larger than the object.
(5) *Arrangement vertically:* erect.
(6) *Arrangement laterally:* reversed.

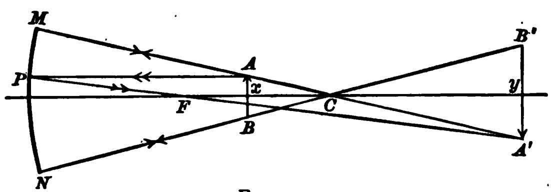


FIG. 114.

305. Relation of Size and Distance of Object and Image. — To determine the law for the relative size of the object

and image, as before, let $A'B'$ be the image of the object AB (Fig. 114). The $\triangle$ CAB and $CA'B'$ are similar. Therefore

$$AB : A'B' = Cx : Cy,$$

or

$$AB : A'B' = R - D_o : D_i - R.$$

But, by a previous proof,

$$D_o : D_i = R - D_o : D_i - R.$$

Hence,

$$AB : A'B' = D_o : D_i.$$

Let S_o represent the *size* (linear) of the object, and S_i the *size* of the image; then

$$S_o : S_i = D_o : D_i.$$

This means that *the size of the object and the size of the image are proportional to their respective distances from the mirror.*

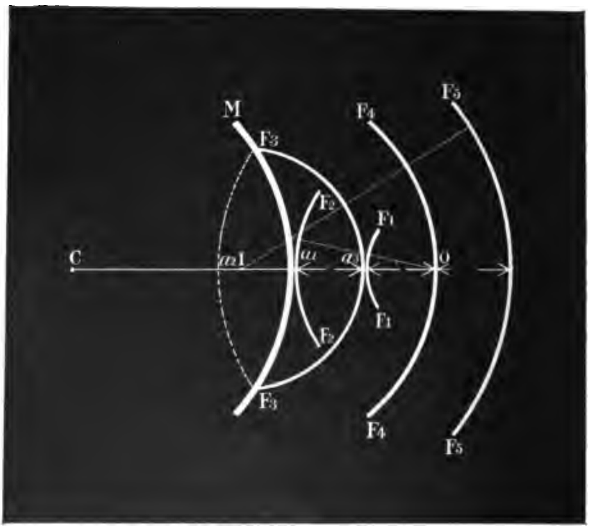


FIG. 115.

306. Reflection by a Convex Mirror. — A convex mirror is the polished *convex* surface of a small portion of a spherical shell.

Let MN (Fig. 115) be a section of a convex mirror with its center

of curvature at C. Let O be the source of a light wave whose successive fronts are F_1, F_2, F_3, F_4. When point a_1 of the wave front F_2 strikes the mirror, it is reflected back toward O. When the point F_2 reaches F_3, point a_1, which would have been at a_2 but for the mirror, has reached a_3, which is situated so that $a_1a_2 = a_1a_3$. The reflected wave front is therefore $F_3a_3F_3$, which is a *sharper* curve than Ma_1N, and this reflected wave consequently appears to diverge from a point I behind the mirror, but *nearer* the mirror than C. I is therefore the *virtual* focus of O.

307. Characteristics of a Convex Mirror Image. — Determining the image of AB (Fig. 116) formed by reflection

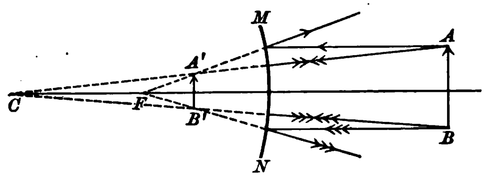


FIG. 116.

from a convex mirror, in a manner similar to that used with concave mirrors, the characteristics of the image are found to be as follows: —

1. *Location:* on the side of the mirror opposite to the object.
2. *Kind:* virtual.
3. *Distance:* nearer the mirror than the object.
4. *Size:* smaller than the object ($S_o : S_i = D_o : D_i$).
5. *Arrangement vertically:* erect.
6. *Arrangement laterally:* reversed.

308. Relation between Conjugate Focal Distances of a Convex Mirror. — As proved in the Appendix, § 461, this relation may be expressed by the equation —

$$\frac{1}{D_o} - \frac{1}{D_i} = -\frac{2}{R} = -\frac{1}{f}.$$

This equation differs from that for concave mirrors in that D_i and f are negative, the image and principal focus

always being situated on the opposite side of the mirror to the object.

309. Comparison of Virtual Images. — Comparing the characteristics of the *virtual* images of a concave mirror with those of a convex mirror and of a plane mirror, it will be seen that they are alike in four of the six characteristics, differing only in *distance* and its dependent characteristic *size*.

PROBLEMS

1. (*a*) What is the height of the smallest plane mirror in which a man 6 ft. tall can see his full-length image ? (*b*) If the man's eye is 68 in. above the floor, how far above the floor should the lower edge of the mirror be placed ? (*c*) If the man stands 5 ft. from the mirror, how far from him is his image ? *Ans.* (*a*) 3 ft. (*b*) 32 in. (*c*) 10 ft.

2. If a man sees his image in a plane mirror inclined 30° forward from a vertical wall, at what angle with the vertical is his image ? *Ans.* 60°.

3. An object 2 cm. high is placed 10 cm. in front of a concave mirror whose radius of curvature is 15 cm. (*a*) How far from the mirror must one be in order to see the image of this object, allowing a distance of 25 cm. between the eye and image to permit of distinct vision ? (*b*) How large is the image ? (*c*) What kind of image is it ? *Ans.* (*a*) 30 cm. + 25 cm. = 55 cm. (*b*) 6 cm. (*c*) Real and inverted.

4. At what two distances from a concave mirror whose radius of curvature is 25 cm. can an object be placed and have formed an image which is twice its size ? *Ans.* 18.75 cm. and 6.25 cm.

5. The focal length of a concave mirror is 12 cm. How far from the mirror must an object be placed to have formed an image of the same size ? *Ans.* 24 cm.

6. How far from a convex mirror whose focal length is 8 cm. should an object be placed so that the image is $\frac{1}{4}$ the size of the object ?

Ans. 24 cm.

CHAPTER XIII

REFRACTION OF LIGHT

310. Refraction.—When a light wave passes obliquely from one medium into another of different density, the direction of the wave is changed owing to the fact that light travels with different velocities in media of different densities. This change of direction of a light wave under these circumstances is called *refraction* of light.

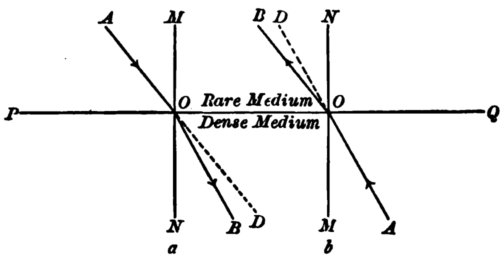


FIG. 117.

In Fig. 117 let PQ represent a plane, perpendicular to the paper, separating two media of different density as indicated. Let AO be an incident ray and MN a normal to the surface of separation at the point of incidence; then OB represents the refracted ray. The acute angle AOM is the *angle of incidence* and is measured between the incident ray and the normal. The acute angle BON is the *angle of refraction* and is measured between the refracted ray and the normal. The angle BOD is the *angle of deviation* and is measured between the refracted ray and the prolongation of the incident ray. If the incident ray is in the rarer medium, the angle of incidence equals the *sum*

of the angles of refraction and deviation; if it is in the denser medium, the angle of incidence equals the *difference* between these angles.

In Fig. 117 *a*, $\measuredangle MOA = \measuredangle NOD = \measuredangle NOB + \measuredangle BOD$.

In Fig. 117 *b*, $\measuredangle AOM = \measuredangle NOD = \measuredangle NOB - \measuredangle BOD$.

311. Illustration of Refraction. — To illustrate how the change of direction of wave front, and consequent change of direction of propagation, is caused, suppose a regiment of soldiers "in column of companies" (Fig. 118), *i.e.* the men in each company in line, shoulder to shoulder, and the companies one behind the other, to march diagonally across a public square one half of which is smoothly paved and the other half roughly paved as indicated in Fig. 118. Since it is more difficult to march on the rough than on the smooth pavement, the rate of progress will be correspondingly slower, and if the cadence is preserved, the steps will be shorter. In the supposed case the *right* of a company front advances upon the rough pavement first and therefore has its progress retarded earlier than its left. This will make the

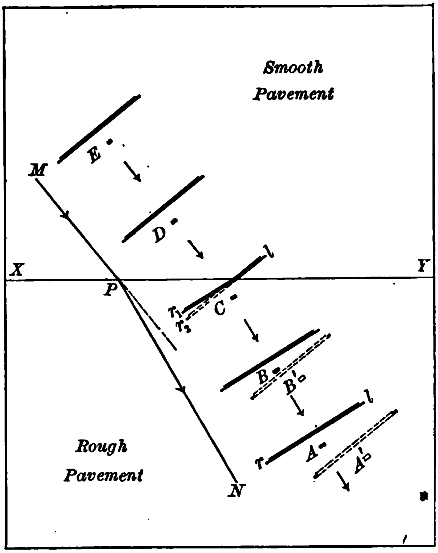


FIG. 118.

company front a broken line while in the act of crossing the line of separation of the two pavements, as company *C* in the diagram. When, however, the whole company has advanced on the rough pavement the company front will again be a straight line, as company *A* or *B*, but not parallel to the original company front, as company *D* or *E*, because the right has been under the retarding conditions longer than the left and as a consequence the "line of march," *i.e.* the direction of forward motion, will have been changed.

312. Index of Refraction. — Suppose a plane light wave to strike a surface of glass obliquely and that the velocity

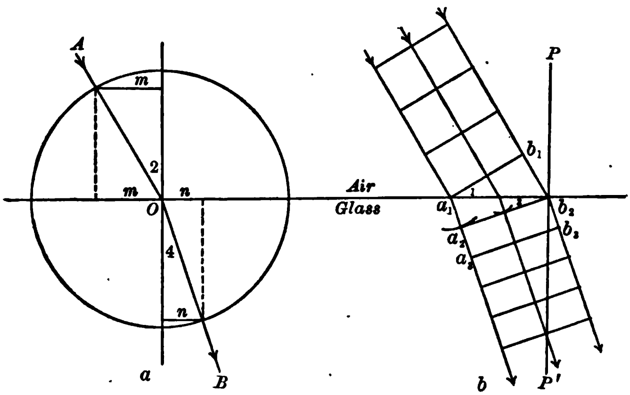


FIG. 119.

of light in glass is two thirds of its velocity in air. The line a_1b_1 (Fig. 119) is the wave front at the instant the point a_1 reaches the glass. While the light travels from b_1 to b_2 the portion which has entered the glass at a_1 will travel a distance equal to two thirds of b_1b_2.

With a_1 as a center and with a radius equal to two thirds of b_1b_2 describe an arc. From b_2 draw a line tangent to this arc. This line b_2a_2 will be the refracted wave front,

and the direction of the light in the glass will be perpendicular to this or along the line a_1a_2, or b_2b_3. Draw the normal PP' to the refracting surface at b_2. The angle b_1b_2P is the angle of incidence. The angle b_3b_2P' is the angle of refraction. The angle $b_1a_1b_2$ equals the angle b_1b_2P and the angle $a_1b_2a_2$ equals the angle b_3b_2P' since the sides are perpendicular to each other.

$$\frac{b_1b_2}{a_1b_2} = \text{sine of } \measuredangle\, b_1a_1b_2 = \text{sine of the angle of incidence.}$$

$$\frac{a_1a_2}{a_1b_2} = \text{sine of } \measuredangle\, a_1b_1a_2 = \text{sine of the angle of refraction.}$$

Therefore

$$\frac{\dfrac{b_1b_2}{a_1b_2}}{\dfrac{a_1a_2}{a_1b_2}} = \frac{b_1b_2}{a_1a_2} = \frac{\text{sine of } \measuredangle \text{ of incidence}}{\text{sine of } \measuredangle \text{ of refraction}} = \frac{3}{2}(=\mu).$$

The ratio of the *sines* of the angles of incidence and of refraction for any two given media is called the *index of refraction* for those two media, and its value is denoted by the Greek letter μ (pronounced *mū*).

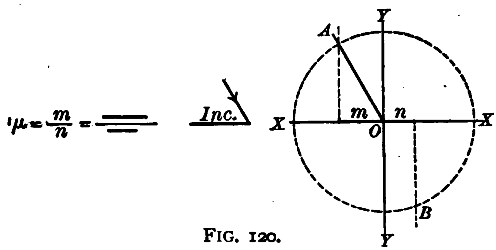


FIG. 120.

313. Graphic Construction of a Refracted Ray. — Representing the incident and refracted rays (Fig. 119 *a*) by single lines, parallel respectively to the incident and refracted paths of the plane wave, it is evident that, since

$\measuredangle 1 = \measuredangle 2$ and $\measuredangle 3 = \measuredangle 4$, $\frac{m}{n} = \frac{b_1b_2}{a_1a_2} = \frac{3}{2}$. This furnishes an easy means for constructing a refracted ray when the angle of incidence is given and the index of refraction, $\mu = \frac{m}{n}$, is known. The steps are suggested by the diagrams in Fig. 120. A line through B from O is the refracted ray.

314. Index of Refraction for Two Given Media. — For any two media the index of refraction for light passing from one into the other is the reciprocal of the index for light passing from the second into the first, *e.g.* the index of refraction from air to glass is $\frac{3}{2}$, from glass to air is $\frac{2}{3}$.

The index of refraction for the same two media is constant, irrespective of the size of the angle of incidence at which the light strikes the surface of separation, provided this angle is not zero. Snell's Law of Sines expresses this fact as follows: If homogeneous light is refracted at a plane surface separating two homogeneous media, the sines of the angles of incidence and of refraction bear a *constant* ratio to one another.

For example, the index of refraction from air to water is $\frac{4}{3}$, and no matter at what angle of incidence the light enters the water from air the sine of the angle of incidence is $\frac{4}{3}$ of the sine of the angle of refraction.

315. The Critical Angle of Refraction. — If a source of light, shrouded so that light can issue from it in but one direction, is placed under water at a point L' (Fig. 121), and the incident light has the direction $L'O$, the direction in the air will be OA such that the sine of angle POA (n_1) is $\frac{4}{3}$ of the sine of angle QOL' (m_1), or $\frac{n_1}{m_1} = \frac{4}{3}$. If the source of light is moved to L'' so that the incident light has the direction $L''O$,

the light in the air will take the direction *OB* so that $\frac{n_2}{m_2}=\frac{4}{3}$. If again the light is moved to L''' so that the ray *OC* on emerging from the water just skims the water surface, taking the direction *OR*, the angle of incidence

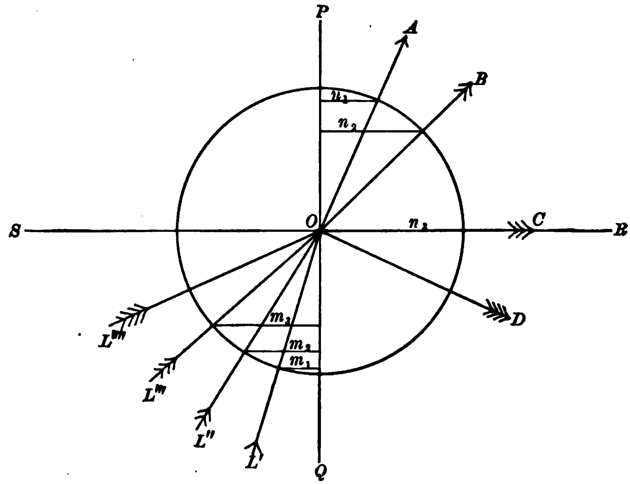


FIG. 121.

QOL''' is then the greatest possible in order that refraction can take place and the angle of refraction is 90°. If the light is moved still farther, to $L''\,''$, the incident light takes the direction $L''\,''O$, but since the light cannot pass into the air, the water surface acts as an opaque plane mirror and totally reflects the light in the direction *OD*, so that $\measuredangle\, QOL'''' = \measuredangle\, QOD$. The maximum angle of incidence in the denser of two media at which refraction can take place is the *critical angle* for these two media.

The value of the critical angle for water and air is 48° 36′ because $\frac{\text{sine } i}{\text{sine } r}=\frac{\text{sine } i}{\text{sine } 90°}=\frac{3}{4}$; the sine of 90° is 1, therefore sine $i = .750$, or i

(the critical angle) = 48° 36′. In a similar way it is found that the critical angle for glass into air is 41° 48′.

316. Relation of the Angle of Deviation to the Angles of Incidence and of Refraction. — The angle of deviation changes in value as the angle of incidence changes, being zero when the angle of incidence is zero and increasing toward a maximum as the angle of incidence approaches the critical angle. If the incident ray is in the denser medium, the maximum value of the angle of deviation is the complement of the angle of incidence, since the angle of refraction in this case is 90°. If the incident ray is in the rarer medium, as the angle of incidence approaches its limit of

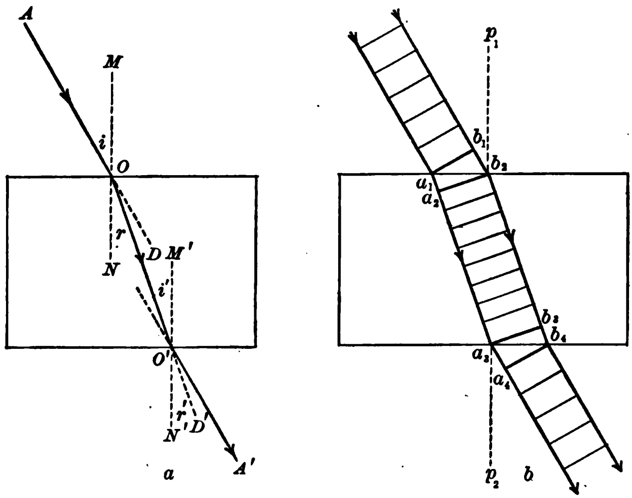


FIG. 122.

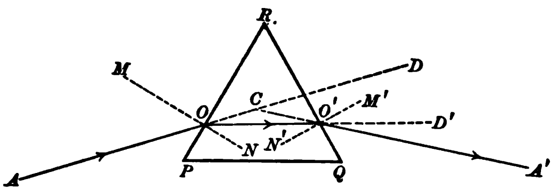
90°, the angle of refraction approaches the critical angle as its limit, and the maximum deviation is the complement of the angle of refraction.

317. Incident and Emergent Rays with a Parallel-faced Medium. — If light passes through a transparent body with

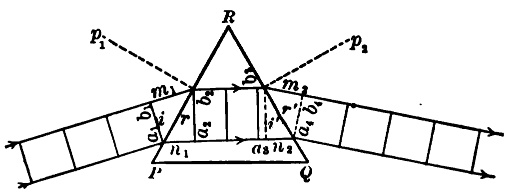
parallel faces, the emergent ray $O'A'$ (Fig. 122 *a*) is parallel to the incident ray AO, because with the same pair of media μ is constant and the angle of refraction at O equals the angle of incidence at O', and since $\mu = \frac{\text{sine } i}{\text{sine } r} = \frac{\text{sine } r}{\text{sine } i'}$ and sine $r =$ sine i', sine $i =$ sine r' or $i = r'$ (if the sines are equal, the angles are equal). The normals MN and $M'N'$ are parallel, therefore i being equal to r', the ray AO is parallel to the ray $O'A'$.

The same thing may be proved with Fig. 122 *b*, which is a diagram of a plane wave through a glass plate.

318. Incident and Emergent Rays with a Triangular Prism. — The path of a plane wave through a transparent



a



b

FIG. 123.

triangular prism is shown in Fig. 123 *a*. If the prism is glass and the surrounding medium air, $\frac{\text{sine} \measuredangle MOA}{\text{sine} \measuredangle NOO'} = \frac{3}{2}$. The wave passes in the direction OO' until it reaches the

other face of the prism, when it is again refracted so that $\frac{\text{sine} \measuredangle N'O'O}{\text{sine} \measuredangle M'O'A'} = \frac{2}{3}$.

319. Angle of Deviation and Refracting Angle of a Prism. — Referring to Fig. 123 *a*, the acute angle $A'CD$ formed by the emergent ray $O'A'$ produced backward and the prolongation of the incident ray AO is the ***angle of deviation of the prism*** and evidently is equal to the sum of the two angles of deviation, $O'OC$ and $A'O'D'$, formed by the passage of the ray through the prism. The amount of deviation depends upon three conditions: the refracting angle (R) of the prism, the substance of which the prism is composed, and the angle of incidence.

For a given prism the deviation is a minimum when the angles of incidence and of emergence are equal.

320. Effects of a Pair of Prisms. — If two prisms are placed base to base as shown in Fig. 124 and light from a source O passes through prism P_1, its path is *Oabc*. Light from the same source passing through prism P_2 will have the path *Odeg*. These two rays intersect at point F, which is therefore the focus of light from O and is the location of the image of O.

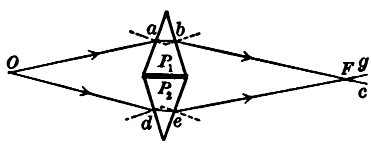


FIG. 124.

If the two prisms are placed with their refracting angles together as shown in Fig. 125, light from O through prism P_1 will take the path *Oabc*, and through prism P_1 the path *Odeg*. The effect is to render the rays still more divergent and no *real* focus of this light exists, but the emergent rays, *bc* and *eg*, seem to come from a point F on the same side of the prisms as the source O, and F is therefore a *virtual* focus of rays from O. These facts

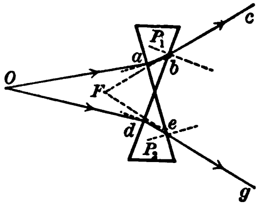


FIG. 125.

must be kept in mind in order to understand the effect of a lens.

321. Lenses.—A *lens* is a solid transparent body bounded by two surfaces, one, or both, of which is curved. Lenses

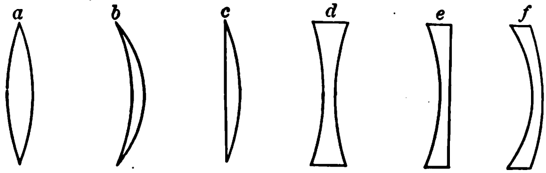


FIG. 126.

are either *converging* or *diverging: a, b,* and *c* (Fig. 126) are *converging* types and, as shown, are thick in the middle; *d, e,* and *f* are *diverging* types and are thin in the middle.

322. Effects of Lenses on Wave Fronts. — The thick-in-the-middle class of lenses retards the *middle* of the wave most since that part traverses more of the lens than the part near the margins and this has the following effects: (1) of making a plane wave front concave in the direction it is moving (Fig. 127 *a*); (2) of increasing the concavity of a concave wave front (Fig. 127 *b*); (3) of decreasing the convexity of a convex wave front (Fig. 127 *c*). The thin-in-the-middle class of lenses evidently produces results exactly opposite, as shown in Fig. 128 *a, b, c*.

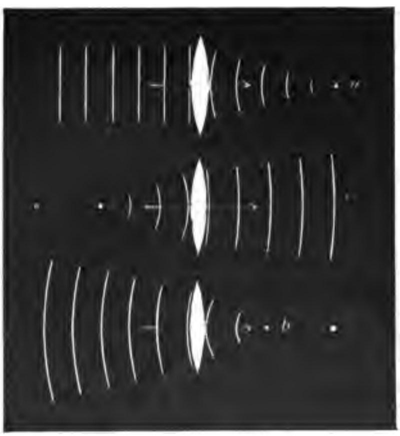


FIG. 127.

323. Optical Center. — The *optical* center of a lens is a point such that rays passing through it are not *deviated* from their paths. The location of this point must then be such that all rays passing through it enter and leave the lens at points on the two surfaces where the tangents are parallel. The point O in Fig. 129 indicates the optical center of various types of lenses; in a double convex or double concave lens it is at the center of the lens.

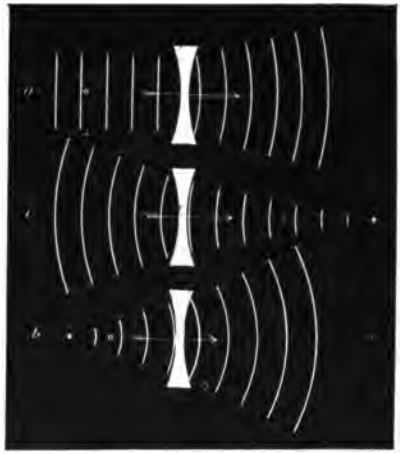


FIG. 128.

Any ray passing through the optical center of a lens is called an *axis*. That axis which also passes through the center of curvature C (Fig. 129) is called the *principal axis*, XY; any other axis, as AA', is

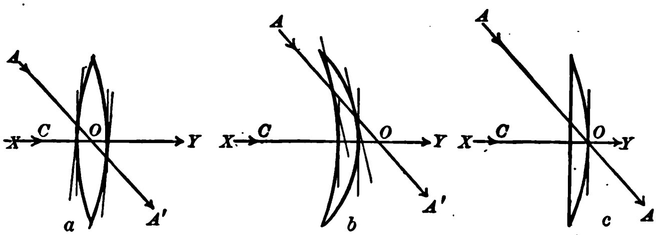


FIG. 129.

called a *secondary axis*. A ray passing through the optical center, upon emerging from the lens, has a direction parallel to its incident direction, *i.e.* is not deviated, and if the

lens is thin the amount of lateral displacement is small and may be neglected.

324. Principal Focus of a Lens. — The point to which a plane wave parallel to the principal axis of a lens is brought to a focus after passing through the lens is the *principal focus* of the lens, and the distance from the lens to this point is the *focal length* (f) of the lens.

If the index of refraction is $\frac{3}{2}$, the principal focus practically coincides with either center of curvature of a double convex lens.

325. Principles of Diagrammatic Construction of Images formed by Lenses. — As in the case of mirrors the image formed by a lens may be readily determined by making use of two simple principles of construction: (1) an *axial* ray, *i.e.* one coinciding with an axis, does not change its direction on passing through a lens; (2) rays parallel to the principal axis on one side of the lens become *focal* rays, *i.e.* pass through the principal focus, on the other side, and *vice versa*.

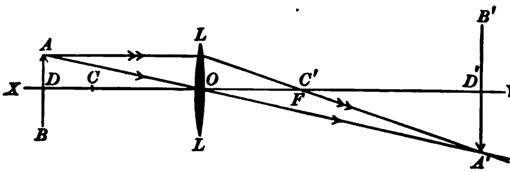


FIG. 130.

326. Application of the Principles. — Let AB, Fig. 130, be an object placed so that $D_o > R$ but $< 2R$. Draw from A an axial ray AOA'. Draw a second ray from A parallel to the principal axis and, after emerging from the lens, as a *focal* ray. These rays intersect at A', which as explained for mirrors is the image of A. By drawing two rays similarly from B the image B' may be found.

327. Characteristics of a Real Image formed by a Lens. — The image $A'B'$ has the following characteristics: —

1. *Location:* on the side of the lens opposite to the object.
2. *Kind:* real.
3. *Distance:* greater than $2R$ but less than ∞.
4. *Size:* larger than the object; the $\triangle AOD$ and $A'OD'$ being similar, $S_o : S_i = D_o : D_i$.
5. *Arrangement vertically:* inverted.
6. *Arrangement laterally:* reversed.

If $A'B'$ is the object, then AB is the image having characteristics differing from the above only in distance and size, D_i being $> R$ but $< 2R$ and the image being smaller than the object. If the object distance equals $2R$, the image distance is $2R$ and the object and image are of equal size.

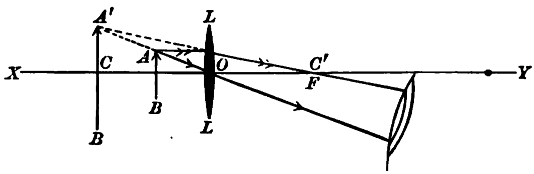


FIG. 131.

328. Characteristics of a Virtual Image formed by a Convex Lens. — Let AB (Fig. 131) be an object where $D_o < R$. Draw as before two rays from A, one an axial ray and the other parallel to the principal axis and, after passing through the lens, as a focal ray. These emergent rays are divergent and will never meet, but they *seem* to come from a point A' on the same side of the lens as the object but farther from the lens. By drawing two similar rays from B its image B' may be located.

The image $A'B'$ has the following characteristics: —

1. *Location:* on the same side of the lens as the object.
2. *Kind:* virtual.
3. *Distance:* farther from the lens than the object.

4. *Size:* larger than the object ($S_o : S_i = D_o : D_i$).
5. *Arrangement vertically:* erect.
6. *Arrangement laterally:* *not* reversed.

The formation of a virtual image with a lens in this way represents the action of a simple microscope or ordinary magnifying glass.

329. Comparison of Images formed by Lenses and Mirrors. — Comparing the kinds of images formed by lenses and mirrors, the following points should be noted: —

1. Only the thick-in-the-middle class of lenses and concave mirrors form *real* images.
2. All other lenses and mirrors form *virtual* images.
3. Light passes *through* a lens, while it does not pass through a mirror but is *reflected* by it. Hence, —
4. In the case of a lens a real image is on the side of the lens opposite to the object, and a virtual image is on the same side as the object.
5. In the case of a mirror a real image is on the same side of the mirror as the object, and a virtual image is on the opposite side of the mirror.

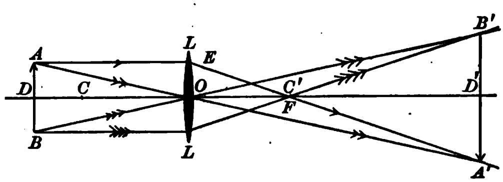


FIG. 132.

330. Relation between the Conjugate Focal Distances and the Focal Length of a Converging Lens. — Let AB (Fig. 132) be an object perpendicular to the principal axis of a thin converging lens LL, O the optical center of the lens, and F its principal focus. Draw from A an axial ray and

one parallel to the principal axis, and locate the point A' as the image of A. Make the same construction for B and B'.

The $\triangle AOD$ and $A'OD'$ are similar, therefore $AD:A'D' = OD:OD'$. The $\triangle EOF$ and $A'D'F$ are similar, therefore $EO:A'D' = OF:FD'$. But $EO = AD$, hence $EO:A'D' = AD:A'D'$ and $OD:OD' = OF:FD'$. Using the symbols D_o, D_i, and f for OD, OD', and OF, respectively,

$$D_o : D_i = f : D_i - f,$$

$$D_oD_i - D_of = D_if,$$

$$D_if + D_of = D_oD_i.$$

Dividing by D_oD_if,

$$\frac{1}{D_o} + \frac{1}{D_i} = \frac{1}{f},$$

i.e. the sum of the reciprocals of the conjugate focal distances equals the reciprocal of the focal length of the lens.

PROBLEMS

1. If the index of refraction from air to glass is 1.5, and light is incident on a glass plate at an angle of 45°, what is the angle of refraction? *Ans.* 28+°.

2. If the index of refraction from air to diamond is 2.5, what is the critical angle of refraction for diamond? *Ans.* 23.5°.

3. If light strikes a glass prism with a refracting angle of 60° at an angle of incidence of 45°, what is the *deviation* if the index of refraction = 1.5? *Ans.* 37.75°.

4. If the focal length of a camera lens is 30 cm., (*a*) how far from the lens should the plate be in order to be in focus when photographing a distant object? (*b*) What is the distance of the plate from the lens if the object to be photographed is 80 cm. from the lens? (*c*) How large an object at the 80 cm. distance can be photographed on an 8 × 10 plate? *Ans.* (*a*) 30 cm. (*b*) 48 cm. (*c*) $13\frac{1}{3} \times 16\frac{2}{3}$ in.

5. (*a*) In focusing a camera for near objects, must the lens be nearer to or farther from the plate than for distant objects? (*b*) Prove by diagram. *Ans.* (*a*) Farther.

6. A photographer places a vase 6 ft. in front of a lens and takes a picture $\frac{1}{4}$ its real height. (*a*) What is the distance of the plate from the lens? (*b*) What is the focal length of the lens? *Ans.* (*a*) 18 in. (*b*) 14.4 in.

7. An object 3 in. high is placed 30 in. in front of a lens whose focal length is 12 in. (*a*) What is the height of the image? (*b*) Will the image be real or virtual? *Ans.* (*a*) 2 in. (*b*) Real.

8. Describe what a fish, at a point some distance below the surface of a clear, quiet pool of water, may be supposed to see on looking upward at the surface, (*a*) of the region above the surface; (*b*) of the region below the surface.

CHAPTER XIV

DISPERSION OF LIGHT; COLOR

331. The Spectrum. — When a beam of ordinary white light passes through a prism it is not only refracted but is also *dispersed*, *i.e.* is separated into what appears to the eye as a series of different hues or colors, called a *spectrum* (Fig. 133). The order of the colors in such a spectrum is red, orange, yellow, green, blue, and violet, the red having the least, and the violet the greatest, angle of deviation, as shown in Fig. 134, in which $\measuredangle DPR < \measuredangle DPG < \measuredangle DPV$.

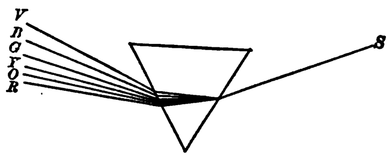


FIG. 133.

332. Angle of Dispersion. — The angle RPV formed by the extreme red and violet rays is called the *angle of dispersion*, and is equal to the difference between the angle of deviation of the violet and that of the red, *i.e.* $\measuredangle RPV = \measuredangle DPV - \measuredangle DPR$. Dispersion, therefore, is due simply to different degrees of deviation of the components of white light, the complexity of whose nature is thereby revealed.

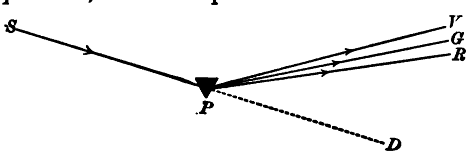


FIG. 134.

333. Cause of Dispersion. — Under the subject of refraction it was shown that deviation is caused by a change of

velocity, and that the degree of deviation depends on the amount of change of velocity.

From this it follows that the velocity of the red component is *retarded* least, and that of the violet most, on entering the glass; while on emerging again into the air and resuming their original equal velocities, the red is *accelerated* the least, and the violet the most of any of the several components.

The explanation of this lies in the fact that ordinary white light is the resultant of waves of infinite variety of length, ranging between the limits of 380 and 720 microns. Since these waves keep abreast of each other in the ether and practically so in the air, the wave lengths are inversely proportional to their frequencies. On entering dense matter, however, the shorter waves meet with greater resistance than the longer ones do, with the result that the shorter waves lag behind the longer ones and are consequently deviated to a greater degree.

334. The Rainbow. — The same scale of colors as that formed by a prism is seen in the *rainbow*, which, in fact, is produced in much the same way, the drops of water taking the place of the prism.

335. Chromatic Aberration. — When refraction only is considered, the mean angle of deviation is taken, *i.e.* the deviation of an intermediate ray, as $\measuredangle DPG$. Close examination, however, will show that whenever light passes through an ordinary lens or prism it becomes more or less fringed with color due to dispersion, which, of course, is a defect so far as the use of a lens is concerned, and is called *chromatic aberration*. In order to explain how this defect is corrected, it is necessary to understand the fol-

lowing facts about *flint* and *crown* glass. Flint glass is a double silicate of lead and potassium, while crown glass, such as window glass, is a double silicate of calcium and sodium. A flint glass prism having a refracting angle of (say) 15° will form an angle of dispersion equal to that formed by a crown glass prism having a refracting angle of about 30°, or double the other; but this equal angle of dispersion in the crown glass will, as a whole, be accompanied by a considerably larger angle of deviation, because of its greater refracting angle.

Let FP (Fig. 135) represent such a flint glass prism, AB being an incident ray; the $\measuredangle RBV$ is the angle of dispersion and DBG is the mean angle of deviation.

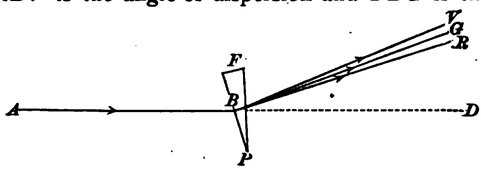


FIG. 135.

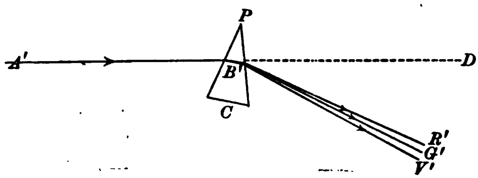


FIG. 136.

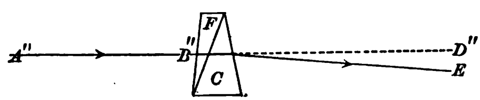


FIG. 137.

Likewise in Fig. 136, let CP represent the crown glass prism mentioned. Here $\measuredangle R'B'V'$ is the angle of dispersion and $\measuredangle D'B'G'$ the mean angle of deviation.

Now if two such prisms are combined, with their refracting angles reversed, Fig. 137, it is evident that the refracting power of the one will counteract that of the other, equal amount for equal amount. The *dispersions* are opposite in order and equal, and will therefore disappear, but the *deviations* while opposite are unequal, and there will remain a deviation equal to the difference between the two mean deviations, *i.e.*, $\measuredangle D''B''E = \measuredangle D'B'G' - \measuredangle DBG$.

336. Achromatic Lens. — A combination lens, one part made of crown and the other of flint glass, so constructed that the dispersions exactly counteract each other, leaving the excess of deviation of the crown glass, is called an *achromatic lens*, *i.e.* without color defect. Figure 138 represents such a lens: *F* is a flint glass lens of the thin-in-the-middle type, *C* is a crown glass lens of the opposite type.

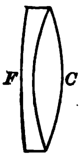


FIG. 138.

337. Direct-vision Spectroscope. — It is evident also that flint and crown glass prisms may be fitted together so as to counteract the mean *deviations* and still have considerable *dispersion*. This principle is made use of in the *direct-vision spectroscope*. A flint glass prism having a refracting angle of 52° will produce the same mean deviation as a crown glass prism having a refracting angle of 60°, but the angle of dispersion for the flint glass prism will be the greater.

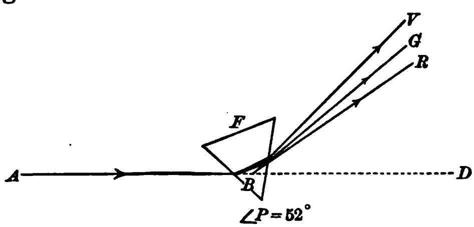


FIG. 139.

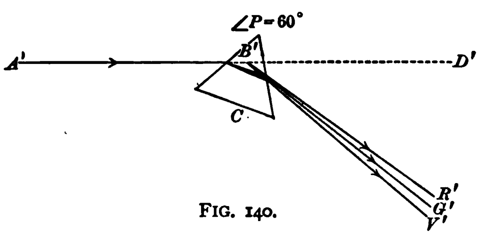


FIG. 140.

Let *FP* (Fig. 139) and *CP* (Fig. 140) represent two such prisms respectively. It is evident that, if the refracting powers of such a pair of prisms are opposed by reversing their refracting angle, the mean deviation will be reduced to zero, while there will remain a small amount of dispersion amounting to the difference between the two separate dispersions. For

$\measuredangle DBG = \measuredangle D'B'G'$, and their algebraic sum is zero, counting plus above and minus below the line of the incident ray, and the combined effect of both prisms makes GB and $G'B'$ both coincide with the prolongation of the incident ray. Since $\measuredangle R'B'V' < \measuredangle RBV$ and $\measuredangle D'B'R' >$

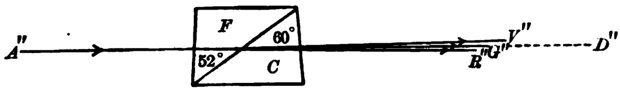


FIG. 141.

$\measuredangle DBR$, the resultant red deviation by the combined prisms is a little below the line of the incident ray, and the resultant violet deviation is a little above this line. One arrangement for producing such a result is shown in Fig. 141, F and C being prisms of flint and crown glass respectively. A train of such prisms increases the amount of dispersion, but the loss of light by absorption limits the number of prisms that may be used to advantage.

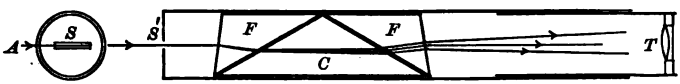


FIG. 142.

A *spectroscope* is an instrument by which a sharp, well-defined spectrum may be formed and observed. A *direct-vision* spectroscope consists of a train of prisms such as shown in Fig. 142, near one end of which, and parallel to the edges of the prisms, is a narrow adjustable slit, S, through which the light enters. At the other end is a telescope, T, for viewing the spectrum.

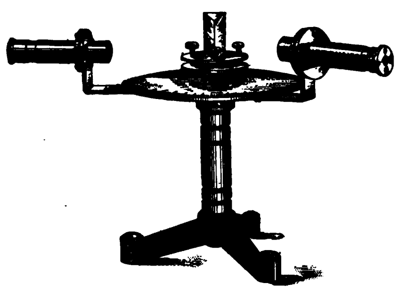


FIG. 143.

The ordinary *prismatic* spectroscope, Figs. 143 and 144, has but one prism, and consequently the tubes carrying the slit and the telescope

form an angle with each other corresponding to that formed by the incident and the mean refracted ray passing through the prism.

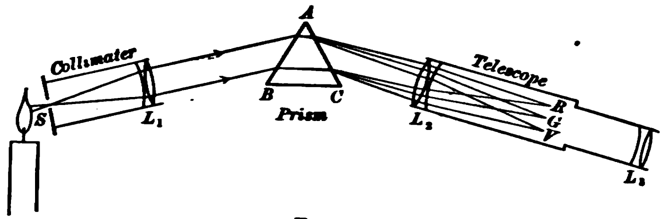


FIG. 144.

338. Different Kinds of Spectra. — The spectrum of sunlight (daylight) is called the *solar* spectrum. This band of colors is interrupted by many narrow black lines, called after the German scientist who first made a careful study of them, *Fraunhofer lines*. The most conspicuous of these lines are designated by the letters *A*, *B*, *C*, etc., as shown in Fig. 145. Each star also has its own set of characteristic dark lines in its spectrum. Such a spectrum is called a *discontinuous spectrum*.

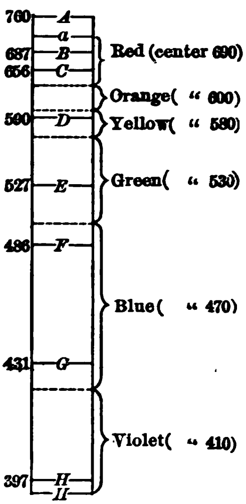


FIG. 145.

The spectrum of light from an incandescent (white hot) solid or liquid has no such lines crossing it and is therefore called a *continuous spectrum*.

The spectrum of light from a luminous gas or vapor consists of one or more very narrow, sharply defined zones of color, the intervening spaces

being dark. This is called a *bright line* spectrum. Each of the chemical elements in the gaseous or vapor condition, when at a sufficiently high temperature to be luminous, gives a spectrum which is different from that of any other element, *i.e.* its spectrum is characteristic of the particular element, and *spectrum analysis*, or the identifying of an element by means of its spectrum, is based on this fact.

The spectrum of light from an incandescent solid or liquid when the light is made to traverse a gas or vapor which is at a lower temperature than the source, is *discontinuous*, *i.e.* is interrupted by dark lines, *and these dark lines correspond exactly to the bright lines which the gas or vapor would give if observed by itself when sufficiently heated.* Such a spectrum is called a *reversed* spectrum. Herein lies the explanation of the existence of the Fraunhofer lines: Light coming from the incandescent nucleus of the sun penetrates the surrounding cooler gases and vapors, which produce a reversal of *their own* spectra, resulting in the dark lines of the solar spectrum. Nearly all of the many thousands of these lines in both the solar and the stellar spectra are found to correspond to the spectra of terrestrial elements, showing beyond question that the earth, sun, and stars are, in general, of the same material composition.

The spectrum of light which has passed through a transparent colored substance generally consists of one or more broad zones, not sharply defined, the colors of the dark intervening zones having been absorbed. This is called an *absorption* spectrum.

339. Color. — By using the term *color* in the subjective sense the subject may be more clearly and accurately discussed than is otherwise possible. This implies, to begin

with, that external to the mind there is no such thing as redness, or greenness, or blueness, etc., but that physical conditions exist there which give rise to such ideas through the sensations they may produce.

The necessity of making a distinction between the cause and the effect in this case lies in the fact that the same sensation may be produced by quite different external physical conditions. For example, when the ordinary spectrum is synthesized, *i.e.* recombined, the light thus formed again produces the sensation of white as it did originally before being dispersed. Again, if red, green, and violet light, and these only, are combined, the same sensation of white results. The sensation of white is also produced by a combination of yellow and blue light, or of blue and purple light, or of purple and yellow light. Furthermore, the sensation of yellow is produced either by waves from a single narrow zone of the spectrum or by the combination of waves from two narrow zones widely separated and totally different from each other, one being red, the other green, when viewed separately.

The sensation of blue is produced either by waves from one narrow zone or from two distinctly different zones, one being the same green zone that enters into the composition of yellow and the other being violet. In fact, *red*, *green*, and *violet* are the only color sensations which can be produced in one and only one way, *viz.* by waves from a single restricted zone of the spectrum; while all other color sensations may be produced by combinations of these three sensations. Hence *red, green, and violet are called the fundamental color sensations* in the subjective sense, or, for convenience, the *fundamental colors.*

340. Trichromatic Theory of Color Vision. — The trichromatic theory of color vision, first suggested by Dr.

Thomas Young, the eminent English scientist, about the year 1800, assumes: —

(1) That the eye is capable of forming *three* and *only three* color sensations. These are *red, green,* and *violet,* and are called the *fundamental color sensations.*

(2) That all other color sensations, including white, are produced by combinations of the fundamental sensations. Black or darkness is the absence of all sensation.

(3) That the fundamental red sensation is most strongly excited by waves about 615 microns long, the effect grow-

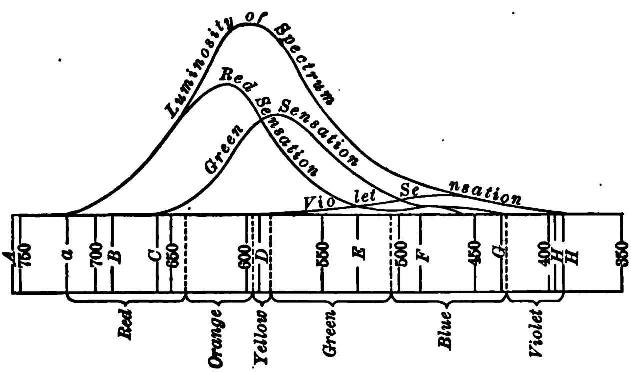


FIG. 146.

ing weaker as the length of the waves increases or decreases from this value, extending, however, over considerable range of wave lengths. In precisely the same way the fundamental green and violet sensations are most strongly excited by waves of 570 microns and 450 microns in length respectively, the effect falling off on both sides of these values.

In Fig. 146 the base line represents the length of a normal solar spectrum with the prominent Fraunhofer lines marked. The numbers are the several wave lengths

in microns. The effectiveness of the several waves in exciting the red, green, and violet sensations is represented by the ordinates of the three curves so marked. It will be observed that these curves overlap each other to a considerable extent and the great complexity of color sensations is due to this fact.

For instance, yellow is always complex, consisting of a combination of red and green sensations, no matter whether produced by waves from the separate red and green zones or by waves 580 microns long from the single intermediate zone, since the same result follows in either case: the red and green sensations are both equally excited. The facts are the same concerning the blue sensation; while the sensation of white results whenever the three fundamental sensations are normally excited together.

341. Combinations of Colored Light — Red, Green, and Violet. — If, by means of a triple projecting apparatus (the von Nardroff Color Mixer, Fig. 147, is well adapted to this experiment), three circles of colored light — one red, one green, and one violet — are thrown on a screen, as shown in Fig. 148, and are then made to overlap, as in Fig. 149, the central area, *W*, common to all three circles appears white. This is what would be expected from the theory, since the light reflected from this area to the eye is such as to excite all three fundamental sensations normally and at the same time. Again, the area,

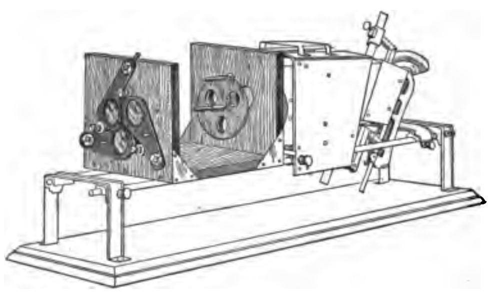


FIG. 147.

Y, common to the red and green circles appears yellow because the light reflected thence into the eye equally excites the fundamental red and green sensations. Likewise the area, *U*, common to the green and violet circles appears blue; while the area, *P*, common to the violet and red appears purple.

342. Combinations of Colored Light — Purple, Yellow, and Blue. — Again, proceeding in the same way, let the three circles be colored with purple, yellow, and blue light,

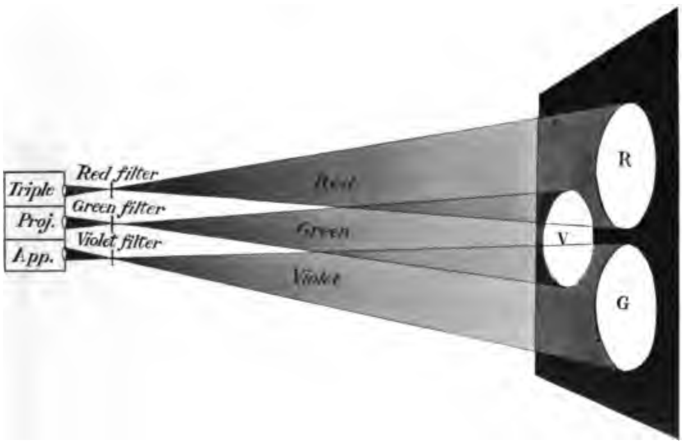


FIG. 148.

which is accomplished by interposing in the beams of light near the lens ray filters, *p-f*, *y-f*, and *v-f*, each of which transmits only the desired zone of the spectrum. Obviously the spectroscope must be used to determine the kind of the light transmitted by such a filter. As shown in Fig. 150, the areas common to any two of the three circles, as well as that common to all three, appear white, which is in accordance with the theory, since all three fundamental

sensations are normally and simultaneously excited by the light from any of these areas.

343. Results of Experiments expressed in Equations. — These results may be expressed in equations by using the initial letter to represent a color, letting U represent blue and B, black. No quantitative value can, however, be attached to the symbols as here used: —

$$R + G = Y;\ G + V = U;\ V + R = P;\ R + G + V = W.$$

$$Y + U = \overline{R + G} + \overline{G + V} = W.$$

$$U + P = \overline{G + V} + \overline{V + R} = W.$$

$$P + Y = \overline{V + R} + \overline{R + G} = W.$$

$$Y + U + P = \overline{R + G} + \overline{G + V} + \overline{V + R} = W.$$

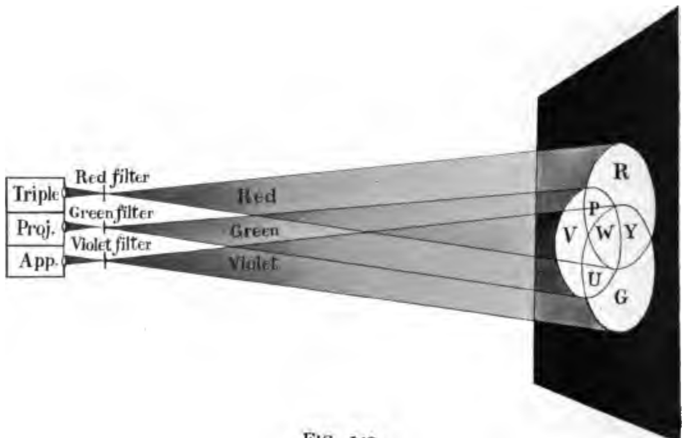


FIG. 149.

344. Composition of White Light so far as the Eye is Concerned. — It will now be readily understood that while the actual number of components of ordinary white light is very great, yet *so far as the eye is concerned* it may be considered that such light is made up of three and only

three components, each of which excites a fundamental color sensation.

345. Application of the Trichromatic Theory in Color Photography. — The most successful results thus far obtained in color photography are based upon the fact that, instead of trying to photograph all the varieties of color in nature, it is only necessary to get photographic effects corresponding to the three fundamental color sensations, and then to synthesize the three fundamental colors in accordance with the effects thus obtained.

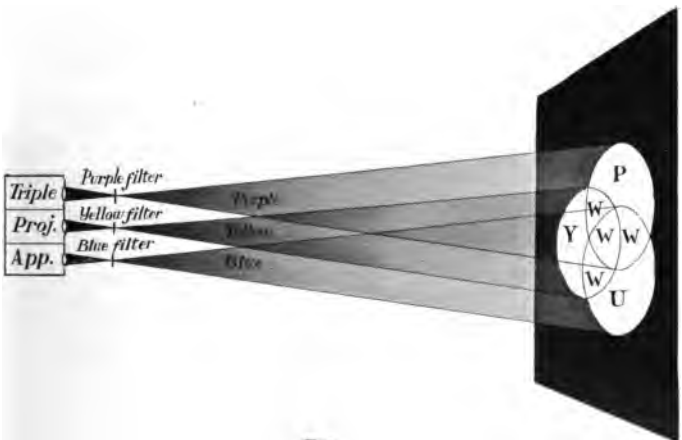


FIG. 150.

346. Color of Opaque Bodies. — A white object owes its appearance to the fact that it reflects the several components of white light equally, the absorption (which is generally considerable) also being equal. If there is complete absorption of the incident light, the body appears black, *i.e.* it is not itself seen, and if distinguished at all it is by the projection of its outline on other reflecting surfaces.

The color of a body, as seen in ordinary white light, is due to the fact that certain components of the incident white light, are absorbed by the body, while the remaining components are either reflected or transmitted wholly or in part. In general, *color results from the suppression of other colors.* A red rose absorbs to a large degree all other components except the red which it reflects; while the green leaf reflects principally green, absorbing the remaining components almost entirely. This may be shown by holding the rose and leaf in the red zone of the spectrum: the flower will have a vivid color, but the leaf will appear almost black; or again, if they are held in the green zone, the conditions are exactly reversed.

347. Color of Transparent Bodies. — Many substances transmit, *i.e.* are transparent to, certain components of white light, absorbing the others, *e.g.* colored glass, many crystals and solutions. Here again the resulting color is due to the suppression of other colors, and if examined with a spectroscope absorption spectra are obtained.

348. Pigments — Opaque and Transparent. — Paints, or pigments, produce their color either by reflecting the components they do not absorb, or else by transmitting them to a white reflecting surface beneath and then retransmitting them. Ordinary paints as a rule act in the first way and are called *opaque pigments;* while stains and dyes and many of the paints used by the artist act largely in the second way and are called *transparent pigments.* It is evident that only those components of white light which remain unabsorbed by a pigment can be effective in producing color.

349. Superposition of Transparent Pigments — Red, Green, and Violet. — To illustrate this point, let three intersecting

FIG. 151

FIG. 152

circles be drawn on the surface of *white* paper (Fig. 151); let the first be colored with red, the second with green, and the third with violet transparent pigments. The result is plainly quite different from that produced when lights having these colors were used; for here the area common to any two, as well as that common to all three circles, is *black*, which means that all the components of the incident white light are absorbed in these areas. Bearing in mind that "*so far as the eye is concerned*" there are but three components in white light, *viz.* red, green, and violet, it is evident that the white paper is white because it reflects to the eye all three components of the incident light. The transparent red pigment, however, absorbs the green and violet components, transmitting only the red which is reflected by the paper beneath and again retransmitted by the pigment so that red only comes from that area to the eye. In like manner the green pigment absorbs the red and violet components, leaving only the green to be transmitted, reflected, and retransmitted to the eye; and so with the violet which absorbs the red and green components and transmits the violet.

Where any two (or all three) of these pigments overlap it is evident that their combined effect is a complete absorption of all the incident light.

350. Results expressed in Equations. — Letting a *small* initial letter be the symbol for the color of a transparent pigment, these results may, as before, be written in the form of equations: —

$$r = W - G - V = R$$

$$g = W - R - V = G$$

$$v = W - R - G = V$$

$$r + g = W - R - G - V = B$$

$$g + v = W - R - G - V = B$$
$$r + v = W - R - G - V = B$$
$$r + g + v = W - R - G - V = B$$

351. Superposition of Transparent Pigments — Purple, Yellow, and Blue. — Again, let three intersecting circles be colored with purple, yellow, and blue transparent pigments (Fig. 152). The result is that only the area common to all three circles is black, while that common to the purple and yellow is red, that common to the yellow and blue is green, and that common to the blue and purple is violet. Evidently the pigments used in this case are quite different in their nature from the other set. "*So far as the eye is concerned*" the yellow pigment absorbs only the violet component, transmitting and retransmitting after reflection by the paper beneath, both the red and green components, which produce in the eye the complex sensation of yellow. In the same way the blue pigment absorbs the red component and transmits the green and violet, causing the complex sensation of blue; while the purple absorbs the green and transmits the red and violet.

The area common to all three pigments is black, because each pigment absorbs one of the three components.

352. Results expressed in Equations. — As before, these results may be shown by equations: —

$$p = W - G = P$$
$$y = W - V = Y$$
$$u = W - R = U$$
$$p + y = W - G - V = R$$
$$y + u = W - V - R = G$$
$$u + p = W - R - G = V$$
$$p + y + u = W - G - V - R = B$$

Fig. 153 A.

Fig. 153 B.

Fig. 153 C.

Fig. 153 D.

Fig. 153 E.

Each of these pigments transmits two components and absorbs one. A combination of any two transmits one and absorbs two components. From this it follows that *purple*, *yellow*, and *blue* transparent pigments may be made to give all the colors that the eye can see. For this reason they are popularly known as the "*primary colors*."

353. Practical Application in the Process of Tricolor Printing. — The explanation of the process of tricolor printing will now be readily understood. Figure 153 is a tricolor print made by superposing the three single color prints shown in *a*, *b*, *c*, one in purple, one in yellow, and one in blue transparent ink. In *d* the yellow and purple prints are shown superposed, and in *e* the superposition of all three gives the complete picture. While the prints, as a whole and in every detail, have exactly the same measurements, the inks are differently distributed on each print so that the combined effect is in accord with the preceding explanation. The printing plates from which the impressions are made are produced by photography, hence this is called a color-photography process.

CHAPTER XV

MAGNETISM

354. Magnet Defined. — A magnet is a substance which will attract certain other substances, such as soft iron and steel, and which, if in the shape of a bar and suspended freely in a horizontal position, will point approximately north and south.

355. Magnets Classified. — Magnets may be classified according to their *origin* as natural or artificial. A natural magnet is an ore called lodestone, an oxide of iron and which, as found in the earth, possesses the properties of a magnet. An artificial magnet is one which has been made to acquire the properties of a magnet by one of the several methods of magnetizing (§ 359).

A magnet which, having been subjected to magnetization, retains its magnetism after the magnetizing force has been removed, is called a *permanent* magnet; while one which, when the magnetizing influence has been removed, loses the properties of a magnet, is called a *temporary* magnet. A piece of hard steel when once magnetized remains a magnet; while soft iron loses nearly all its magnetism as soon as the magnetizing influence is removed.

356. Magnetic Substance Defined. — A substance which can be made to acquire the properties of a magnet is called a *magnetic substance.* Those substances which a magnet will attract are few in number, steel, soft iron, nickel, and cobalt being the most common magnetic substances.

357. Properties of a Magnet. — Any bar magnet if freely suspended in a horizontal position will point very nearly north and south; at some places on the earth's surface exactly so. The end of the magnet which points north is the north (N) or positive (+) pole; the other end is the south (S) or negative (−) pole. It is this directive property of a suspended or pivoted magnet that gives the magnetic compass its value.

If a magnet is placed in iron filings and raised, the quantity of filings clinging to the various parts of the magnet shows that the force of attraction is not exhibited uniformly along the magnet, but is strongest near each end, and diminishes rapidly toward the middle, where the attraction is zero.

This is shown in Fig. 154, where the distance above or below the horizontal reference line representing the magnet indicates the force of attraction at various points along the magnet. The poles of a magnet are the points where the force of attraction is strongest, and are near but not at the ends of the bar.

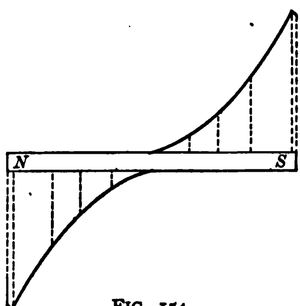


FIG. 154.

358. Law of the Action of Magnet Poles. — If the N pole of a magnet is brought near the N pole of a freely suspended magnet, it repels that pole, but will attract the S pole. Similarly, the S pole of the first magnet will attract the N pole of the suspended magnet and repel the S pole. From these facts is derived the law: *Like poles of magnets repel each other; unlike poles attract.*

359. Methods of Magnetizing. — *First Method.* — If a piece of steel is rubbed several times from one end to the other, always in the same direction, with either the N or S pole of a magnet (Fig. 155), it becomes a magnet with the end last touched of opposite polarity to the rubbing pole. The last fact results from the attraction of unlike poles. This method is called magnetization *by contact.*

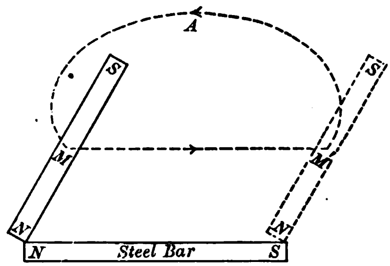


FIG. 155.

Second Method. — If a piece of iron or steel is placed near but not in contact with one pole of a magnet, it becomes a magnet, and that part of it which is nearest the pole of the magnet is of opposite polarity. This fact indicates why a magnet will attract a magnetic substance; for the magnetic substance, when within the influence of a magnet, first becomes magnetized with its dissimilar pole nearest the inducing pole, and then attraction results. This method is called magnetization *by induction.*

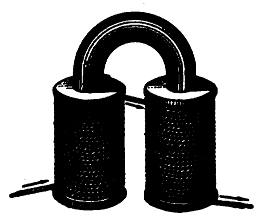


FIG. 156.

Third Method. — If a coil of insulated wire is wrapped around a magnetic substance and an electric current is passed through the wire, the magnetic substance becomes magnetized (Fig. 156). Such a magnet, when the core is of soft iron, so that it is magnetized only while the current is flowing around it, is called an *electro-magnet.* The kind of pole at each end is determined by the direction in which the electric current flows around it.

360. Theory of Magnetism.—If a magnet is broken in two, each half is a complete magnet with two poles, one at each end as shown in Fig. 157.

If each half is again broken in two there will be four magnets, each with two poles, one at each end. This process may be continued indefinitely, and each piece broken from a magnet is found to be a magnet.

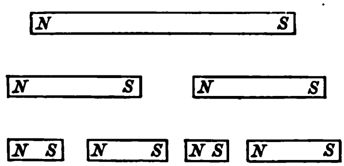


FIG. 157.

Upon these facts is based the theory that each molecule of a magnetic substance is a complete magnet with an N and S pole. In the case of an unmagnetized body these molecular magnets by their mutual action form groups of closed magnetic circuits as shown in Fig. 158 *a*, so that no free magnetism is manifested by the body as a whole. When, however, a magnet is brought near this body, or is rubbed over it, the mutual action of these molecular magnets is overpowered by the greater force of the inducing magnet and the molecules assume parallel directions (Fig. 158 *b*) with all like poles pointing in one direction, giving resultant N and S poles at the ends of the body which thus becomes a magnet. Additional evidence that magnetism is due to molecular arrangement is found in the fact that a magnetic substance if hammered or violently jarred while under magnetizing influence becomes

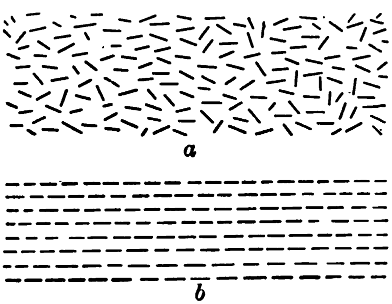


FIG. 158.

a magnet more readily; also the fact that a magnet if heated to a red heat, in which condition its molecules have greater freedom of motion among themselves, becomes demagnetized, adds weight to the theory.

361. Lines of Magnetic Force. — Since attraction or repulsion exists between two magnet poles, this force must be exerted through the medium separating the poles. The direction and intensity of magnetic force in the medium about a magnet are represented by lines, the intensity of the force being represented by the number of lines passing through 1 sq. cm. in a plane at right angles to the direction of the force. The space around a magnet traversed by these lines of force is called the *magnetic field*. It must be understood that although the lines are conventional, they represent real forces both as to direction and intensity.

362. Law of Magnetic Force. — A long slender magnet acts as if all of its force was exerted from the two points called the poles. From each of these poles lines of force radiate in every direction. At a distance of a cm. from either pole the area of a spherical shell is $4\pi a^2$. The intensity of the magnetic force I_1, or the number of lines per sq. cm., at any portion of this shell is the total force F divided by the area, $4\pi a^2$, or $\frac{F}{4\pi a^2}$.

At a distance of b cm. from the same pole the intensity of the force I_2 is $\frac{F}{4\pi b^2}$. Therefore $I_1 : I_2 = \frac{F}{4\pi a^2} : \frac{F}{4\pi b^2}$, or $= \frac{1}{a^2} : \frac{1}{b^2}$.

$$\left(\frac{1}{a^2} \div \frac{1}{b^2} = b^2 \div a^2\right).$$

$$\therefore \quad I_1 : I_2 = b^2 : a^2,$$

which means that the intensity of the magnetic force in a magnetic field about either pole is inversely proportional to the square of the distance from that pole. This relation is another case of "the law of inverse squares."

363. Direction of Lines of Force. — The direction of a *line of force* in any portion of the field of a magnet is the direction in which an N pole of another magnet would tend to move if placed in that part of the field. This direction may be determined by applying the law of magnet poles and the law of magnetic force.

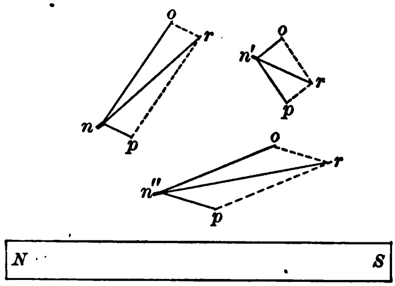


FIG. 159.

Suppose a small magnetic compass needle *n* (Fig. 159) is placed in the field of a bar magnet *NS* so that the distance *nN* is *one half* as great as the distance *nS*; because of the less distance the repulsion between the N poles is greater than the attraction between the S pole of the magnet and the *n* pole of the compass needle in the inverse ratio of the squares of the distances *nN* and *nS* or $2^2 : 1^2 = 4 : 1$.

Draw then from the *n* pole of the needle the line *no* in a direction *away* from the N pole of the bar magnet to represent its repulsion and the line *np toward* the S pole of the bar magnet to represent its attraction, making the line *no* four times as long as the line *np*.

Since these two lines, *no* and *np*, represent the two magnetic forces acting upon the *n* end of the needle, the needle must take the direction of their resultant *nr* obtained by completing the parallelogram and drawing the diagonal through *n*. In the same way the direction in which the compass needle would point in any other portion of the field may be found, as *n'* or *n''*.

364. Various Magnetic Fields. — If iron filings are scattered over a sheet of paper laid upon a bar magnet, since each filing within the field of the bar magnet becomes a

magnet by induction, they will take the direction of the magnetic force which will be for each filing the direction

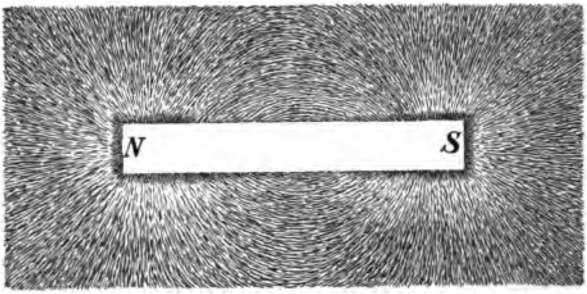


FIG. 160.

of the resultant of the forces of attraction and repulsion of the poles of the bar magnet for it as shown in the preceding paragraph.

Figure 160 shows the disposition of iron filings in the field of a bar magnet.

Figures 161 *a* and *b* show the direction of the lines of force between two dissimilar poles of two magnets placed near each other. In Fig. 162 the parallelograms are drawn for three separate filings, *a*, *b*, and *c*, and show why the filings are arranged as in Fig. 161. The filing *a* will take the direction *am*; the filing *b*, the direction *bk*; the filing *c*, the direction *ch*.

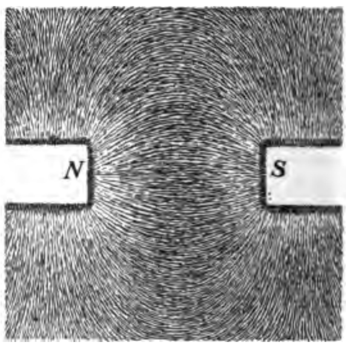


FIG. 161 *a*.

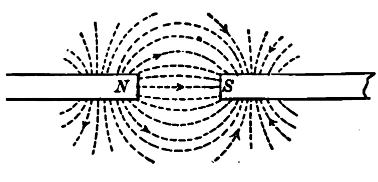


FIG. 161 *b*.

Figures 163 *a* and *b* show the magnetic field in the space between two parallel magnets with like poles adjacent, and in Fig. 164 are drawn parallelograms for three separate filings, *a*, *b*, and *c*, showing that *a* takes the direction *af*, *b* the direction *bg*, and *c* the direction *ch*, which is according to the arrangement of the filings as indicated in Fig. 163.

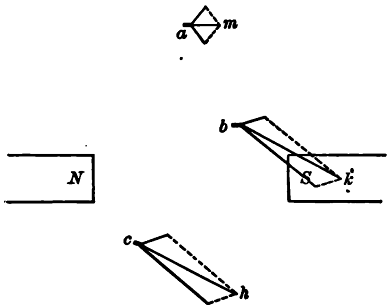


FIG. 162.

365. Magnetic Lines do not Intersect. — Since the direction of the magnetic force at any part of the field of a magnet is the direction of the resultant of the magnetic forces of the two poles, and since there can be for each point but one resultant, *no two lines of force can intersect.*

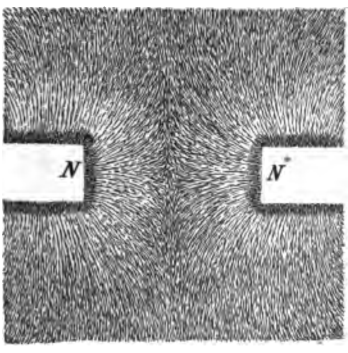


FIG. 163 *a*.

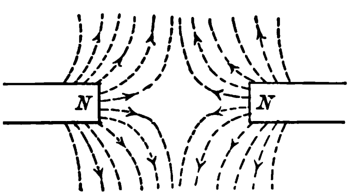


FIG. 163 *b*.

366. Lines of Magnetic Force are Curved. — Since the direction of the resultant force at any point is determined by the inverse ratio of the squares of the distances of the poles from that point, and since at no two successive points can this ratio be the same, it follows that in passing

from each point in a line of force to the one next to it the direction of the resultant force changes; hence a line of magnetic force is, as a rule, a *curved line*. The only exception to this rule is when the two component forces are in the same straight line, for example, directly between the poles of a strong U magnet.

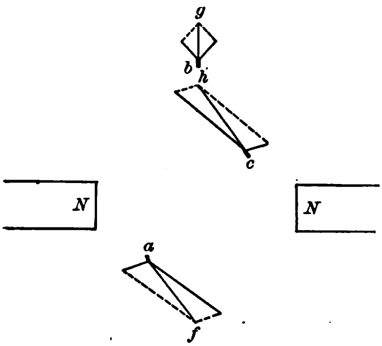


FIG. 164.

367. Magnetic Permeability. — The permeability of a substance is its ability to transmit magnetic force. The permeability of

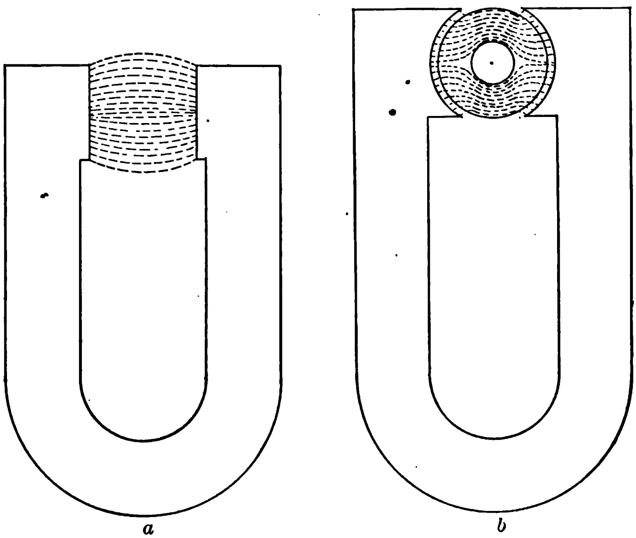


FIG. 165.

all non-magnetic substances such as air, wood, glass, copper, etc., is practically 1; the permeability of the highly magnetic substances iron and steel is many times as great, the value for some grades of iron being as high as 400.

Figures 165 *a* and *b* show the difference in the field of a U magnet produced by interposing an iron cylinder in the space between the poles. Nearly all the lines of force instead of passing through the air space pass through the iron because of the less opposition it offers to the transmission of the force.

368. Terrestrial Magnetism. — 1. At any point on the surface of the earth a magnet freely suspended in a horizontal position points approximately north and south, the exceptions being noted below.

2. Whenever a body of hard iron or steel has been fixed for a long time in a vertical position it is found to be magnetized. In the northern hemisphere its lower end will have become a + pole, while in the southern hemisphere that end would have become a – pole.

3. If a magnetic compass is carried along the Atlantic coast from the Equator northward, the needle points slightly east of north up to the coast of South Carolina, and thence it deviates slightly to the west, its deviation gradually increasing until at about lat. 70° N. it points due west. At still higher latitudes it points south of west. Again, if a magnetic compass is carried northward along the Pacific coast the needle points somewhat east of north, and this deviation increases until at lat. 70° N. it points due east. If a compass is carried along a somewhat broken line connecting Charleston, S.C., with the eastern end of Lake Superior, the needle continually points due north.

4. If a magnetized needle suspended from its center of gravity is at the Magnetic Equator, it is horizontal, but

upon proceeding north the N end of the needle dips more and more until at a certain place in lat. 70° N. it becomes vertical. Upon moving southward from the Equator the S end of the needle dips.

These four sets of phenomena indicate that the earth is a magnet with its – pole in the northern hemisphere, but not coincident with the geographic pole. The location of this magnetic pole is lat. 70° N., long. 96° W.

The angle between the magnetic meridian (which is the direction of a magnetic needle) and the geographic meridian passing through any point on the earth's surface, or the variation of a compass needle at that point from true north, is called the *angle of magnetic declination* for that point. This declination in New York at the present time is about 9° W.

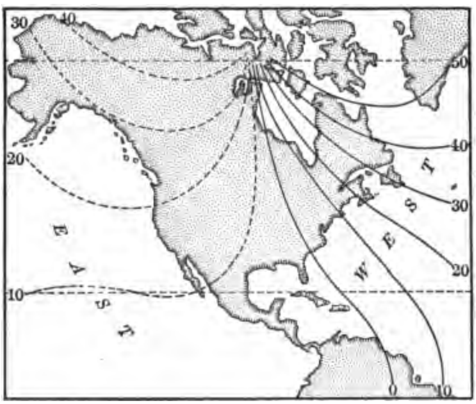


FIG. 166.

A line passing through all points where a compass points due north is called the *agonic line*. Places east of this line have a west declination; places west of the line have an east declination (Fig. 166). The declination at Portland, Me., is about 20° W; at Portland, Ore., is about 20° E.

The angle which a magnetized needle, pivoted at its center of gravity and free to swing in a vertical plane, makes with a horizontal plane through its point of support is called the *angle of inclination* or the *angle of dip*. This inclination at New York is about 73°.

CHAPTER XVI

STATIC ELECTRICITY

369. Electrification by Contact. — If a glass rod is rubbed with a piece of silk, both being dry, and the rod is then brought near bits of paper, straw, pith, etc., these light bodies are attracted, but soon after coming in contact with the rod are repelled. The glass rod is said to be *electrified* or to have a *charge of electricity* upon it. This property of being electrified by contact is common to all substances; but with some bodies, as metals, the electric charge developed at the part of the body rubbed spreads all over it; while with others, such as glass, vulcanite, sealing wax, etc., the electric charge does not spread beyond the part where it is developed. Bodies of the first kind are electric *conductors;* bodies of the second kind are *nonconductors* or *insulators*.

If a conductor is held in the hand while being rubbed, the electric charge spreads over the conductor and is carried through the hand and body to the earth over which it spreads, so that the quantity of the charge remaining on the conductor is so small a fraction of the quantity developed that the conductor will exhibit none of the properties of a charged body. To electrify a conductor it must first be insulated from the earth by supporting it upon some nonconducting substance.

370. Two Kinds of Electric Charge. — If a small pith ball, suspended by a silk thread, is approached by a glass

rod which has previously been electrified by being rubbed with a piece of silk, the pith ball will be attracted and after becoming electrified by contact with the glass rod will be repelled. If now, a rod of vulcanite which has been electrified by being rubbed with a piece of flannel is brought *near* the pith ball which the electrified glass rod repels, the pith ball will be attracted. It will, however, soon be repelled by the vulcanite rod when it will again be attracted by the glass rod if that is brought near. These facts indicate a *difference* in the electric charges on the glass and on the vulcanite. The glass is said to be *positively* charged, the vulcanite *negatively*.

From the mutual action of the charges it is evident that *like* kinds of electric charges *repel*, and *unlike* kinds *attract* each other.

371. Electrostatic Series. — Any substance in the following *electrostatic series* is positively electrified when rubbed by any substance following it in the list, and negatively electrified when rubbed by any preceding substance.

1. Catskin	6. Linen	11. Iron
2. Flannel	7. Silk	12. Copper
3. Feathers	8. The hand	13. Silver
4. Glass	9. Wood	14. Sulphur
5. Cotton	10. Shellac	15. Gutta percha

372. Gold-leaf Electroscope. — An *electroscope* is an instrument which shows the presence of an electric charge. A more sensitive electroscope than the pith-ball pendulum is made by suspending within a jar (Fig. 167) two strips of metal foil at the lower end of a metal rod which passes through the cover of the jar. The outer end of the metal rod terminates in a knob or disk.

If the electrified glass rod is brought in contact momentarily with the knob of the electroscope, some of its positive

charge passes down the metal rod and spreads over the two pieces of foil which then, by the repulsion of like electric charges, diverge widely. If the electrified glass rod is again brought *near* the charged electroscope, the leaves diverge still more widely, owing to the repulsion by the electrified glass rod of the positive charge on the knob and rod sending it into the leaves, thus increasing their charge and consequently their divergence.

FIG. 167.

If, however, an electrified piece of sealing wax or vulcanite is brought near the knob of the positively charged electroscope the leaves collapse, owing to the attraction between their charge and the negatively electrified sealing wax, resulting in the withdrawal of their charge to the knob.

373. Electric Field. — Since electrically charged bodies attract or repel one another according to the kind of their charge, there must exist in the space between two electrified bodies a *field of force*, which is similar in direction to that obtained with magnets under similar conditions.

374. Electric Potential. — A body when raised from the earth acquires potential energy by virtue of the work done against gravity which resists the motion. Similarly, a charged body acquires *electric potential* energy when moved against the electric forces acting upon it.

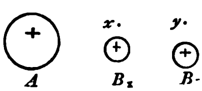


FIG. 168.

If a body A (Fig. 168) is positively charged, it requires an expenditure of energy to bring a body B, also positively charged, toward A against the

electric repulsion of *A* for it. A unit quantity of electricity is that quantity which will repel a like quantity when it is 1 cm. distant, with the force of 1 dyne. If there is unit quantity of electricity on *B*, the quantity of energy necessary to bring it from outside of the field of *A* to its present position *y* is its *electric potential.*

375. Difference of Potential. — To move *B* to the position *x*, still nearer *A*, requires an additional expenditure of energy; hence the potential at *x* is higher than at *y*, and the *difference of potential* between *x* and *y* is equal to the work done in moving the unit quantity of electricity on *B* against the electric force of the field, from *y* to *x*.

The **difference of potential** *between any two points in an electric field is equal to the energy expended by or against the electric forces in moving unit quantity of electricity from the one point to the other.*

On account of the electric repulsion of *A*, the electric charge on *B* tends to move from *x* toward *y*; hence, if a potential difference (*P. D.*) exists between two points, that point has the higher potential from which the positive electricity tends to move, *i.e.* electricity moves from points of high potential to points of lower potential.

CHAPTER XVII

CURRENT ELECTRICITY

376. Effects of Electric Current. — The presence of a moving charge of electricity in a conductor is manifested by certain energy changes, two of which take place within the conductor, while a third occurs outside of the conductor.

I. *Effects within the Conductor*

1. *Heating Effect.* — If a *thin* wire is properly connected in circuit with a generator such as that which supplies electric current for commercial purposes, this wire is observed to become hotter and hotter until, if the current is strong enough, it becomes incandescent, or may even melt.

2. *Electrolytic Effect.* — If a current is made to pass through acidulated water contained in a suitable vessel, shown in Fig. 169, by inserting in the vessel the terminals of two wires connected with such a circuit, a gas appears at each of these terminals. These gases upon being tested prove to be hydrogen and oxygen, of which water is composed. Any liquid compound which acts as a conductor of electricity is called an *electrolyte*, and the effect of an electric current in decomposing an electrolyte is called the *electrolytic* effect. The terminals of the wire placed in the electrolyte are called *electrodes*.

FIG. 169.

II. *Effect outside of the Conductor*

Magnetic Effect. — If a section of the wire connected in such a circuit is placed in a north and south direction over and near a magnetic compass needle (Fig. 170), the N end of the needle is deflected east or west, of north; if the wire is placed under the needle, the deflection is in the opposite direction. This *magnetic effect* indicates that a *magnetic field* exists about a wire carrying a current. The lines of magnetic force, whose direction is indicated by the deflected needle, must be concentric circles about the wire as an axis, since opposite deflections are obtained on the two sides of the wire.

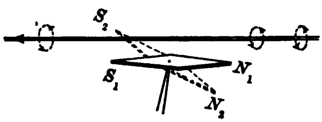


FIG. 170.

377. Simple Voltaic Cell. — The history of current electricity dates from the discovery of Volta, an Italian, in the latter part of the eighteenth century, that when strips of two unlike metals, such as copper and zinc, are placed in dilute sulphuric acid and connected by a wire, a charge of electricity moves continually through the *circuit* thus formed. This combination of zinc and copper in sulphuric acid is called a *simple voltaic cell.*

378. Chemical Action in a Simple Cell. — By chemical action is meant any action which results in the production of substances *differing in composition* from those with which the action begins.

Copper and zinc are elements, *i.e.* the molecules of each of these substances are made up of like atoms. Sulphuric acid, however, is a compound of three substances: hydrogen (H), sulphur (S), and oxygen (O), each molecule of

the acid containing two atoms of hydrogen, one of sulphur, and four of oxygen, as indicated by its formula, H_2SO_4.

If a piece of ordinary commercial zinc is placed in dilute sulphuric acid there is chemical action between the zinc and the acid, resulting (1) in the production of hydrogen (H) gas, coming from the acid molecules, appearing as bubbles on the surface of the zinc and then rising through the liquid; (2) in the wasting away of the zinc, which, combining with the SO_4 of the acid, forms a white salt called zinc sulphate, whose composition is indicated by its formula, $ZnSO_4$, in which Zn stands for zinc.

This white salt is soluble in the liquid present, but may be detected by evaporating some of the liquid to dryness, when the salt remains as a white solid. The wasting away of the zinc, if not otherwise apparent, may be shown by weighing it before and after the action has taken place.

If a piece of copper is placed in sulphuric acid, there is no evidence of action either by the evolution of gas, change in weight, or solid remaining after evaporating the liquid.

When the two strips are placed in the acid and brought in contact, or connected, action occurs, but the hydrogen gas appears on both strips. By weighing these strips before and after the action, it is found, however, that the zinc alone decreases in weight, and the same white solid, zinc sulphate, is obtained by evaporating some of the liquid.

If the conditions are modified by *amalgamating* the piece of zinc, *i.e.* rubbing mercury over its surface thus forming a pasty *alloy* with the zinc, no action takes place either when the zinc is *alone* in the acid or when both zine and copper are in the acid but *unconnected* with each other. However, as soon as the circuit is "*closed*" by connecting the metal strips, chemical action begins, but the hydrogen gas now appears only at the surface of the copper.

379. Local Action. — Any action that takes place in a cell when the strips are not connected, and which is evidenced by the appearance of gas on the zinc strip, is called *local action*. From the foregoing it appears that amalgamating the zinc prevents this local action.

If the amalgamated zinc strip and the copper strip are weighed after being connected for some time in the acid, it is found that the zinc alone has decreased in weight, showing that although hydrogen appears at the surface of the copper, the copper is not chemically acted upon by the acid.

The fact that amalgamated zinc is acted upon by the acid only when connected with the copper strip, shows the importance of such a connection.

After *ordinary* zinc has been acted upon by the acid for some time, it turns dark in color and black particles are seen in the bottom of the cell, which upon examination prove to be particles of carbon (or iron) that were not completely removed when the zinc ore was smelted in the process of the manufacture of the zinc. It was the contact of these dissimilar substances with the zinc in the presence of the acid that brought about the local action referred to above. Amalgamating the zinc covers up these impurities and the strip then acts like *chemically pure* zinc, which is not acted upon by the acid unless connected with some dissimilar substance.

380. Electric Current from a Simple Cell. — If a section of the wire connecting the zinc and copper strips in a simple cell is given a north and south direction, and is then held near a magnetic compass needle, the needle is deflected, showing that an electric current is flowing through the wire which connects the strips in the acid.

If a current flowing *north over* a compass needle produces a west deflection of the N end, a current flowing *south under* the needle will also produce a west deflection. Hence, by looping the wire many times around the needle in a north and south direction (Fig. 171), the magnitude of the deflection is increased, because the magnetic force on both the upper and lower sides of the loop has the same direction *within* the loop.

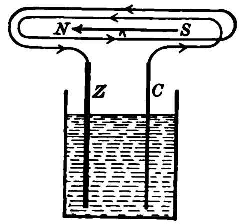


FIG. 171.

A testing instrument called a *voltmeter* (Figs. 172 *a* and *b*) consists of a coil of many turns of fine *insulated* wire within which is pivoted a magnetic needle, so that when a current flows through the coil, the deflection of the needle causes a pointer attached to it to move over a graduated scale. If the copper and zinc strips of a simple cell are connected to the ends of the coil of a voltmeter, a deflection is produced. If, however, this connection is continued for several minutes, a considerable decrease in this deflection is noticeable, indicating a weakening of the current flowing from the cell.

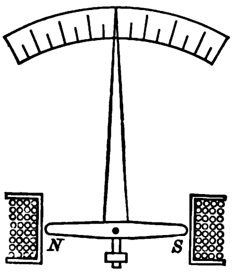


FIG. 172 *a*.

From the definition of *potential difference* it follows that the two strips, copper and zinc, in the sulphuric acid, are at different electric potentials, since a charge of electricity is moved

FIG. 172 *b*.

from one to the other; and since the charge moves in the connecting wire from the copper to the zinc, the copper is at higher potential than the zinc.

The potential difference (P. D.) between the copper and the zinc, as measured (for a reason to be given later) by the magnitude of the deflection of the voltmeter needle, appears to decrease during the first few minutes after the circuit is closed.

381. Source of the E. M. F. of a Cell. — If two metals are placed in an acid which acts upon one and not upon the other, or more upon one than upon the other, the metals are of different electric potentials and the more soluble metal is of the lower potential. By the continued chemical action between the zinc and the acid this potential difference is maintained, and a current of electricity moves through the wire.

Whenever in any circuit there is maintained a continual potential difference, that circuit is said to contain an **electromotive force.**

382. Electric Circuit. — An *electric circuit* is the series of conductors through which the current flows, including the conductors within the generator. In the case of the circuit of the cell, the current flows through the wire from the copper to the zinc, and then through the liquid from the zinc to the copper. That terminal of a cell from which the current flows into the connecting wire is called the *positive* terminal, and the other the *negative* terminal. In the simple cell the upper end of the copper is the positive (+) terminal, and the upper end of the zinc is the negative (−) terminal.

383. Conduction of Current. — Electric charges move in two different ways. In the case of *solid conductors* the

current passes *through* them, the parts of the conductor themselves not moving; but in the case of *liquid conductors* (except elements, such as mercury) the electric charge is carried *upon and with* moving portions of the matter. A liquid compound which will conduct an electric current is an *electrolyte*. The aqueous solutions of the common acids, bases, and salts are the most important electrolytes.

When an acid, base, or salt is dissolved in water, it becomes to a certain extent *dissociated*, *i.e.* some of the molecules break up into what are called *ions*, each of which is an atom, or group of atoms, *which is electrified*. When sulphuric acid, H_2SO_4, is dissociated, there are formed two hydrogen ($\overset{+}{H}$, $\overset{+}{H}$) ions and one sulphuryl ($\overset{--}{SO_4}$) ion, the hydrogen ions being positively charged, the sulphuryl ion negatively charged.

All bodies under ordinary conditions have equal quantities of positive and negative charges which neutralize one another, thus rendering them non-electrified. If, however, some of the positive electricity is removed from a body, the negative charge then being in excess makes the body negatively electrified. When a metal plate dissolves in an acid, the *metal* ions leaving the plate carry with them *positive* electric charges — which leaves the plate negatively charged.

As the zinc plate begins to dissolve in sulphuric acid, the positive zinc ($\overset{++}{Zn}$) ions leaving this plate attract the negative sulphuryl ($\overset{--}{SO_4}$) ions of the dissociated acid and repel the positive hydrogen ($\overset{+}{H}$) ions toward the copper plate. The $\overset{++}{Zn}$ ions and $\overset{--}{SO_4}$ ions ultimately combine to form zinc sulphate ($ZnSO_4$), and the $\overset{+}{H}$ ions give up their positive charge to the copper plate, rendering it positively

electrified, while they themselves become molecules of ordinary hydrogen (H_2).

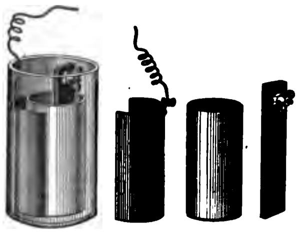


FIG. 173.

In consequence of the positive charge on the copper plate and the negative charge on the zinc plate there is a flow of electricity *through* the wire from the copper to the zinc. Through the liquid the charge is carried from the zinc to the copper *upon* and *with* the hydrogen ions.

384. Polarization of a Cell. — The explanation of the decrease in the potential difference of the copper and zinc strips of a simple cell after being connected for several minutes is based upon the fact that if the copper strip is shaken while in the liquid the voltmeter reading at once increases;

FIG. 174.

FIG. 175.

also if the copper is removed from the cell and held for a few seconds in a Bunsen flame, and is then returned to the cell and connected as before, the voltmeter reading again has the maximum value. Thus it is evident that this de-

crease in the potential difference of the metal strips of the cell, which is called *polarization* of the cell, is due to the deposit on the copper plate of something that is removable in this manner. As has been shown, it is hydrogen that is deposited on the copper of a simple cell when the strips are connected. The remedy for polarization, therefore, is to surround the copper (or other insoluble element which may be used) in the cell with some substance which combines readily with hydrogen, such as copper sulphate ($CuSO_4$), nitric acid (HNO_3), potassium bichromate ($K_2Cr_2O_7$), manganese dioxid (MnO_2), etc.

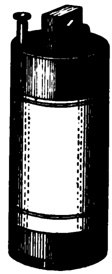


FIG. 176.

All of these compounds are rich in oxygen (as is seen by their formulæ), which combines with the hydrogen thus forming water (H_2O). These substances when used to prevent polarization are called *depolarizers*.

FIG. 177.

385. Explanation of Polarization. — The theoretical explanation of the polarization of a cell caused by hydrogen accumulating on the insoluble plate, is that a layer of gas on that plate constitutes a layer of *nonconducting* material, and that the positively charged hydrogen ($\overset{+}{H}$) ions, thus being unable to give up their charge to the copper and then combine with each other to form neutral molecules of hydrogen, *remain* near the plate and repel the like

FIG. 178.

positive charges of the hydrogen ($\overset{+}{H}$) ions moving toward the copper plate. This electric repulsion being in the opposite direction to the electromotive force of the cell lowers the potential difference of the terminals.

386. Some Common Cells. — The following are the constituents of several well-known cells: —

Name	Elements	Exciting Electrolyte	Depolarizer
Daniell (Fig. 173)	Zinc-copper	Sulphuric acid (H_2SO_4)	Copper sulphate ($CuSO_4$)
Gravity (Fig. 174)	Zinc-copper	Zinc sulphate ($ZnSO_4$)	Copper sulphate ($CuSO_4$)
Leclanché (Fig. 175)	Zinc-carbon	Ammonium chlorid (NH_4Cl)	Manganese dioxid (MnO_2)
Dry (Fig. 176)	Zinc-carbon	Ammonium chlorid (NH_4Cl)	Manganese dioxid (MnO_2)
Bunsen (Fig. 177)	Zinc-carbon	Sulphuric acid (H_2SO_4)	Nitric acid (HNO_3)
Edison-Lalande (Fig. 178)	Zn-copper oxid	Potassium hydroxid (KOH)	Oxygen in the copper oxid

387. Electrical Units. — There are five electrical quantities whose values must be accurately measured in order to obtain intelligible results with regard to the transformations of energy that take place in an electric circuit. They are (1) *electromotive force* (E. M. F.), (2) *potential difference* (P. D.), (3) *current strength* (C.), (4) *electrical resistance* (R.), (5) *electric power* (P.).

1. *Electromotive force is that which maintains continually a difference of potential in an electric circuit.* It arises in that portion of the circuit where there is a transformation into electrical energy of some other form of energy, such as chemical affinity, mechanical energy, heat, etc.

There is no E. M. F. if there is no source of energy in a circuit. Electromotive force is measured by the *total* potential difference in the circuit, *i.e.* the P. D. of the terminals of the generator *plus* the drop in potential *within* the generator.

2. *Potential difference*, as previously defined, *is that condition by virtue of which electricity tends to move from one point to another in the circuit*, and its value is the energy expended in moving a unit of positive electricity from the one point to the other against the E. M. F. of the circuit which maintains this potential difference.

The relation between electromotive force and potential difference may be illustrated by the following analogy:—

Suppose two tanks, *A* and *B*, containing water at different levels, are connected by a pipe *CD*, as shown in Fig. 179. If the water stands higher in *A* than in *B*, *the difference of pressure* or *head* of water tends to move it from *A* to *B*, and if the valve in the pipe is opened, the water flows until it is at the same level in both tanks. If now a pump *P* is connected to the tanks so that it pumps water from tank *B* into tank *A*, it will maintain a continual head of water in *A*, *i.e.* the gravitation potential of the water in *A* will be continually higher than in *B*.

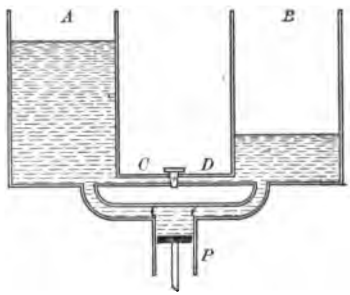


FIG. 179.

The *head of water* is analogous to *electric potential difference*, and the *ability of the pump*, through the energy applied to it, *to maintain this head* is analogous to *electromotive force*.

The *unit* of electromotive force and of potential difference, called a *volt*, is the $\frac{1000}{1434}$ part of the E. M. F. of a standard Clark cell. The E. M. F. of a Clark cell is 1.434 volts at 15° C. A Clark cell is a zinc-mercury cell, with the zinc in zinc sulphate and the mercury in mercury sulphate, all of the materials being chemically pure.

Electromotive force and potential difference are both measured in the same units, since they are both determined by the expenditure of energy per unit quantity of electricity.

3. *Current strength* is the quantity of electricity that passes a given point in the circuit per second. The unit of current strength, called an *ampere*, is that current which, if passed through a given solution of silver nitrate, will deposit silver at the kathode (negative plate) at the rate of .001118 gm. per sec.

4. *Electrical resistance* is the opposition a substance offers to the passage of an electric current through it. The unit of resistance, called an *ohm*, is the resistance offered to an unvarying current by a column of mercury 106.3 cm. long, 1 sq. mm. in cross section, at a temperature of 0° C. An ohm in practical use is represented by the resistance of a certain coil of wire whose resistance has been compared with that of such a mercury column. A piece of apparatus consisting of various coils of wire, whose resistances are multiples or submultiples of an ohm, is called a *rheostat*.

5. *Electric power* is the energy expended per second by the electric current. The unit of power, called a *watt*, is the energy expended by a 1-ampere current in that portion of a circuit the terminals of which have a potential difference of 1 volt. Since E. M. F. (E) equals the expenditure

of energy per unit quantity of electricity, and C equals the quantity of electricity moving in the circuit per second, their product, or EC, is the total expenditure of energy per second in the circuit, or the power P, *i.e.* $P = EC$. To determine the energy expended per second in any portion of a circuit, it is only necessary to multiply the P. D. of that portion of the circuit by the current flowing through it, the product of volts and amperes being watts. A kilowatt (kw.) is 1000 watts.

If in a portion of an electric circuit energy is expended at the rate of a kilowatt for the period of one hour, the total energy expended is a kilowatt-hour. Electric power is furnished to consumers at from 5 cents to 15 cents per kilowatt-hour.

For example, if the P.D. of an incandescent lamp is 110 volts and $\frac{1}{2}$ ampere flows through the lamp, it expends energy at the rate of $110 \times \frac{1}{2} = 55$ watts $= .055$ kilowatt. At 15 cents per kilowatt-hour it would cost $.055 \times 15 = .825$ cent to use the lamp for one hour. Again, if the P.D. at the terminals of a street-car motor is 500 volts and 20 amperes flow through it, the rate of expenditure of energy is $500 \times 20 = 10{,}000$ watts $= 10$ K.W. At 5 cents per kilowatt-hour it would cost $5 \times 10 = 50$ cents to run the motor for one hour.

388. Essential Conditions for the Production of an Electromotive Force in a Cell. — By experiment it may be proved that the essential conditions for the production of an E. M. F. in a cell are: (1) the plates, or elements, must be of different substances; (2) they must be electrical conductors; (3) they must be placed in an electrolyte which acts more upon one than upon the other. However, it is found that if in various cases these conditions are fulfilled the magnitude of the E. M. F. is different for each different combination of elements. If the five elements, zinc, lead, iron, copper and carbon, are chosen and the ten possible

combinations of these in pairs are taken with sulphuric acid as the electrolyte, the potential difference of the first and last of the elements in this series, or zinc and carbon, is the greatest, and the potential differences of a combination of a pair of these substances decrease the nearer they are to one another in the series.

389. Current Strength in a Circuit.—The current strength in a circuit may be determined by introducing into the circuit an instrument called an *amperemeter* (or *ammeter*), which is constructed similar to a voltmeter except that the coil of wire used is of such size as to offer extremely little resistance to the current. If ammeters are inserted in two different electric circuits identical in every other respect except that the generator in one has a higher E. M. F. than in the other, the *current strength* in the circuit of higher electromotive force is proportionally greater. Also, if in any given circuit various resistances are introduced, provided the E. M. F. of the circuit remains unchanged, the *current strength* varies inversely as the total resistance in the circuit. These two relations are expressed in the form of a law, known as Ohm's Law:—

The current strength in any circuit varies directly as the electromotive force and inversely as the total resistance.

If C equals the current strength in amperes, E equals the E. M. F. in volts, R equals the resistance in the circuit *external* to the generator, and r equals the *internal* resistance of the generator, then —

$$C = \frac{E}{R + r}.$$

For example, if the E. M. F. (E) of a generator is 10 volts, the external resistance (R) is 4 ohms, and the internal

resistance (r) of the generator is 1 ohm, then the current strength (C) in the circuit is

$$C = \frac{10}{4+1} = 2 \text{ amperes.}$$

390. Ohm's Law applied to a Portion of a Circuit. — The relation between volts, amperes, and ohms, as expressed by the equation, $C = \frac{E}{R}$, where E is the electromotive force and R is the resistance of the entire circuit, holds good also for any portion of a circuit and may be expressed: $C = \frac{PD_1}{R_1}$, where PD_1 is the potential difference in volts between the terminals *of the given portion* of the circuit, R_1 is the resistance in ohms of that portion, and C is the current strength in amperes.

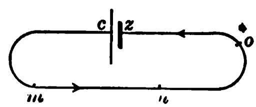


FIG. 180.

In any single circuit the current strength has the same value throughout the circuit. Consequently in the circuit, *cmnozc* (Fig. 180), the current strength in the portion *mn* of the circuit, or C_{mn}, equals the current strength C_{no} in the portion *no* of the circuit, and $C_{mn} = \frac{PD_{mn}}{R_{mn}}$, and $C_{no} = \frac{PD_{no}}{R_{no}}$; therefore, $\frac{PD_{mn}}{R_{mn}} = \frac{PD_{no}}{R_{no}}$, and $PD_{mn} : PD_{no} = R_{mn} : R_{no}$, or *the potential difference between any two points in an electric circuit is directly proportional to the resistance between those two points.* This actually is the form in which G. S. Ohm, the German scientist, expressed his law.

391. Illustration. — To illustrate this relation which is the most important and universal relation in electricity, suppose that in the given circuit, *cmnozc* (Fig. 180), the E. M. F. is 2 volts, the resistance (r) of the generator is 1 ohm, the resistance between c and m is 1 ohm,

between m and n is 2 ohms, between n and o is 3 ohms, between o and z is 3 ohms. The total resistance is $1 + 1 + 2 + 3 + 3 = 10$ ohms. The current strength is $C = \frac{2}{10} = .2$ ampere.

The potential difference between c and m, PD_1, divided by the resistance between c and m, 1 ohm, equals the current strength, .2 ampere,

$$\frac{PD_1}{1} = .2, \quad PD_1 = .2 \text{ volt.}$$

If the ends of a voltmeter coil are connected to the points c and m, the voltmeter reading will be .2 volt.

Similarly, the potential difference between m and n, PD_2, divided by the resistance between m and n, 2 ohms, equals the current strength, $\frac{PD_2}{2} = .2$ ampere, from which $PD_2 = .4$ volt, *i.e.* if the ends of a voltmeter coil were connected to m and n, the reading would be .4 volt.

Finally, since the resistance of the *external* circuit, *cmnoz*, equals 9 ohms, if the voltmeter was connected to the points c and z, the reading PD_3 would be 1.8 volts, since $\frac{PD_3}{9} = .2$ ampere.

It is important to note that since the E. M. F. is 2 volts and the potential difference of the terminals of the generator is 1.8 volts, there is a drop in potential *within* the generator of .2 volt which equals the current strength, .2 ampere, times the generator resistance, 1 ohm.

This fall of potential in a circuit may be graphically represented as in Fig. 181, where the circuit *zcmnoz* is plotted

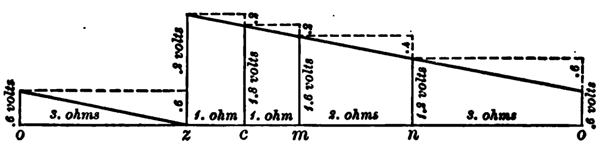


FIG. 181.

along the horizontal line oo, the electric potential of any point being represented by the vertical distance of the sloping line above oo at that point.

The electromotive force, 2 volts, is measured by the maximum potential at *z*, and is located at the zinc because the chemical energy is there transformed into electrical energy. There is a drop in potential of .2 volt within the cell represented by the difference in the ordinates at *z* and *c*. The potential difference of the terminals of the cell, 1.8 volts, is equal to the *sum of the potential differences* of the various external portions of the circuit.

392. Voltmeter and Ammeter Contrasted.—It may now be shown why it is that if the external conductor *cmnoz* is disconnected from the generator *cz*, and the terminals of the generator are connected directly to a voltmeter coil whose resistance is, say, 999 ohms, the reading of the voltmeter would be practically equal to the E. M. F. of the circuit, 2 volts.

For the whole resistance now is $999 + 1 = 1000$ ohms. The current strength $C = \frac{2}{1000} = .002$ ampere. The reading of the voltmeter, PD_1, divided by its resistance 999 ohms or $\frac{PD_1}{999} = .002$.

$$PD_1 = 999 \times .002 = 1.998 \text{ volts.}$$

The drop in potential within the generator, PD_1, divided by its resistance 1 ohm, equals the current strength, .002 ampere.

$$\frac{PD}{1} = .002, \text{ from which } PD = .002 \text{ volt.}$$

Since the resistance of the voltmeter is many hundred times as large as the resistance of the generator, the fall of potential in the voltmeter (1.998 volts) represents in this case to within $\frac{1}{10}$ of one per cent the E. M. F. of the generator.

If the resistance of the voltmeter coil had been some small resistance, as 1 ohm instead of 999 ohms, the fall of potential in the voltmeter would no longer represent the E. M. F. of the generator. For in this case the total resistance would be $1 + 1 = 2$ ohms. The current $C = \frac{2}{2} = 1$ ampere. The potential difference of the voltmeter terminals PD_1 divided by its resistance 1 ohm or $\frac{PD_1}{1} = 1$ ampere. Therefore

$PD = 1$ volt, which is but one half of the E. M. F. of the circuit. Hence the measurement of the needle of an ammeter which has a small resistance cannot be taken as a measure of E. M. F. or of P. D.

393. Magnetic Effect of Electric Current. — As was previously stated, if a magnetic compass is held near a wire traversed by an electric current, the needle is, as a rule, deflected, and the deflection, when the needle is above the wire, is in the opposite direction from that when the needle is below the wire. The direction of the deflection of the N end indicates the direction of the magnetic force acting on it; hence it is inferred that the lines of force of the magnetic field produced by the current take the direction of concentric circles in a plane perpendicular to the wire, which is their center.

Let the central portion in Fig. 182 *a* represent a cross section of a wire perpendicular to the plane of the paper with the current flowing down toward the paper; experiment shows that the lines of force have the direction indicated by the arrows in the concentric circles. As is apparent, this force has a clockwise direction, or the direction in which a right-handed screw must be rotated to move it down in the direction the current flows.

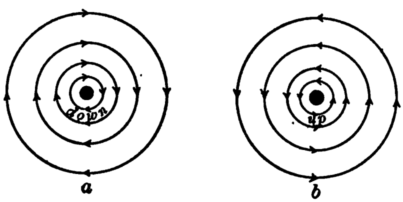


FIG. 182.

In Fig. 182 *b* the current is represented as flowing up through the plane of the paper, in which case the direction of the magnetic force is counter-clockwise, as indicated in the concentric circles. This direction is also the same as that in which a right-handed screw must be rotated in order to move it in the direction of the current.

The relation between the direction of magnetic force and direction of current may also be expressed in the form of a right-hand rule:—

If the conductor is grasped in the right hand (Fig. 183), *with the thumb extended in the direction of the current, the direction of the lines of magnetic force is around the conductor in the direction of the fingers.*

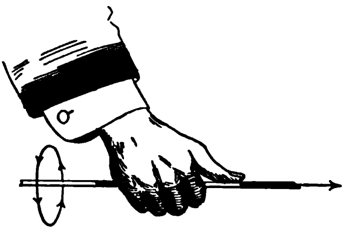


FIG. 183.

If a current flows *north over* a compass needle, the deflection of the N end is toward the west; if the current flows *south under* the needle, the deflection is also toward the west; consequently, if the wire is bent in the form of a vertical loop (Fig. 184) around the needle, the direction of the lines of force *within* the loop produced by the current in the upper side *m* is the same as that of those produced by the current in the under side *n* of the loop, and the deflection is thereby increased. If instead of using a loop of one turn the needle is placed within a coil of many turns, the lines of force within the coil are increased an equal number for each turn of the coil; hence the deflection increases with an increase in the number of turns in the coil.

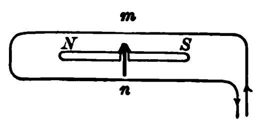


FIG. 184.

If an insulated wire is wound about a soft iron rod as a core, and an electric current flows through the wire, the iron rod acts like a magnet the intensity of which depends upon the strength of the current and the number of turns of the coil. The coil itself without the iron core will also act as a magnet when a current is flowing through it; for the lines of force running lengthwise within the coil, upon emerging

from one end, move around through the outside space, reëntering at the other end of the coil, just as in a magnet the lines of force within the magnet move from the S toward the N end, and upon emerging from this end they circulate through the surrounding space, reëntering near or at the S pole. The effect of the iron core is to increase the number of lines of force for a given magnetizing force by increasing the magnetic permeability within the coil. The right-hand rule previously stated and slightly modified to fit the present case shows how to find the position of the N end of the electromagnet, provided the current direction is known: —

If the electromagnet coil is grasped in the right hand so that the current flows toward the finger ends, the extended thumb points toward the N pole (Fig. 185).

If the core of an electromagnet is soft iron of good quality, it will be found upon opening the current, or decreasing the strength of current in the coil, that the intensity of the magnetic force decreases almost to zero, or decreases with the decrease in current. It is because such an electromagnet is so completely under control that it is used in a great majority of the practical applications of electricity, — such as electric motors, telegraph, telephone, bells, annunciators, arc lamp, etc.

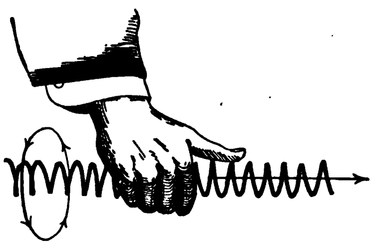


FIG. 185.

394. Heating Effect of Electric Current. — In every portion of an electric circuit a part or all of the electrical energy therein expended is transformed into heat, causing a raise

in temperature of the conductor and the contiguous medium. If in any portion of a circuit there is also a transformation of electrical energy into chemical energy, as in the process of electrolysis, or into mechanical energy, as in an electric motor, then only a part of the total electrical energy therein expended is converted into heat; in any other portion of the circuit *all* of the expended electrical energy is so converted.

If V represents the potential difference in volts between the ends of any given portion of a circuit and C the current strength in amperes, the total electrical power of that portion of the circuit is VC watts. Since $C=\frac{V}{R}$, or $V=CR$, the total power may be expressed as $CR \times C = C^2R$ watts. The quantity of heat produced increases, therefore, with an increase in the current strength, being directly proportional to the square of the current; and also with an increase in the resistance of the conductor, being directly proportional to that resistance. In that part of an electric circuit where there is the greatest resistance (the current being the same), there will be the greatest quantity of heat produced. The unit of heat, called a calorie, is the quantity of heat necessary to raise the temperature of 1 gm. of water 1° C. As stated previously in the discussion of mechanical energy, a watt equals the expenditure of energy at the rate of 1 joule per sec. The relation between the unit of heat and the unit of energy is expressed by the equation, 1 calorie $= 4.2$ joules. The product C^2R is expressed in watts or in joules per second; to convert this into calories, divide by 4.2. Therefore the quantity of heat, H, obtained per second from a given quantity, C^2R, of electrical energy is expressed by the equation, $H=\frac{C^2R}{4.2}=C^2R \times .24$ calo-

ries. The total quantity of heat produced in a given time, t seconds, by passing a current of C amperes through a conductor whose resistance is R ohms, equals $C^2Rt \times .24$ calories.

The heating effect of the electric current is usefully applied in electric lamps, electric heaters, electric welding, and in the use of fuse wire for protecting a given portion of a circuit from excessive current.

395. Electrolytic Effect of a Current. — An *electrolyte* is a compound which when changed to the liquid state, either by being dissolved or by being fused, becomes more or less dissociated, *i.e.* some of its molecules separate into parts, each of which carries an electric charge. These charged parts of molecules are called *ions*, and consist either of a single atom or of a group of atoms. Ions consisting of *hydrogen* or of any *metal* are always *positively charged;* all other ions are *negatively charged.*

The ability of a liquid compound to conduct a current of electricity is due to dissociation, and the extent of dissociation may be measured by the conductivity of the liquid.

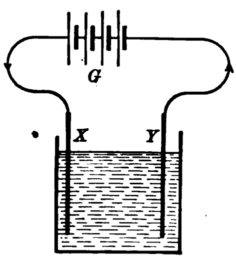


FIG. 186.

In the process of *electrolysis*, *i.e.* the chemical decomposition of a compound by means of the electric current, the *terminals* X and Y (Fig. 186), called the *electrodes*, are placed in the electrolyte in order to introduce it in circuit with the generator G. The electrode X through which the current *enters* the electrolyte is the *positive* (+) electrode, or *anode;* the electrode Y through which the current *leaves* the electrolyte is the *negative* (−) electrode, or *kathode*. The *ions* of

the electrolyte which carry positive electric charges move toward the kathode, while the negatively charged ions move toward the anode, where they give up their charges and are generally set free, becoming ordinary molecules of that substance of which the ions consist. Hence by the electrolysis of any compound containing a metal, that metal is liberated at the kathode.

This effect is usefully applied in the processes of electrotyping and electroplating, and in the production on a commercial scale of many metals, such as aluminum and sodium.

Michael Faraday, a noted English scientist, who first made a thorough study of electrolysis, expressed his results in the form of two laws: —

1. *The mass of any substance liberated at the electrodes is proportional to the quantity of electricity that passes through the electrolyte.*

2. *For the same current, the mass of any substance liberated at the electrodes is proportional to its chemical equivalent.*

The chemical equivalents of a few common elements is given in the following table: —

Element	*Equivalent*
Copper	31.8
Hydrogen	1
Silver	107.94
Zinc	32.7

From the definition of an ampere it is known that 1 ampere current will deposit at the kathode per second .001118 gm. of silver. The quantity of any other substance than silver, such as zinc, whose equivalent is 32.7, which will be deposited per second by a current of 1 ampere, is found by the proportion, equivalent of silver, 107.94, is to equivalent of zinc, 32.7, as the quantity of silver deposited per second by 1 ampere, .001118 gm., is to the quantity of zinc so deposited, *i.e.*

$$107.94 : 32.7 = .001118 \text{ gm.} : x \text{ gm.},$$
$$107.94x = .0365586,$$
$$x = .000339 \text{ gm. of zinc,}$$

deposited per second by 1 ampere current. Similarly, it may be found that .000329 gm. of copper is deposited at the kathode per second by a current of 1 ampere.

This quantitative relation is usefully applied in the chemical electric meters, by means of which the quantity of electricity supplied to a consumer is determined from the increase in weight of the kathode of an electrolytic cell placed in the circuit. This quantity of electricity multiplied by the constant voltage maintained gives the energy expended.

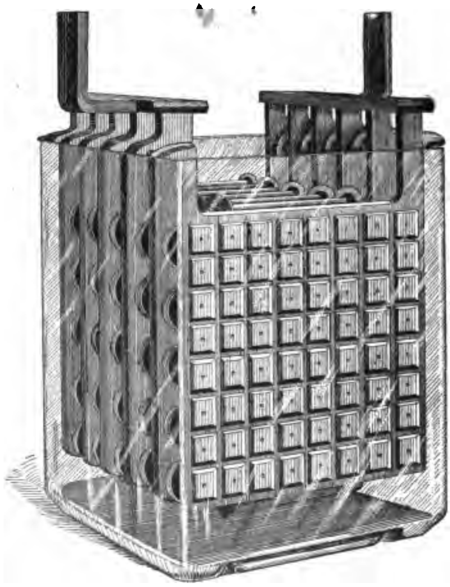


FIG. 187.

395 a. Storage Cells.—Another important application of the electrolytic effect is in the manufacture of *storage cells* (Fig. 187), sometimes called *accumulators*.

If two grids made of lead (Pb) and filled with litharge (PbO) are placed in dilute sulphuric acid (H_2SO_4) and a current of several amperes is passed for a number of hours through this cell, the hydrogen evolved at the kathode

reduces the litharge there to metallic lead (Pb), while the oxygen liberated at the anode *oxidizes* the litharge there to lead peroxide (PbO_2). The two grids, alike at the start, thus become essentially different, and this difference is manifested as a difference of electric potential. The potential difference of the terminals of a storage cell when thus charged is usually about 2 volts.

When this "charged" cell is used to send a current through a given circuit, the current flows *outside* of the cell from the PbO_2 grid to the Pb grid and back through the liquid *in* the cell from the Pb grid to the PbO_2 grid. Hydrogen will then be formed at the PbO_2 grid, just as it is at the copper plate in a simple cell, which gradually *reduces* the lead peroxide to its original condition, litharge, while the oxygen liberated at the Pb grid *oxidizes* it to the original condition. The cell is then completely discharged, and to obtain a current from it again it must be recharged as before.

While the cell is being charged, the current *enters* at the plate, which is changed from PbO to PbO_2. When the cell is being discharged, its current *leaves* at the PbO_2 plate; therefore it follows that the potential difference established in charging a storage cell is in the *opposite* direction to the E. M. F. of the charging current, and thus constitutes a counter E. M. F. Since, as stated above, this counter E. M. F. equals about 2 volts per cell, the generator used to charge such a cell must have an E. M. F. higher than 2 volts to drive the current through the cell against this opposition.

396. Arrangement of Cells to form a Battery. — From Ohm's law, $C = \frac{E}{R + r}$, it follows that to increase the current through a given external resistance R, it is neces-

sary to increase the voltage E, or to decrease the internal resistance r, or to do both. To produce these results cells may be connected to form a battery in three different arrangements: (1) all *in series;* (2) all *in parallel;* (3) in a number of rows, the cells in each row being in series, but the rows connected in parallel.

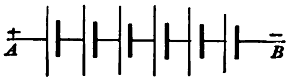


FIG. 188.

Figure 188 shows 6 cells connected all in series; A and B are the terminals of the battery.

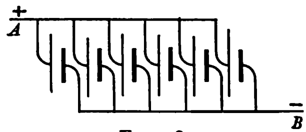


FIG. 189.

Figure 189 shows 6 cells connected all in parallel; A and B are the terminals.

Figure 190 shows 6 cells connected in 2 rows of 3 cells each; A and B are the terminals.

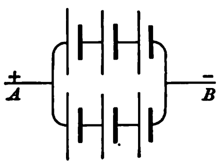


FIG. 190.

Figure 191 shows 6 cells connected in 3 rows, 2 in a row; A and B are the terminals.

397. Electromotive Force of a Battery. — Figure 192 shows two cells connected in series. The potential difference between points A and B is the E. M. F. of one cell. The potential difference between points B and C is likewise the E. M. F. of one cell. Therefore the potential difference between A and C is the sum of the potential differences between A and B and between B and C, or equal to twice the E. M. F. of one cell. If, then, s cells of e volts each are connected in series, the E. M. F. of the battery is se volts.

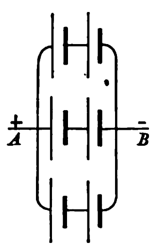


FIG. 191.

Figure 193 shows two cells connected in parallel. The

potential difference between B and C is the E. M. F. of one cell. The potential difference between points D and E is likewise the E. M. F. of one cell. But B and D are connected to the same point A; hence A, B, and D are of the same potential. Likewise C, E, and F are of the same potential. Therefore the potential difference between points A and F, which are the terminals of the battery, is only equal to that of one cell. Hence the E. M. F. of a battery of p cells of e volts each, all connected in parallel, equals e volts, the E. M. F. of one cell.

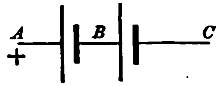


FIG. 192.

If n cells are arranged in p rows with s cells in each row, $n = sp$. The cells in each row being connected in series, the E. M. F. of one row equals se volts. Since, by connecting in parallel the voltage is not increased, the E. M. F. of p such rows connected in parallel continues to be se volts, or the E. M. F. of the battery of n cells so arranged equals the E. M. F. of one row, se volts.

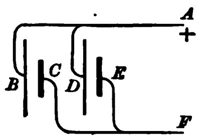


FIG. 193.

398. Internal Resistance of a Battery of Cells. — When cells are connected in series, since the entire current passes through each cell the resistance of s such cells of r ohms each is the *sum* of the resistances, or sr ohms.

If v represents the drop in potential in each cell of a battery connected in parallel, the current through each cell is $\frac{v}{r}$ amperes. The total current through p cells connected in parallel is $\frac{pv}{r} = \frac{v}{\frac{r}{p}}$ amperes. Therefore the total resistance of a battery of p cells in parallel is the resistance of one cell divided by the number of cells, or $\frac{r}{p}$ ohms.

When n cells are connected in p rows of s cells each, the resistance of each row, since the cells are in series, equals sr ohms. Then the resistance of p such rows connected in parallel equals $\frac{sr}{p}$ ohms.

399. Current Strength of a Battery.—From this it follows that by connecting cells in series the voltage is increased as many times as there are cells so connected, and that the internal resistance is likewise increased in the same ratio. Consequently if the value of R, the *external* resistance, is a *small* quantity, there is no advantage in connecting cells in series. For $\frac{se}{R + sr}$ approaches the value of $\frac{se}{sr} = \frac{e}{r}$ as R approaches the value of zero. But $\frac{e}{r}$ equals the current strength from *one* cell if R equals zero, hence, in such a case, the current from a battery of cells in series is no greater than would be obtained from one cell.

If, however, the value of the external resistance is large, the value of the denominator of the fraction, $\frac{se}{R + sr}$, is not increased s times by adding sr to R for s cells, but the voltage or numerator is thus increased s times. Therefore the current from cells in series is greater than from one cell when there is large external resistance, and very nearly proportionally greater if the value of r is relatively small.

By connecting cells in parallel, although there is no increase in voltage over that of one cell, there is a *decrease* in the resistance of the battery, which renders it advantageous to connect cells in parallel if the external resistance is comparatively small. For $\frac{e}{R + \frac{r}{p}}$ approaches the value of $\frac{e}{\frac{r}{p}} = \frac{pe}{r}$

as R approaches the value of zero, and $\frac{pe}{r}$ is p times $\frac{e}{r}$, which is the current obtainable from *one* cell when R equals zero.

If, however, the external resistance is large, the current from cells in parallel is not much stronger than would be obtained from one cell, because decreasing the internal resistance from a value of, say, $\frac{1}{2}$ ohm to $\frac{1}{10}$ ohm will not greatly change the relative value of the denominator, $R + r$, if R is a large resistance.

From the foregoing the following equations are obtained: —

1. For s cells connected in series, $C = \frac{sE}{R + Sr}$. (1)

2. For p cells connected in parallel, $C = \frac{E}{R + \frac{r}{p}}$. (2)

3. For n cells arranged in p rows with s cells in each row, $C = \frac{sE}{R + \frac{sr}{p}}$, where $n = sp$. (3).

It can be proved by an algebraic solution of equation (3) that the conditions under which the value of C is a maximum is when the cells are so arranged that the internal resistance, $\frac{sr}{p}$, of the entire battery most nearly equals the external resistance, R.

PROBLEMS

1. The P. D. of the terminals of an arc lamp is 50 volts, the current flowing in the lamp is 10 amp. (*a*) How many watts are expended in the lamp? (*b*) Since 1 watt = 1 joule per sec., how many joules of energy are expended in the lamp in 1 hr.? *Ans.* (*a*) 500 watts. (*b*) 1,800,000 joules.

2. How many joules of energy does a kilowatt-hour represent?
Ans. 3,600,000 joules.

3. The P. D. of the terminals of an incandescent lamp is 115 volts, and .45 amp. flows through the lamp; what is the resistance of the hot filament of the lamp? *Ans.* 255.5 ohms.

4. If 10 amp. flow through a motor whose resistance is $1\frac{1}{2}$ ohms, what is the P. D. of its terminals? *Ans.* 15 volts.

5. The E. M. F. of a generator is 115 volts, and its internal resistance is 1.8 ohms; the terminals of the generator are connected by 3 resistances, *A*, *B*, *C*, in series thus: o- - -*A*- -o- -*B*- - -o- - -*C*- - - - -o. The resistance *A* is 5 ohms, *B* is 8 ohms, *C* is 10 ohms. Between *A* and *B* and between *B* and *C* is introduced an ammeter whose resistance is .1 ohm each.

(*a*) What will the ammeter between *A* and *B* read?

(*b*) What will the ammeter between *B* and *C* read?

(*c*) If a voltmeter were connected at the terminals of resistance *A*, what would be its reading?

(*d*) What is the P. D. of the terminals of resistance *B*?

(*e*) What is the P. D. of the terminals of resistance *C*?

(*f*) If the voltmeter were connected across the terminals of the generator, what would it read?

(*g*) If the voltmeter resistance is 1000 ohms, and while it is connected across the generator terminals the connection between resistance *B* and *C* is broken, what will the voltmeter read?

Ans. (*a*) 4.6 amp. (*b*) 4.6 amp. (*c*) 23 volts. (*d*) 36.8 volts. (*e*) 46 volts. (*f*) 106.72 volts. (*g*) 114.8 volts.

6. (*a*) What is the E. M. F. of 4 cells connected in series, if the E. M. F. of each cell is 1.4 volts? (*b*) E. M. F. of 4 cells in parallel? *Ans.* (*a*) 5.6 volts. (*b*) 1.4 volts.

7. (*a*) What is the battery resistance of 4 cells connected in series if the internal resistance of each cell is .3 ohm? (*b*) Battery resistance of 4 cells in parallel? *Ans.* (*a*) 1.2 ohms. (*b*) .075 ohm.

8. (*a*) If the terminals of the battery of the 4 cells in (6*a*) and (7*a*) were connected by a wire whose resistance is 12.8 ohms, what current would flow through this wire? (*b*) If the external resistance were .128 ohm, what current would flow through it? *Ans.* (*a*) .4 amp. (*b*) 4.22 amp.

9. (*a*) If the terminals of the battery of 4 cells in (6*b*) and (7*b*) were connected by a wire whose resistance is 12.8 ohms, what current

would flow through this wire? (*b*) If the external resistance were .128 ohm, what current would flow through it? *Ans.* (*a*) .108 amp. (*b*) 6.9 amp.

10. What is the greatest number of amperes to be obtained from 20 gravity cells, the E. M. F. of each being .9 volt and internal resistance 3 ohms, with an external resistance of 500 ohms? *Ans.* .032 amp.

11. (*a*) Find the current strength from 4 cells in series, each cell having an E. M. F. of 1.5 volts and an internal resistance of 2 ohms with an external resistance of 10 ohms. (*b*) If the cells are in parallel, what is the current? *Ans.* (*a*) $\frac{1}{3}$ amp. (*b*) $\frac{1}{7}$ amp.

12. What is the greatest current obtainable from 6 cells with an external resistance of $\frac{1}{2}$ ohm, if the E. M. F. of each cell is 2 volts and internal resistance .3 ohm? *Ans.* 6.32 amp.

13. A current of 2 amp. is passed for 15 min. through a coil of wire whose resistance is 7 ohms, placed in 150 gm. of water whose temperature is 10° C., what will its temperature be at the end of 15 min., assuming that no heat is lost by radiation? *Ans.* 50.3° C.

14. An incandescent lamp whose resistance is 250 ohms, and in which flows a current of .45 amp., is submerged for 10 min. in 200 gm. of water whose initial temperature is 18° C., what is the resulting temperature? *Ans.* 54.4° C.

15. Since 1 amp. current deposits silver on the kathode in silver nitrate solution at the rate of .001118 gm. per sec., what mass of silver will be deposited in $\frac{1}{2}$ hour by a current of $\frac{1}{2}$ amp? *Ans.* 1.006 gm.

16. The chemical equivalent of copper is 31.8. Define the value of 1 amp. in terms of the mass of copper deposited per sec. by that current on a kathode in copper sulphate solution. *Ans.* .000329 gm.

CHAPTER XVIII

ELECTRICAL INSTRUMENTS AND MEASUREMENTS

400. Measurement of Current Strength. — In order to determine the current strength in a circuit, any one of the three effects, heating, electrolytic, or magnetic, may be used, since the magnitude of each effect can be measured, and from this value the current strength estimated, according to the formulæ which have been given.

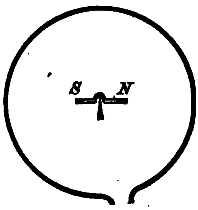


FIG. 194 *a*.

401. Galvanometers. — A *galvanometer* is an instrument which is used to measure current strength by the magnetic effect produced by it. Galvanometers are of various types: —

402. Tangent Galvanometer. — A *tangent* galvanometer (Fig. 194 *a* and *b*) consists of a vertical circular coil of insulated wire placed in the plane of the magnetic meridian, in the center of which coil is supported a short magnetic needle, free to move on a horizontal plane. While a current is passing through the coil, the forces acting upon the needle are (1) the magnetic force NC (Fig. 195) of the current whose direction is *perpendicular* to the plane

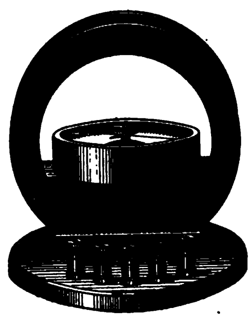


FIG. 194 *b*.

of the coil, and (2) the earth's magnetic force NH whose direction is *parallel* to the plane of the coil, since it is placed in the magnetic meridian. The position taken by the deflected needle is in the direction of the resultant of these two forces, or the diagonal ND of the parallelogram $NHDC$, constructed upon the forces NH and NC as adjacent sides.

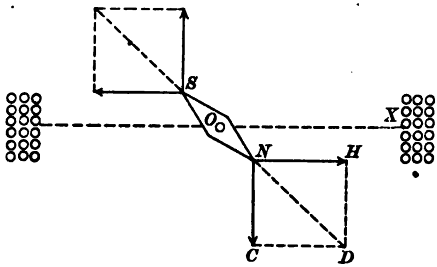


FIG. 195.

If from any point in one side of any angle α (Fig. 196) a perpendicular is dropped upon the other side, the *ratio* of the side thus formed opposite the angle to the adjacent side is called the *tangent* of that angle.

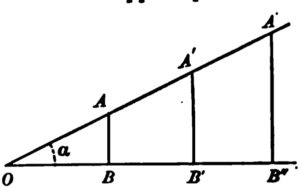


FIG. 196.

$$\tan.\ \alpha = \frac{AB}{BO} = \frac{A'B'}{B'O} = \frac{A''B''}{B''O},$$

i.e. the tangent is a constant ratio for the same angle.

In the preceding diagram the angle XON is the angle of deflection, δ (pronounced delta), and equals the angle HND. Hence, by definition, $\tan \delta = \frac{HD}{NH}$.

At the same place on the earth, the value of NH, the earth's magnetic force, remains constant. If the line NC (Fig. 197) represents in a given case the magnetic force due to a current in the coil, the needle takes the direction ND, or the angle of deflection is HND. If, however, the current strength is doubled, its magnetic force is doubled, and is

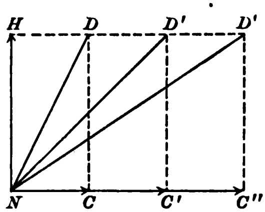


FIG. 197.

represented by NC'; the angle of deflection then becomes HND'. By doubling the current the angle of deflection has not been doubled, but the tangent of the second deflection, $\frac{HD'}{NH}$, is twice the tangent of the first deflection, $\frac{HD}{NH}$.

If the current strength is increased threefold, its magnetic force is represented by the line NC'', which is three times as long as NC. Here, also, it is evident that the deflection HND'' is not three times the deflection HND, but the tangent of HND''; $\frac{HD''}{NH}$ equals three times the tangent of HND, $\frac{HD}{NH}$.

Hence it follows that with a tangent galvanometer the current strength in the coil is directly proportional to the *tangent* of the deflection produced.

If for a particular instrument the deflection, δ_1, obtained by passing a *known* current, C_1, through its coil has been determined, the value of any other current, C_2, which produces a deflection, δ_2, may be obtained from the proportion,

$$C_1 : C_2 = \tan \delta_1 : \tan \delta_2.$$

403. Ammeter.—Instead of marking the scale in degrees of arc, a current known to be one ampere may be passed through the coil and the place on the scale where the needle comes to rest marked accordingly; likewise, currents of 2, 3, 4, etc., amperes are passed through the coil, and the position of the needle marked in each case. Such a galvanometer may be used to indicate directly the current strength in amperes, and is called an *amperemeter* or *ammeter.* The coil of an ammeter is usually made of thick copper wire of few turns, so that the resistance of the coil is small, and consequently does not decrease a current

appreciably when the instrument is introduced into an electric circuit.

404. Voltmeter. — If the coil of a galvanometer consists of many hundreds of turns of fine wire, so that its resistance is very large, the current that flows through the coil will be a measure of the potential difference of the points with which the coil is connected, as was shown in the example given in § 392.

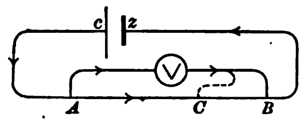


FIG. 198.

This may also be shown as follows: If a voltmeter is connected at two points, A and B (Fig. 198), in a circuit, the current flowing in the voltmeter is $C_1 = \frac{PD_{AB}}{R_V}$, where R_V represents the resistance of the voltmeter coil. If the voltmeter is connected at the two points, A and C, the current flowing in the voltmeter is $C_2 = \frac{PD_{AC}}{R_V}$.

Hence $$C_1 : C_2 = \frac{PD_{AB}}{R_V} : \frac{PD_{AC}}{R_V} = PD_{AB} : PD_{AC},$$

i.e. the current that flows in a voltmeter is directly proportional to the potential difference at its terminals.

If such a galvanometer is connected successively to pairs of points in a circuit whose potential differences are known to be 1, 2, 3, 4, etc., volts respectively, and the points on the scale where the needle comes to rest each time are marked to indicate these potential differences, the galvanometer becomes a *voltmeter.*

405. D'Arsonval Galvanometer. — A D'Arsonval galvanometer consists of a coil of fine wire, suspended by a thin metallic ribbon between the poles of a powerful U magnet, as shown in Fig. 199 *a* and *b*. If a current enters at A and passes as indicated around the coil, and then out at B, from the right-hand rule of electromagnets, it will be

seen that the side of the coil facing *front* acts as an N pole. This side will therefore be repelled by the N pole of the U magnet, and attracted by the S pole, and in consequence will twist around until the side now facing front faces the S pole. If a plane mirror, *M*, is attached to the wire suspending the coil, a beam of light reflected by the mirror is deflected by its rotation. This deflection of the reflected light is used to determine

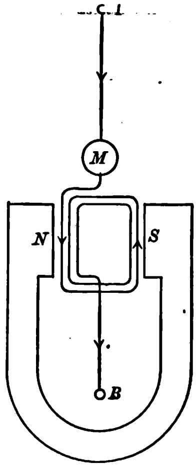


FIG. 199 *a*.

FIG. 199 *b*.

the amount of deflection of the coil. To concentrate the lines of force of the U magnet within the suspended coil, the coil is wound around a soft iron cylinder, which is fixed in position; *i.e.* it does not rotate with the coil. With a D'Arsonval galvanometer the deflection is directly proportional to the current in the coil, since the effect of the earth's magnetic force is inappreciable as compared with the controlling force of the U magnet.

406. Measurement of Resistance. — In order to measure the resistance offered by any given conductor, it is necessary to have a coil, or a series of coils, of wire whose resistances

have been determined by comparison with the resistance offered by the column of mercury, the dimensions of which were given in the definition of an ohm. If a number of pieces of metal, *A*, *B*, *C*, separated from one another by small spaces, are placed as shown in Fig. 200 *a* and *b* upon the cover of a box, inside of which is a coil of wire whose resistance equals 1 ohm connecting *A* with *B*, and a 2-ohm coil connecting *B* with *C*, and if these metal pieces may also be connected

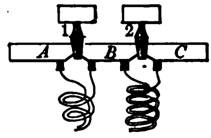


FIG. 200 *a*.

by inserting tapering metal plugs, 1 and 2, in the spaces between the pieces, this instrument is a *rheostat* (resistance box, plug form). If the rheostat is introduced into an electric circuit, and if both plugs are pushed well in place, the current passes from *A* to *C* with practically *no* resistance; but if plug 1 is removed, the current, in order to pass from *A* to *C*, must pass through the 1-ohm coil connecting *A* and *B*, thus introducing a resistance of 1 ohm into the circuit. If, however, plug 1 is reinserted between *A* and *B* and plug 2 is removed, a resistance of 2 ohms is introduced into the circuit. If both plugs are removed, the resistance introduced is 3 ohms.


FIG. 200 *b*.

407. Current Reverser. — It is often desirable to reverse the direction of the current through a particular instrument, as, for instance, a galvanometer, in which case a current reverser, one type of which, consisting of a 2-pole, double-throw knife switch, is shown in Fig. 201. The cur-

rent enters the reverser at hinge x and leaves at hinge y. The galvanometer is connected at the split contacts c and d, which are also connected *diagonally* with the split contacts, b and a, as shown. When the knife blades are thrown so as to connect x with c and y with d, the current passes from x to c, to m, to n, to d, to y, and out. If, however, the blades are thrown so as to connect x with a and y with b, the current flows from x to a, to d, to n, to m, to c, to b, to y, and out. Thus, by moving the blades from the contacts c and d to a and b, the current in the galvanometer is reversed, since in the latter position it flows from n to m, instead of from m to n, as in the first position.

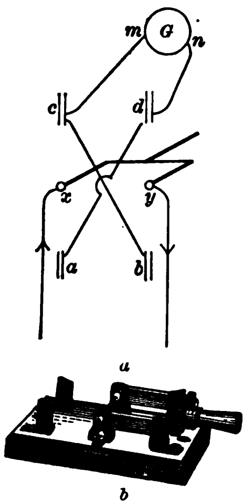


FIG. 201.

408. Measurement of the Resistance of a Cell. — If a cell connected in circuit with a rheostat and an ammeter gives a current C_1 when the external resistance is R ohms, and if by increasing this external resistance by n ohms, the current C_2 then obtained, is one half of C_1; then, since

$$C_1 = \frac{E}{R+r}, \quad C_2 = \frac{E}{R+n+r}, \text{ and } C_1 = 2\,C_2,$$

$$\frac{E}{R+r} = \frac{2\,E}{R+n+r},$$

$$2(R+r) = R+n+r,$$

$$2\,R + 2\,r = R+n+r,$$

$$r = n - R,$$

or the resistance of the cell equals the difference between the increase, n, in the resistance which reduces the current value one half, and the original external resistance R.

409. Measurement of the Resistance of a Wire.—(*Substitution Method.*) The resistance of a wire, or a group of wires, is determined by the substitution method as follows: —

The wire whose resistance is to be found is connected at *r* and *s* (Fig. 202) of a 2-pole knife switch, so that when the blades are in contact with *r* and *s*, the wire is connected in circuit with a cell and a galvanometer. After the galvanometer deflection has been observed, the knife blades are thrown over into contact with *t* and *u*, thereby substituting a rheostat, connected at these points, in place of the wire of unknown resistance. By means of the rheostat sufficient resistance is introduced into the circuit to reproduce the deflection previously obtained. This resistance then equals the resistance of the given wire.

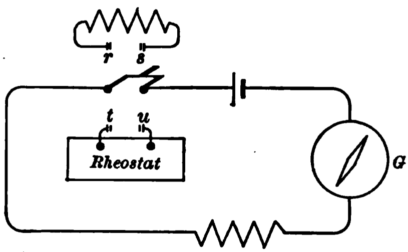


FIG. 202.

Since the deflections are the same, the value of C is the same in both cases. $C = \frac{E}{R+r}$, and, since the same cell is used, the values of E and r will not change appreciably in the few minutes required to perform the experiment; therefore the values of R must be equal. The external resistance, R, includes the resistance of the galvanometer coil and that of the connecting wires; but since these remain unchanged in both parts of the experiment, the rheostat resistance must equal that of the wire whose resistance is to be determined.

410. Laws of Resistance of Wires. — From results obtained by this method, the following laws of resistance may be derived: —

1. The resistances (R) of wires of the same material and diameter are directly proportional to their length (l).

$$R_1 : R_2 = l_1 : l_2.$$

2. The resistances of wires of the same material and length are inversely proportional to the *squares* of their diameters (d), or to their cross sections (S).

$$R_1 : R_2 = d_2^2 : d_1^2, \text{ or } = S_2 : S_1.$$

3. The resistance varies with the material of the wire.

The specific resistance of a substance, denoted by the letter k, is the resistance of a wire of that substance having *unit* length and *unit* diameter.

If the lengths and diameters are the same, the resistances of any two wires are directly proportional to their specific resistances.

$$R_1 : R_2 = k_1 : k_2.$$

Sometimes the value of k is given to represent the resistance of a cube of the given substance, 1 cm. on each edge. Its value is also given as the resistance of a wire 1 m. long, 1 mm. in diameter, or of a wire 1 ft. long, .001 in. (called 1 mil) in diameter. In each case the dimensions of k must be stated. For instance, if the dimensions of k are 1 m. long, 1 mm. in diameter, its value for copper is .021 ohm; for German silver, .4 ohm; for iron, .125 ohm. Hence for the same length and diameter, the resistance of a German silver wire is approximately 19 times that of a copper wire, and the resistance of an iron wire 6 times that of a copper wire.

Combining these three laws in one equation, $R = \frac{kl}{d^2}$, where, if k is the resistance of a wire 1 m. long, 1 mm. in diameter of the given material, l is the length of the wire

in meters, d^2 is the square of the diameter measured in millimeters.

For example, given that k for German silver is .4 ohm (1 m., 1 mm). Find the resistance of 2 m. of No. 30 German silver wire. (Diameter of No. 30 wire = .25 mm.)

$$R = \frac{.4 \times 2}{.25^2} = \frac{.8}{.0625} = 12.8 \text{ ohms.}$$

Again, given that k for copper = .021 ohm (1 m., 1 mm.). Find the resistance of 20 m. of No. 30 copper wire.

$$R = \frac{.021 \times 20}{.25^2} = \frac{.42}{.0625} = 6.72 \text{ ohms.}$$

411. Variation of Resistance with Temperature. — The resistance of a wire increases with a rise in its temperature.

If 20 m. of No. 30 copper wire is kept in water whose temperature is 0° C. while the resistance is being measured, the resistance is found to be 6.72 ohms. If the wire is again kept in water whose temperature is 100° C. while its resistance is being measured, the resistance is found to be 9.40 ohms. The increase in resistance (9.40 − 6.72) is 2.68 ohms. The increase in resistance per degree of rise of temperature is $\frac{2.68}{100}$, or .0268 ohm. The ratio of the increase in the resistance per degree of rise in temperature, to the resistance at 0° C. equals $\frac{.0268}{6.72} = .004$, which is the *temperature coefficient* of resistance for copper. Carbon is an exception to the rule that an increase of temperature produces an increase of resistance. The hot resistance of the carbon filament in an incandescent lamp is only about $\frac{2}{3}$ of its resistance when cold.

412. Wheatstone Bridge. — A *Wheatstone bridge* is an instrument by which resistance may also be measured. It

consists of a divided circuit of two branches, ACB and ADB (Fig. 203), in which a point C in one branch is connected with some point D in the other branch, thus forming a "bridge" extending from the one branch to the other. A galvanometer, G, is inserted in this bridge. The electric potentials of the points A and B are the same for branch ACB as for branch ADB; hence, there must be some point D in the branch ADB that has the same potential as that of a given point C in the branch ACB. This point D is experimentally found either by varying one of the four resistances r_1, r_2, r_3, r_4, or by sliding the contact D along the conductor ADB until no current flows across the "bridge" through the galvanometer. Since C and D are at the same potential, no current flows from C to D, and the current in AC, C_1, equals the current in CB, C_2, also the current in AD, C_3, equals the current in DB, C_4. The potential difference of A and C, PD_1, equals the potential difference of A and D, PD_3; similarly, PD_2, the potential difference of C and B, equals PD_4, the potential difference of D and B.

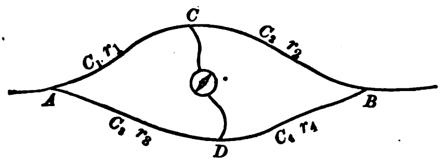


FIG. 203.

$$PD_1 = PD_3 = r_1C_1 = r_3C_3$$
$$PD_2 = PD_4 = r_2C_2 = r_4C_4.$$

Therefore, $$\frac{r_1C_1}{r_2C_2} = \frac{r_3C_3}{r_4C_4}.$$

Hence, $$r_1 : r_2 = r_3 : r_4.$$

Thus, if in a two-branch circuit two points are selected which have the same potential, the four resistance arms, into which the two branches are divided by these two points, are in proportion.

A common form of bridge is the slide wire bridge, shown in Fig. 204, *a* and *b*. One branch, AK_2B, is a single wire 1 m. long; the other branch, AxK_1RB, contains two resistance arms, one, x, unknown, the other, R, a known rheostat

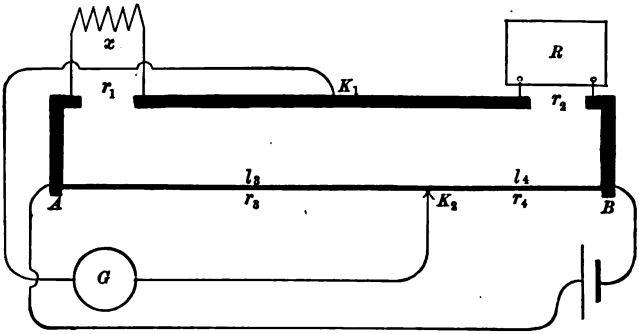


FIG. 204 *a*.

resistance. The point K_2 is a sliding contact, and when it is so located that it has the same potential as K_1, then $x : R = l_3 : l_4$. The ratio $l_3 : l_4$ can be used in place of $r_3 : r_4$, since AK_2B is a uniform wire, and the resistances of any two portions of it are proportional to their lengths.

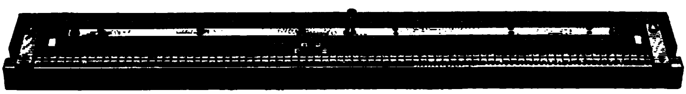


FIG. 204 *b*.

Since in the proportion $x : R = l_3 : l_4$ the three terms R, l_3, and l_4 are known, the unknown resistance x may be calculated $\left(x = \frac{l_3}{l_4} \cdot R\right)$.

The degree of accuracy to which resistances may be determined depends upon the correctness of the rheostat resistance R and upon the sensitiveness of the galvanometer

used to determine when points K_1 and K_2 are equipotential. The advantage of the bridge method in determining resistances over the substitution method is that the proportionality of the arms of the bridge is not affected by any change in the E. M. F. of the generator used in the circuit.

413. Resistance of Parallel Conductors. — If two points, A and B, in a circuit are connected by three wires (Fig. 205) whose resistances are r_1, r_2, r_3 ohms respectively, their *combined* resistance is determined as follows:

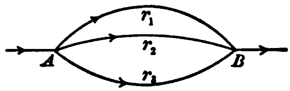


FIG. 205.

Let e be the potential difference between points A and B, then the current strength in branch r_1, or $c_1 = \frac{e}{r_1}$, in branch r_2, $c_2 = \frac{e}{r_2}$, and in branch r_3, $c_3 = \frac{e}{r_3}$.

The total current $C = \frac{e}{R}$, where R is the *combined* resistance.

But $$C = c_1 + c_2 + c_3 \cdots$$

Then $$\frac{e}{R} = \frac{e}{r_1} + \frac{e}{r_2} + \frac{e}{r_3} \cdots,$$

or $$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} \cdots,$$

i.e. the ***reciprocal*** of the combined resistance equals the ***sum of the reciprocals*** of the resistances of the several branches.

For example, if three wires whose resistances are 2, 3, and 4 ohms respectively are connected in parallel, their combined resistance R is found from the equation,

$$\frac{1}{R} = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{6}{12} + \frac{4}{12} + \frac{3}{12} = \frac{13}{12};$$

therefore, $$R = \frac{12}{13} \text{ ohm}.$$

Again, if four wires, each of which has a resistance of 100 ohms, are

connected in parallel, their combined resistance R is found from the equation

$$\frac{1}{R} = \frac{1}{100} + \frac{1}{100} + \frac{1}{100} + \frac{1}{100} = \frac{4}{100} = \frac{1}{25};$$

$R = 25$ ohms, or one fourth of the resistance of one wire. Hence, if any number, n, of *equal* resistances, such as incandescent lamps, are connected in parallel, their combined resistance is $\frac{1}{n}$th of the resistance of *one* of them.

414. Current in a Divided Circuit.—The current strength throughout a circuit of single conductors connected in series is the same at any point; but if the conductors are joined in parallel, this is no longer true. Suppose that two points, A and B (Fig. 206), in a circuit are connected by two wires in parallel whose separate resistances are r_1 and r_2 ohms respectively. Letting e represent the potential difference of the points A and B, the currents in the branches are, respectively,

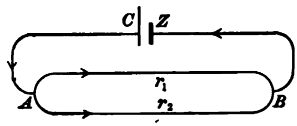


FIG. 206.

$C_1 = \frac{e}{r_1}$, and $C_2 = \frac{e}{r_2}$.

Then $$C_1 : C_2 = \frac{e}{r_1} : \frac{e}{r_2} = \frac{1}{r_1} : \frac{1}{r_2}.$$

Therefore $$C_1 : C_2 = r_2 : r_1,$$

or the currents in the branches of a divided circuit are inversely proportional to the resistances of the branches.

That part of the whole current C, or $C_1 + C_2$, which flows in each branch, may also be calculated:—

For $$C_1 + C_2 : C_1 = r_1 + r_2 : r_2;$$

therefore, $$C_1 = \frac{r_2}{r_1 + r_2} C.$$

Similarly, $$C_1 + C_2 : C_2 :: r_1 + x_2 : r_1;$$

therefore, $$C_2 = \frac{r_1}{r_1 + r_2} C.$$

For example, in a divided circuit of two branches whose resistances are 24 ohms and 1 ohm respectively; find what fraction of the whole current flows through the 24 ohm branch.

The current $C_1 = \frac{1}{24 + 1} = \frac{1}{25}$ of the whole current.

For $$C_1 = \frac{e}{24} \text{ and } C_2 = \frac{e}{1};$$

the whole current $C_1 + C_2 = \frac{25\,e}{24}$.

Therefore $$C_1 = \frac{\frac{e}{24}}{\frac{25\,e}{24}} = \frac{e}{24} \times \frac{24}{25\,e} = \frac{1}{25} \text{ of the whole current.}$$

If the 24 ohm branch is the coil of a galvanometer and the 1 ohm branch is a wire connecting the galvanometer terminals outside of the galvanometer, this 1 ohm wire is called a *shunt* to the galvanometer because it allows a portion of the current to pass around. In the given case it would be called a $\frac{1}{25}$ shunt because it allows only $\frac{1}{25}$ of the whole current to pass through the galvanometer coil. Its resistance, however, is only $\frac{1}{24}$ of the resistance of the coil. Similarly a $\frac{1}{10}$ shunt would have $\frac{1}{9}$ of the resistance of the galvanometer coil, and a $\frac{1}{100}$ shunt would have $\frac{1}{99}$ the resistance of the coil.

In general, either branch of a two-branch circuit is a shunt to the other.

PROBLEMS

1. Given two cells, A and B, having an E. M. F. 1.5 volts and 2 volts respectively, and .5 ohm and .1 ohm internal resistance respectively. Each cell is connected successively to a voltmeter whose resistance is 200 ohms, and to an ammeter whose resistance is .01 ohm. Compare the readings of each instrument for each cell, and state in which instru-

ment the readings are practically proportioned to the E. M. F. of the circuit.

Ans.

Voltmeter readings	$A = 1.496$ volts. $B = 1.999$ volts.	Ammeter readings	$A = 2.94$ amp. $B = 18.18$ amp.
Ratio of E. M. F.'s			$A:B = .750$
Ratio of Vm. readings			$A:B = .748$
Ratio of Am. readings			$A:B = .162$

2. A current of .45 amp. in the coil of a certain tangent galvanometer produces a deflection of 41°; what current will produce a deflection of (*a*) 10°, (*b*) 20°, (*c*) 30°? *Ans.* (*a*) .091 amp. (*b*) .188 amp. (*c*) .299 amp.

3. A copper wire 12 m. long, .5 mm. diameter, offers 1 ohm resistance; find the resistance of 20 m. of copper wire, .25 mm. diameter.
Ans. 6.66 ohms.

4. If 2 m. of German silver wire, .25 mm. in diameter, offers 12.8 ohms resistance, what is the specific resistance of German silver in terms of the units 1 m. and 1 mm.? *Ans.* .4 ohm.

5. If the arm r_1 of a slide wire Wheatstone bridge is an unknown resistance, arm r_2 is 10 ohms, and the equipotential point, K_2, of the slide wire is 79.2 cm., what is the unknown resistance (assuming that the *zero* end of the meter wire is at A)? *Ans.* 38.08 ohms.

6. If the arm r_1 of a slide wire bridge is 2 ohms, arm r_2 is 3 ohms, and the point, K_2, of the slide wire, which is connected through the galvanometer with K_1, is 50 cm., (*a*) which point, K_1 or K_2, is at the higher potential? (*b*) If the P. D. of the main terminals A and B of the bridge is 2 volts, what is the P. D. of points K_1 and K_2? *Ans.* (*a*) K_1. (*b*) .2 volt.

7. What is the relative resistance of 90 cm. of platinum wire, .4 mm. in diameter, and the same length of copper wire, .33 mm. in diameter, the specific resistance of platinum being 7 times that of copper?
Ans. 4.76 : 1.

8. Six incandescent lamps with a resistance of 240 ohms each are connected in parallel with two points whose P.D. is 115 volts. (*a*) What is the joint resistance of the lamps? (*b*) What current flows between the two points? *Ans.* (*a*) 40 ohms. (*b*) 2.875 amp.

9. A D'Arsonval galvanometer, whose resistance is 240 ohms, has its terminals connected also by a branch wire or shunt whose resistance is 10 ohms. When the galvanometer is connected to two points whose

P. D. is .2 volt, what current flows (*a*) through the galvanometer coil, (*b*) through the shunt? *Ans.* (*a*) .00083 amp. (*b*) .02 amp.

10. A cell whose E. M. F. is 1.5 volts and internal resistance .25 ohm has its terminals connected by three wires in parallel whose separate resistances are 3, 5, and 8 ohms respectively; (*a*) what current flows through each of these wires? (*b*) what is the joint resistance of the three wires?

Ans. (*a*) .43 amp. in 3 ohm wire. .26 amp. in 5 ohm wire. .16 amp. in 8 ohm wire. (*b*) 1.519 ohms.

11. A storage battery is connected in series with an incandescent lamp and milliammeter (an ammeter whose divisions indicate $\frac{1}{1000}$th amp.). In parallel with the lamp is connected a voltmeter. The milliammeter reads .020 amp. and the voltmeter 7.2 volts, what is the resistance of the lamp? *Ans.* 360 ohms.

12. A D'Arsonval galvanometer, whose resistance is 240 ohms when connected in series with a certain cell and a rheostat resistance of 100 ohms, deflects 50 scale divisions. Since the deflection is proportional to the current, what resistance in series with the galvanometer and cell would reduce the deflection to 5 scale divisions? *Ans.* 3160 ohms.

CHAPTER XIX

INDUCED ELECTRO-MOTIVE FORCE

415. Electrical and Magnetic Conditions Related. — It has previously been shown that when a current passes around a bar of soft iron, the iron becomes magnetized, and that when a wire carrying a current is brought near a suspended magnetized needle, the needle is deflected, showing that there is a magnetic field produced around a wire when a current is passing through it. These and similar phenomena indicate an intimate relation between electrical and magnetic conditions.

416. Production of Induced E. M. F. — The space about a magnet is traversed by lines of force which pass out from the N pole and enter the magnet again at the S pole. If a coil of insulated wire, *A* (Fig. 207), held perpendicular to the plane of the paper is moved to the left so that the coil cuts the lines of force emanating from the N pole of a bar magnet placed in the plane of the paper, it is found that an E. M. F. is generated in the coil which lasts only as long as the coil is cutting the lines of magnetic force. This electro-motive force is called an *induced* E. M. F. because it is produced by the *influence* of variations in the magnetic field through which the coil of wire is passing.

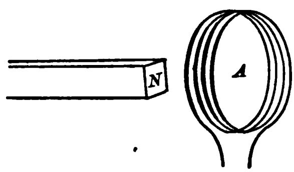


FIG. 207.

417. Transformation of Mechanical Energy into Electrical Energy. — It requires *more* mechanical energy to move a closed coil across these lines of force than to move it through space not traversed by such lines. It is this additional mechanical energy which is transformed into electrical energy in the coil, and whose value is measured by the product of the E. M. F. produced and the resulting current.

418. Strength of Current. — If the terminals of the coil are connected, a current will flow in the coil during the time the E. M. F. is being induced therein, and the strength of this current will depend, as always, both upon the magnitude of the induced E. M. F. and the resistance of the coil.

419. Magnitude of Induced E. M. F. — The magnitude of the induced electro-motive force depends upon three things: (1) the *intensity of the magnet field* across which the coil passes — the stronger the magnet, the greater is the induced E. M. F.; (2) the *speed* with which the coil is moved across these lines of force — the greater the speed, the greater is the induced E. M. F.; (3) the *number of turns* in the coil which is moved across the magnetic field — the greater the number of turns, the greater is the induced E. M. F.

In brief, then, *the magnitude of induced electro-motive force is determined by the* **rate** *at which lines of magnetic force are cut*, i.e. *the number of lines cut per second.*

420. Direction of Induced E. M. F. — The direction of an induced E. M. F. depends upon two other directions: (1) the direction of the lines of magnetic force, (2) the direction in which the coil is moved across these lines.

If the coil *A* is moved on toward the left across the lines of force passing into the S pole of a magnet, the direction

of the E. M. F. is opposite to that produced when the coil was moved toward the left across the lines of force at the N pole. The direction of a line of magnetic force in any portion of space is the direction in which the N pole of a needle tends to move if placed in that space. Since when near the N pole of a magnet another N pole is repelled, the direction of lines of force at the N pole of a magnet is *away from* that pole; conversely the lines of force at the S pole of a magnet have a direction *toward* that pole.

If, after the coil *A* (Fig. 207) has been moved to a position around the N pole of the bar magnet, it is then moved toward the right *away from* the pole, the direction of the E. M. F. produced is opposite to the direction of the E. M. F. induced while the coil was being moved to the left *toward* the pole.

The direction of the motion of the coil was in the direction of the length of the magnet, the direction of the lines of force at the pole are perpendicular to this length, and since the coil (its circumference) was held so as to be perpendicular to both of these directions, there are here three directions each perpendicular to each of the others. These three directions may be represented by the thumb and first two fingers of the *right* hand extended so that each is perpendicular to each of the others, as shown in Fig. 208.

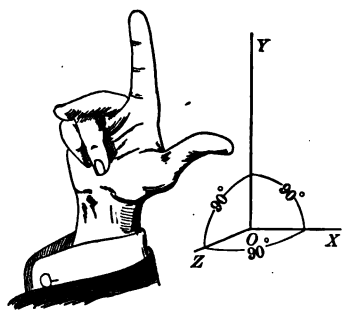


FIG. 208.

421. Three-finger Rule. — If the direction X (Fig. 208) of the extended thumb of the right hand indicates the

direction of the motion of the coil across lines of magnetic force, and the direction Y of the extended forefinger indicates the direction of the lines of force at a given point in the space, the direction Z of the extended middle finger will indicate the direction of the E. M. F. induced in the coil as it passes this given point in space.

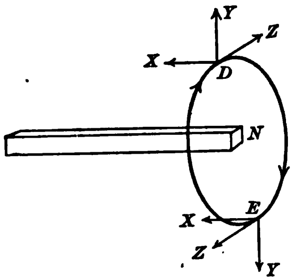


FIG. 209.

For example, if a coil (Fig. 209) is *moved toward and over the* N pole of a magnet, the direction of the E. M. F. in the upper part D of the coil is in the direction Z perpendicular to the plane of the paper and into it, the direction of the E. M. F. at the lower part E of the coil is in the direction Z perpendicular to the plane of the paper and outward from it. In this case the direction of the current flowing in the coil, if looked at so as to *face* the pole of the magnet, is *clockwise*, as shown.

422. Mechanical Drag on Conductors moving in a Magnetic Field. — In Fig. 210 is shown a square wire frame rotated in the direction of the arrow about an axis perpendicular to the lines of force between two unlike poles, N and S, of a magnet. By means of the three-finger rule it may be found that the direction of the E. M. F. induced in part P of the wire frame is from a to b. In part Q of the wire frame the induced E. M. F. is from c to d. At b (Fig. 211), when

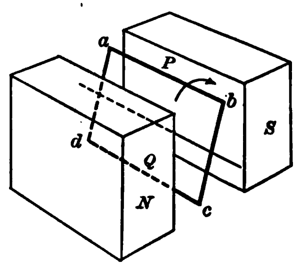


FIG. 210.

the current is flowing toward you, the magnetic field produced by this current about the conductor ab is counterclockwise. Hence the lines of force from the field magnet

are opposite in direction to these lines on the upper side of *b* and in the same direction on the lower side. This causes a weakening of the field on the upper side of *b*, as shown, and a strengthening on the lower side, resulting in a tendency to move the conductor *ab upward* in opposition to the direction (shown by the arrow) in which it is being rotated. Conversely, for *c* there is a confluence of magnetic lines on its upper side and a decrease on its lower side resulting in a tendency to force the conductor *cd downward* in opposition to the direction it is being rotated. It is in overcoming this mechanical *drag* on the conductor that the mechanical energy of rotation is transformed into electrical energy.

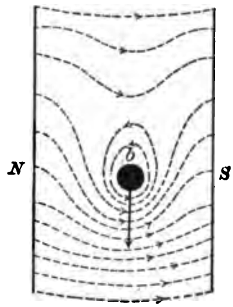


FIG. 211.

423. Induction Coil.—It has so far been assumed that a permanent magnet is used to produce the magnetic field across which the coil passes, but this need not be, and usually is not, the case. If a coil of wire PP' (Fig. 212), wound about an iron core cc', is connected with some electric generator B, so that a strong current flows through the coil, there is produced an intense magnetic field about this electro-magnet. If another coil SS' is wound about this electro-magnet, it is possible to induce an E. M. F. in coil SS' by varying the strength of the electro-magnet either by introducing resistance in the

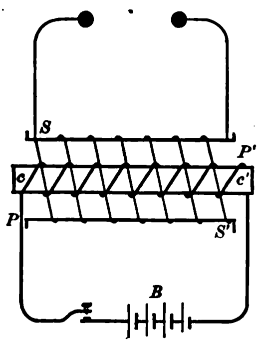


FIG. 212.

circuit of the electro-magnet, or by alternately opening and closing its circuit, or by moving the core into or out of the coil. Whenever the electro-magnet or *primary* circuit, as it is called, is *closed*, the result is the same as if a magnet was suddenly thrust into the coil SS', and for that instant an E. M. F. is induced in the secondary coil. The direction of the E. M. F. in the secondary circuit is *opposite* to that in the primary circuit. Whenever the primary circuit is *opened*, the result is the same as if a magnet was suddenly withdrawn from the coil SS', and for that instant an E. M. F. is induced in the secondary circuit whose direction is *the same* as that in the primary circuit. If the terminals of the secondary coil are connected, there will circulate in it a current during the instants of opening and of closing the primary circuit, but *alternating* in direction.

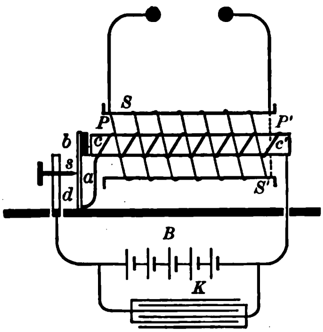


FIG. 213*a*.

424. Induction Coil with Automatic Interrupter.—This opening and closing of the primary circuit may be done by an automatic interrupter, shown in Fig. 213 *a* and *b*. This device is used also in an electric bell to keep the clapper in vibration.

FIG. 213*b*.

The current from the generator B, after passing around the iron core cc', is led to an upright piece of spring brass a,

at the upper end of which, and facing the core cc', is a piece of soft iron b. Thence the current flows through the screw s, down through the metal support d, and back to the generator. While the current is thus flowing, the core cc' is magnetized and attracts the iron piece b, pulling the spring away from contact with screw s, thus breaking the primary circuit. The core then becomes demagnetized, releases the iron piece, which, by the recovery of the brass spring, is brought again into contact with screw s, thus closing the primary circuit, and the former operation is repeated.

Whenever the primary circuit is thus opened or closed, an E. M. F. is induced in the secondary coil SS', which alternates in direction with each change in the primary circuit.

If the number of turns of wire in the primary coil is sufficient to magnetize the core to saturation when a few amperes of current are flowing through the coil, and if there is a rapid-action interrupter in the primary circuit, it is possible with a secondary coil of several thousand turns to generate in it an alternating E. M. F. of great intensity, running into thousands of volts. If the terminals of the secondary coil are separated a few centimeters, this E. M. F. is powerful enough to send a charge of electricity across the air space between the terminals, heating the air to incandescence in its passage, and producing what is known as an electric spark. In this manner are constructed the spark coils used in X-ray experiments and in wireless telegraphy.

425. Capacity. — The electric potential to which a given charge of electricity on a body will raise it depends upon the capacity of the body — the greater its capacity, the less

is its electric potential for the same charge. This is expressed in the form $V = \frac{Q}{K}$, when V is the potential, Q the quantity of electricity, and K the capacity of the body. The capacity of a body is increased by increasing the extent of its surface, and also by bringing near it a conductor charged with the opposite kind of electricity. The *condenser* K, introduced in parallel with the interrupter (Fig. 213 *a*), consists of a number of sheets of tinfoil insulated from each other, the alternate sheets being connected together. If at any given instant an induced current of high voltage is produced in the primary circuit, the condenser, by increasing the capacity of the portion of the primary circuit embracing the interrupter, reduces very greatly the voltage or P. D. of the terminals connected with it.

426. Self-induction. — While the interrupter is in action, a spark is seen at the point of screw *s* at the instant the primary circuit is broken. This is due to the fact that the change in the electric condition of the various turns of the primary coil is not instantaneous. There seems to be a sort of magnetic inertia, in virtue of which a given magnetic field tends to persist after the current which caused it ceases. When the circuit is broken at the interrupter, the change of magnetic field produces, by induction, in the primary coil a high E. M. F. in the same direction as the primary E. M. F. It is this E. M. F. which produces the spark at the interrupter and it is the P. D. produced by this E. M. F. at the interrupter which is reduced by means of the condenser K. At the instant the circuit is closed at the interrupter, this self-induction produces in the primary coil an E. M. F. opposed to the primary E. M. F. This counter electro-motive force retards the establishment of current in

the primary. Thus by using a condenser the delay in the change of magnetic condition of the core is decreased; hence the action of the interrupter is more rapid and the induced E. M. F. correspondingly greater.

In every circuit of alternating E. M. F. in which there are coils of wire, this retarding or choking effect acts as additional resistance to the passing of the current. This *reactance*, so called, of an alternating circuit may be estimated in ohms just as in the case of ordinary resistance.

Self-induction may be realized in the following manner. When several storage cells are connected in series, no unpleasant effect is experienced if the circuit is interrupted while grasping the terminals of the battery in the hands; but if in circuit with these cells there is a high resistance telegraph relay or several telegraph sounders, or any other high resistance electro-magnets, a very perceptible shock is felt when the circuit is interrupted.

427. Static Transformer. — If around an iron ring (Fig. 214) are wound a primary coil P of a few turns of course wire, and a secondary coil S of many turns of fine wire, and if the primary coil is connected in a circuit containing an *alternating* E. M. F. (*i.e.* an E. M. F. whose direction is reversed many times per second), the lines of magnetic force produced in the ring by this primary current are reversed as many times per second as the primary E. M. F. is reversed.

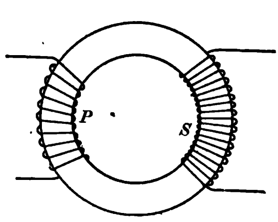


FIG. 214.

If there are n_1 turns of wire in the primary coil and n_2 turns in the secondary, since the lines of force generated by the current in each of the n_1 turns cuts each of the n_2 turns in the secondary, there

is induced in the secondary coil S an alternating E. M. F. as many times as great as the primary E. M. F. as the number of turns of wire in the secondary coil is as great as the primary turns.

If the primary E. M. F. is 110 volts and the secondary coil has 20 turns to every 1 in the primary, since the lines of force generated by 1 primary turn cut 20 secondary turns, the induced E. M. F. in the secondary is 20 times 110 volts or 2200 volts.

A transformer so connected is called a "step up" transformer. If the connections are changed so that there is but 1 secondary turn for each 20 primary turns and the primary E. M. F. is 2200 volts, the E. M. F. induced in the secondary is $\frac{1}{20}$ of 2200 volts, or 110 volts. A transformer so connected is said to "step down" the voltage.

Except for loss due to heating the iron core, the electrical energy, EC, supplied to the primary equals the energy in the secondary coil; hence, if the voltage is "stepped up," the secondary current is reduced proportionally, and if the voltage is "stepped down," the current is proportionally increased.

In many electric light plants the alternating E. M. F. generated at the power house is over 2000 volts; the voltage required to operate incandescent lamps does not exceed 110 volts, so that it is customary to introduce a "step down" static transformer into the circuit in the immediate locality of a group of lamps, the lamps being fed with the secondary current delivered by the transformer.

428. The Dynamo. — A *dynamo* is a *machine* for the production of an induced electro-motive force. It consists of a number of connected coils of wire which are wound about an iron core and revolved within the field of a pow-

erful electro-magnet by the expenditure of mechanical energy. Lines of magnetic force are thus being continually cut by some of the coils, and in consequence an E. M. F. is constantly being generated within some portion of this series of coils.

429. Dynamo with a Single Coil Armature. — The simplest form of a dynamo (Fig. 215) consists of a powerful U magnet between poles of which is revolved a single coil of wire. The terminals of the coil are connected to two metal rings, *c* and *d*, which revolve with the armature about the axis XX'. Held in contact with these two rings are two strips, *e* and *f*, of metal or carbon, which are stationary and form the terminals of the dynamo. To these is connected the external portion of the circuit *GH*.

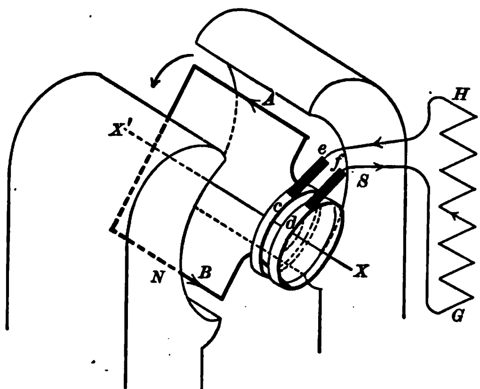


FIG. 215.

Suppose the coil to be rotated in the direction indicated by the arrow. As the part *A* of the coil passes up in front of the S pole of the U magnet there is induced in the wire an E. M. F. whose direction (by the three-finger rule) is found to be as indicated by the arrowheads. At the same time part *B* passes down in front of the N pole, and there is induced in it an E. M. F. in the indicated direction. For this instant a current will flow out from strip *f*,

through the external circuit *GH* in the direction indicated, reëntering the coil at strip *e*. When the plane of the revolving coil becomes vertical, the parts of the coil are moving parallel with the lines of magnetic force between the poles of the magnet; therefore, since the coil is not then cutting any lines, the value of the E. M. F. drops to zero. As the part *A* of the coil takes the position indicated in the diagram by *B*, and *B* takes the position indicated by *A*, an E. M. F. is again generated in the coil in the indicated direction, but as *B* is connected with ring *d* the current flows out of strip *e*, through the external circuit *HG* in the opposite direction to that indicated, reëntering the coil again by strip *f*. As the plane of the revolving coil again becomes vertical, the value of the E. M. F. again drops to zero. Thus, it is seen that in one complete revolution of the coil the E. M. F. rises to a maximum, decreases to zero, rises to a maximum in the *opposite* direction, and again decreases to zero. This may be represented graphically by a curve (Fig. 216) in which the distance from the horizontal axis at any point represents the value of the E. M. F. at that instant, the E. M. F. in one direction being above the horizontal axis while that in the opposite direction is below it. Such a dynamo is called an *alternator*.

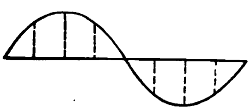


FIG. 216.

430. Direct Current Dynamo with a Single-coil Armature. — If the terminals of the revolving coil are connected as shown in Fig. 217 *a* to the two parts, *c* and *d*, of one split ring which revolves with the coil about the axis XX' and the strips, *e* and *f* (Fig. 217 *b*), rest upon these two segments, as the part *A* moves up in front of the S pole and the

part B down in front of the N pole the E. M. F. induced in the coil is in the indicated direction, a × in the circle

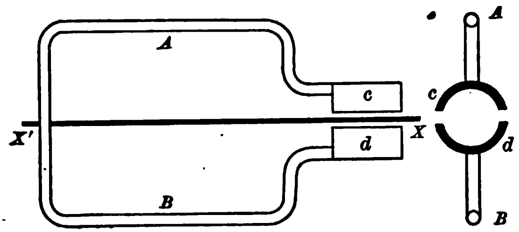


FIG. 217 *a*.

meaning current flowing from you, and a dot in the circle, current flowing toward you. For this instant the current flows from d through strip f, through the external circuit GH in the indicated direction, reëntering the coil through strip e and segment c. As the plane of the coil becomes vertical this E. M. F. decreases to zero. When A and B interchange positions, the two segments of the split ring change also, so that strip f is in contact with segment c,

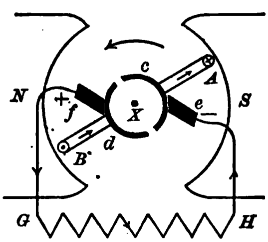


FIG. 217 *b*.

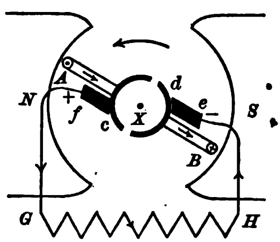


FIG. 217 *c*.

and the E. M. F. generated in A when in the position shown in Fig. 217 c sends a current to c, out through strip f, through the external circuit GH in the *same* direction as before, reëntering the coil through strip e and seg-

ment *d*. As the plane of the coil becomes vertical this E. M. F. decreases to zero. Hence, it is seen that in one revolution of the coil there is produced a P. D. whose value for the external circuit *GH* may be graphically represented by the curve (Fig. 218), the P. D., when there is any, always being in the same direction.

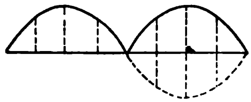


FIG. 218.

431. Dynamo with a Ring Armature. — If instead of one revolving coil there are two coils whose planes are perpendicular to each other, when the E. M. F. is a maximum in one, it is zero in the other (Fig. 219), and the P. D. in the external circuit will be represented by a curve (Fig. 220) compounded of the two curves, *abcde* and *fghij*, similar to the above. As the number of coils used is greater, this line approaches a straight line as a limit.

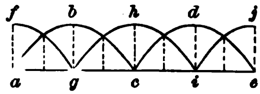


FIG. 219.

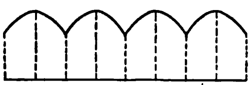


FIG. 220.

Such a direct current dynamo may be constructed as shown in Fig. 221 *a*, or as shown in the diagram (Fig. 221 *b*). Around an iron ring are wound eight coils, 1, 2, 3, 4, 5, 6, 7, 8, all in the same fashion as shown in the diagram. The split ring has the same number of segments, *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*. One end of coil 1 is joined to segment *a*, as is also one end of coil 8; the other end of coil 1 and one end of coil 2 is joined to the next segment *b*. The coils are similarly connected all around the ring. The strips B_1 and B_2, called *brushes*, rest on the segments at the end of the vertical diameter of the split ring. As the coils are revolved in the direction of the arrow, the coils 1 to 4 inclusive are moving down in front of the S pole, and by the three-

finger rule the induced E. M. F. in them has the direction indicated by the small arrowheads. The coils 5 to 8 inclusive are moving up in front of the N pole, and the induced E. M. F. in them is in the direction marked. It is seen that the E. M. F. in the two sides of the ring both send a current to the top segment of the split ring, out through brush B_1 through the external circuit GH in the indicated direction to brush B_2 in contact with the bottom segments, where on reëntering, the current divides, passing up through the coils on the two sides of the ring. The ring revolves with the coils, but the brushes B_1 and B_2 remain stationary, so that as coil 8 takes the position of coil 1 and coil 4 that of 5, although

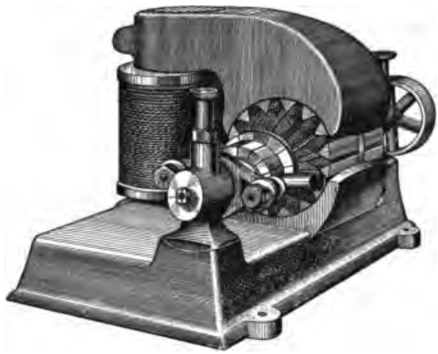


FIG. 221 *a*.

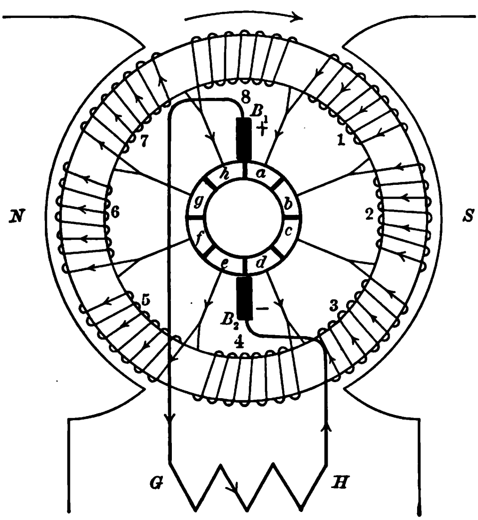


FIG. 221 *b*.

the E. M. F. in each is reversed as it passes the vertical plane, the direction of the current will still be out through brush B_1, and through the external circuit GH in the same direction as before.

432. Parts of a D. C. Dynamo.—The split ring of a dynamo is called the *commutator* because, as shown, by means of it the alternating current in the revolving coils is changed to a direct current in the external circuit. For this reason this form of dynamo is called a *direct current* (D. C.) dynamo. The revolving coils and iron core constitute what is called the *armature*. The coils of the armature are wound about an iron core in order to increase the number of lines of force which pass through it from the one pole to the other, because iron is a much better conductor of magnetic flux than air. This core is usually not one piece but is built up of iron disks insulated from each other to prevent the flow of induced currents in it. The strips which rest on the commutator and which form the terminals of the dynamo are called the *brushes*. The magnet which produces the field of force is known as the *field* magnet. There are, then, four principal parts to a direct current dynamo: the *field magnet*, the *armature*, the *commutator*, the *brushes*.

433. Residual Magnetism of the Field Magnet.—In order to furnish a field of force to start the generation of E. M. F. in the armature when it first begins to revolve, it is necessary that the field magnet should always be magnetized to some extent. When the dynamo is manufactured, its magnet is energized from some outside source, and as has been seen in the experiments with electro-magnets, after they have once been strongly magnetized from 3 to 5% of this magnetism remains after the current is cut off

from its coil. This *residual magnetism* is available to start the generation of E. M. F. in the revolving armature. Some of the current issuing from brush B_1 is diverted, as shown in Fig. 222, so as to pass through the coil MN wound around the core of the field magnet, so that as its magnetization is built up the value of the induced E. M. F. in the armature rises until the field magnet is saturated. When this condition is reached, no increase in the field of the magnet is produced by a further increase in the current flowing around the field coil MN. Hence, if the speed of revolution of the armature remains constant, the E. M. F. then generated has reached its highest value for this dynamo working under the given conditions.

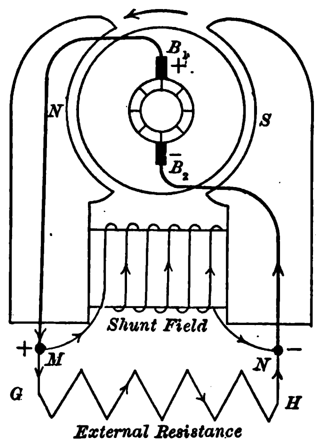


FIG. 222.

434. Classification of D. C. Dynamos. — According to the manner in which the field coil and external circuit are connected relative to each other direct current dynamos are divided into three classes: —

1. *Shunt Dynamo.* — In this dynamo (Fig. 222) the field coil MN and the external circuit GH are connected *in parallel* with each other. The positive brush B_1 is connected both to the terminal M of the field coil and the terminal G of the external circuit, and the terminal N of the field coil and terminal H of the external circuit are both joined to the negative brush B_2. Hence the current in the field coil is not the same as that in the external circuit unless they are of the same resistance. If the

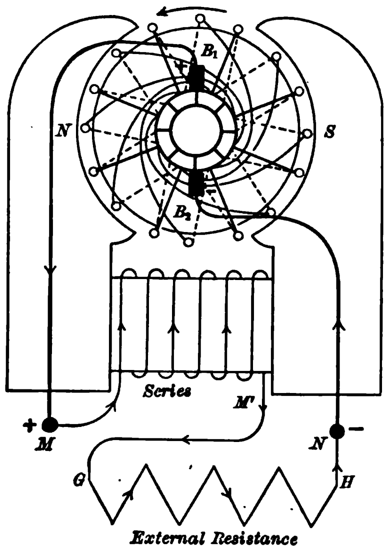


FIG. 223.

external resistance is one fourth of the field coil resistance, the current in the external circuit is four times the current in the field coil, *i.e.* one fifth of the current flows through the armature.

2. *Series Dynamo.*—In this dynamo (Fig. 223) the field coil MM' and external circuit GH are connected in *series* with the armature. The positive brush B_1 is connected with the end M of the field coil the other end of which is connected with the terminal G of the external circuit, and the terminal H of the external circuit is joined to the negative brush B_2. The current flowing in the field coil is, therefore, the same as that in the armature, or in the external circuit.

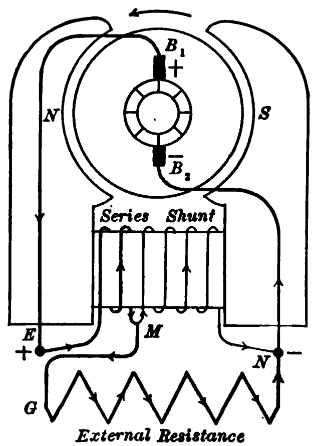


FIG. 224.

3. *Compound Dynamo.*—In this dynamo (Fig. 224) there are two field coils; one coil, EM, is connected *in series* with the external circuit GH, and the other coil, MN, is connected *in shunt* with GH. Hence the current in

coil *EM* is the sum of the currents in *GH* and in *MN*, but the currents in *GH* and *MN* are inversely as their resistances.

435. The Characteristics of the Series Dynamo. — In this dynamo the same current flows through the armature, the field coil, and the external circuit. If the external resistance is large and the current flowing small, the magnetic field produced is weak and therefore the electromotive force is small. As the external resistance is decreased the current increases and also the strength of the magnetic field. This causes the electro-motive force to increase rapidly with the increasing current until the magnetic field becomes saturated. When this point is reached, a large increase in the current produces only a very small increase in the strength of field and the electro-motive force increases very little. Owing to the resistance of the armature a current flowing through it causes a drop in potential and the potential difference at its terminals is less than the electro-motive force developed. After the saturation point of the iron of the magnetic field is passed, the increasing drop in the armature may cause the potential difference at the terminals of the machine to decrease somewhat as the current increases. Thus in a series dynamo as the *load increases the EMF increases*, rapidly at first, then more slowly until when overloaded it decreases.

436. The Characteristics of the Shunt Dynamo. — In the shunt-wound dynamo the terminals of the field coil are connected directly to the brushes and the current flowing in the coil depends only on its resistance and the potential difference at the brushes. When the machine is running and no current flows in the external circuit, the electro-motive force generated is a maximum, the current in the field coil being at its highest value.

When current flows in the external circuit, the increased armature current causes a larger D. P. in the armature and a less P. D. at the brushes for the same E. M. F. This causes a slight decrease in the *field* current and lowering of the strength of the field and of the E. M. F. As the resistance in the external circuit is gradually decreased with the increase in the current, the P. D. at the brushes continues to decrease, slowly at first, but as the iron of the field falls below saturation, the drop in the field strength and in the E. M. F. developed becomes more rapid, the decrease in the P. D. at the brushes being more noticeable with each small increase of current. Thus in a shunt dynamo *as the load increases, the E. M. F.*, starting at its maximum value, *decreases*, slowly at first, then more rapidly.

437. Characteristic of a Compound Dynamo. — As has been shown, as the external resistance of a shunt dynamo is decreased, the current in the shunt-field coil decreases a little, thus weakening the magnet and lowering the E. M. F. generated. If, then, a second coil of few turns is wound around the core of the field magnet, and this coil is connected so that it carries the larger current, passing through the external circuit, the magnet may be strengthened to its full value and the E. M. F. remain constant. Hence the effect of compounding the field coils of a dynamo is to keep the P. D. at the terminals of the machine more nearly constant.

438. Action of an Alternator. — In one form of *alternator* or alternating current (A. C.) dynamo (Fig. 225) the coils of the armature are wound alternately in opposite fashion around successive cores and connected in series, one terminal of the series of coils being connected at ring K_1, on which rests brush B_1, the other terminal being connected to the

other ring K_2, on which rests brush B_2. If there are 8 coils in the armature, there are 8 poles of the field magnet, alternately N and S. This field magnet is separately excited by a direct current from some outside source not shown in the diagram. For an instant coil 8 is moving under an S pole and coil 1 under an N pole, the generated E. M. F. has the direction indicated by the small arrowheads, and the current in the external circuit flows, as shown, from brush B_1 through GH to brush B_2 and back through the armature coils. The next instant, however, as coil 8 moves under the next N pole and coil 1 under an S pole, the generated E. M. F. in coil 8 will be as shown for coil 1, and in coil 1 as shown for coil 2. Hence the current will flow from brush B_2 through HG to brush B_1. Thus, in the case shown, the current will alternate in direction 8 times for every revolution of the armature, or there will be 4 complete cycles in the reversals of the E. M. F. (Fig. 216 shows one cycle.) If the armature is revolved

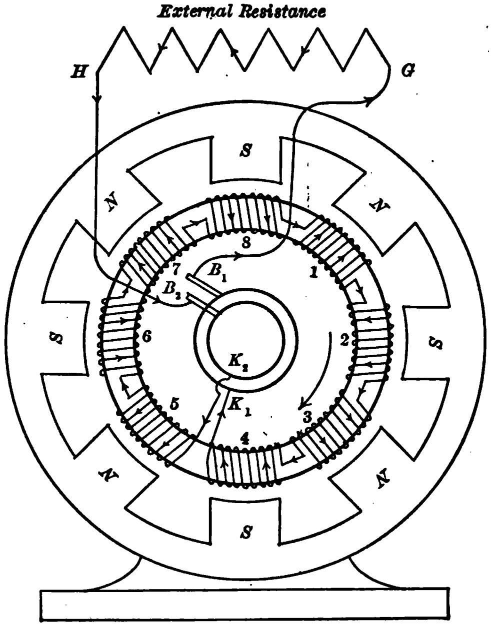


FIG. 225 a.

900 times per minute or 15 times per second, there will be produced 60 cycles per second in the reversals of electromotive force.

In many of the large alternators the armature is stationary and the field magnets revolve, the result being in no wise different. The rotating part is often called the *rotor*, and the stationary part the *stator*.

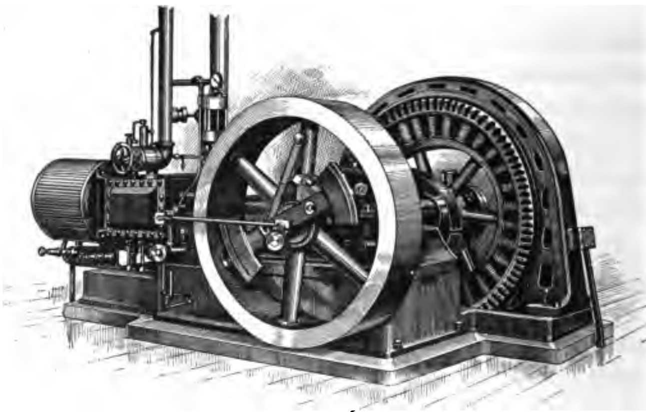


FIG 225 *b*.

Such an alternating current could not be used to energize the field magnets, because their polarity would change with each reversal in the direction of the current, so that a smaller direct-current dynamo is generally used in connection with an alternator to excite its field.

PROBLEMS

1. If the primary coil of a transformer has 200 turns and the secondary coil 20 turns, and if an alternating current with a voltage of 5000 is passing through the primary, what is the voltage induced in the secondary coil? *Ans*. 500 volts.

2. If in a direct-current dynamo a single coil armature of one turn is revolved 15 times per second between the poles of a field magnet having a magnetic flux of 500,000 lines, what E.M.F. is induced in the armature?

Ans. Since each half of the coil cuts the flux twice in each revolution, the number of lines cut per revolution is 2,000,000. The lines cut per second is therefore 30,000,000. Since 1 volt equals 10^8 C.G.S. units of E.M.F., the E.M.F. generated is .3 volt.

3. A drum armature of a direct-current dynamo has 36 coils with 4 turns each. If when rotated with a speed of 2000 revolutions per minute it is to develop an E.M.F. of 150 volts, what must be the magnetic flux of the field magnet?

Ans. The two sides of a drum armature are connected to the brushes *in parallel*, hence the E.M.F. $= [\frac{1}{2} \cdot (144 \times 2 \times \frac{2000}{60} \times N)] \div 10^8 = 150$ volts. $N = 3,125,000$ lines.

4. The E.M.F. of a series direct-current dynamo is 125 volts. The armature resistance is 1 ohm, the series field-coil resistance is 2.5 ohms, and the terminals of the dynamo are connected by 20 lamps in parallel, each having a resistance of 240 ohms. (*a*) If a voltmeter were connected across the terminals of the dynamo, what would be its reading? (*b*) What current flows through the armature? (*c*) What current flows through the field coil? (*d*) What current flows through each lamp?

Ans. (*a*) 96.8 volts. (*c*) 8.06 amp.
(*b*) 8.06 amp. (*d*) .403 amp.

5. The E.M.F. of a shunt direct-current dynamo is 125 volts. The armature resistance is 1 ohm, the shunt field resistance is 200 ohms, and the terminals of the dynamo are connected by 20 lamps in parallel, each having a resistance of 240 ohms. (*a*) If a voltmeter were connected across the terminals of the dynamo, what would be its reading? (*b*) What current flows through the armature? (*c*) What current flows through the field coil? (*d*) What current flows through each lamp?

Ans. (*a*) 114.85 volts. (*c*) .574 amp.
(*b*) 10.15 amp. (*d*) .478 amp.

CHAPTER XX

PRACTICAL APPLICATIONS OF ELECTRICITY

439. Electric Motor. — An *electric motor* is a machine constructed like a dynamo with field magnet, armature, commutator, and brushes, the armature of which revolves when a sufficiently strong electric current is passed through the field coils and the armature.

In this machine electrical energy is transformed into mechanical energy, while in a dynamo mechanical energy is transformed into electrical energy. In other words, the intake of a motor is electrical energy, and its output is mechanical energy; while the intake of a dynamo is mechanical energy, and its output is electrical energy.

If a shunt dynamo, *G*, is driven by a steam engine or other prime mover, and if the current from this dynamo is led into the field coils and armature of a similar machine, *M*, as shown in Fig. 226, the current will make the right side of the field magnet of *M* an N pole and the left side an S pole. By using the right-hand rule for electro-magnets with the current in the armature, it is seen that an N pole is produced at the lower extremity of the vertical diameter of the armature, and an S pole at the upper extremity. By the mutual action of the poles of the field magnet and the poles of the armature, it is evident that there is a system of forces acting which will rotate the armature clockwise, and if the dynamo is placed so that its *field poles are in reverse order to those of the motor* its armature must also be revolved clockwise in order to produce a current in the same direction as that in the motor.

The attractions of the unlike poles of field and armature constitute a mechanical couple whose effect is to produce rotation. Likewise the repulsions of the like poles of field and armature constitute a similar couple whose effect is rotation in the same direction.

As coil 1 in the motor armature moves into the position of coil 4, the current in it reverses; likewise with coil 3 as it moves to the position of coil 2. Hence the N pole of

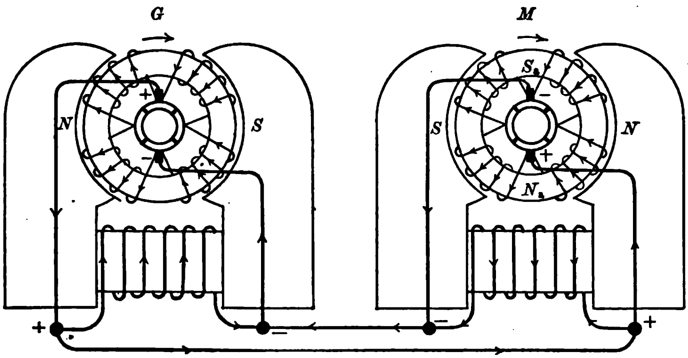


FIG. 226.

the armature continues to be at the upper extremity of the vertical diameter of the armature; or, as the armature revolves, its poles do not revolve with it; therefore the tendency for rotation continues to exist.

If, however, the armature is of the drum type as shown in Fig. 222 or Fig. 253, the formation of these poles in the armature is not so apparent. In § 422 it was shown that if in a conductor in front of the N pole of a field magnet a current is sent in a direction toward you, the lines of force due to the field magnet reënforce those on the upper side and oppose those on the under side of this conductor due to the current in it. This difference tends to move the conductor downward in front of the N pole. Similarly in

the conductor in the other side of the armature in front of the S pole where the current flows away from you, this reaction of the magnetic field of the conductor and of the field magnet tends to move the conductor up in front of the S pole. It is this mechanical drag of the field magnet on the conductors of the drum armature of a motor that produces its rotation.

440. Speed of Electric Motor. — A constant force acting upon a body produces accelerated speed, so that under the continued action of the poles of field and armature the armature revolves faster and faster until brought to uniform speed, for the following reason: —

The field magnet causes a magnetic flux through the armature, and the coils of the armature as it revolves cut these lines of force, and there is consequently generated in them, just as in a dynamo, an E. M. F., which by the three-finger rule is found to be opposite in direction to the driving current in the armature. This counter electro-motive force opposes the flow of current in the armature. As the armature revolves faster, this counter E. M. F. continues to increase until, even if there were no load on the motor pulley and no frictional resistance in its bearings, the counter E. M. F. would come to equal the driving E. M. F., and no current would flow in the motor. The forces being in equilibrium, the motion of the motor would continue uniform at the high speed necessary to make the counter E. M. F. equal to the driving E. M. F.

As the load on the motor pulley increases, the speed of the armature slackens. This will decrease the counter E. M. F., and the driving E. M. F. then being the greater, will send sufficient current into the motor to rotate the armature against this additional load.

The ability of the armature to rotate against a given mechanical resistance, *i.e.* the rate of work the motor is capable of, depends both on the *mutual attraction and mutual repulsion* of the poles of the field and of the armature and on the *speed* of the armature. The mutual force of the poles depends on the current C in the armature and field, and the counter E. M. F., E_M, depends on the speed of the armature for a given strength of field. Hence the output of the motor is the product E_MC. The energy supplied by the dynamo to the motor, or its intake, equals the driving E. M. F., E_D, times the current C, or E_DC. The efficiency of the motor is $E_MC : E_DC = E_M : E_D$. As the counter E. M. F., E_M, decreases, the current, C, increases, and the product E_MC is a maximum when E_M is one half of E_D; that is, a motor performs the greatest quantity of work per second when the counter E. M. F. developed in it is one half of the driving E. M. F. of the dynamo.

441. Magnetic Drag in a Dynamo. — Referring back to the series-wound dynamo in § 435, it will be noticed that the current produced in the armature is in such a direction as would form an S pole at the lower extremity, and an N pole at the upper extremity of the vertical diameter of the armature. The mutual action of these poles and of the field-magnet poles would tend to rotate the armature in an opposite direction to which it is being revolved to generate in it the indicated E. M. F. This magnetic action constitutes what is called *magnetic drag*, and it is in overcoming this drag that the mechanical energy supplied at the armature pulley is transformed into electrical energy. That this drag exists may be experienced by turning the armature of a small hand dynamo, first when the external circuit is open, and then when it is closed. The great

resistance offered to being turned in the latter instance is sufficient evidence of the existence of a magnetic opposition to the rotation.

It is evident also that as the external resistance grows less, the greater becomes the current passing through the armature, and the more powerful do its poles become; hence the greater becomes the magnetic drag, and the greater the mechanical energy necessary to be supplied at the armature pulley to produce rotation. Consequently the greater the output of the dynamo in electrical energy, the greater must be its intake of mechanical energy. It is evident also that the efficiency of a dynamo, or the ratio of its output of energy to its intake, can never become 100%, for some of the mechanical energy is expended in overcoming the frictional resistance at the bearings of the armature shaft and in churning the air in the interpolar space. Also, a certain amount of the electrical energy produced is transformed into heat within the armature and field coils as a current is forced through them. The heat produced by an electric current flowing through a wire is .24 C^2Rt calories; hence this loss of energy in the dynamo increases as the square

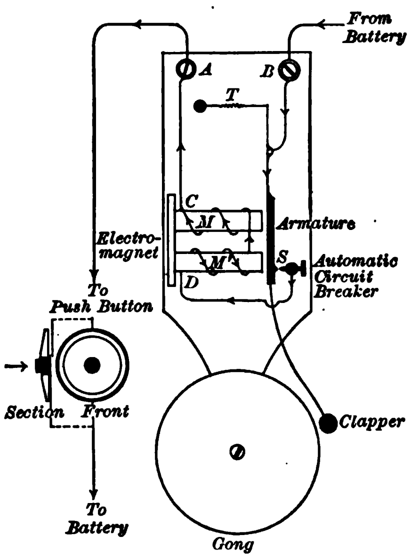


FIG. 227 *a*.

of the current produced, the internal resistance, R, of the dynamo being constant.

442. Electric Bell. — An electric bell consists of an electro-magnet MM' (Fig. 227 *a* and *b*) in front of whose poles is a soft iron armature pivoted so as to be free to move toward the poles. The clapper of the bell is fastened to the armature and moves with it. In contact with the armature when at rest is a platinum-tipped screw S. The spring T is adjusted so that its tension is sufficient to keep the armature in contact with S when no current is flowing around the electro-magnet coils. When the push button is pressed, the current from the battery enters the bell at one binding post (say B), passes to the armature support, and along the armature to screw S, thence through screw S to its support which is connected with the electro-magnet coils.

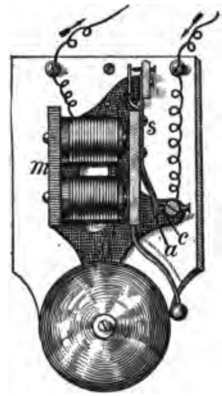


FIG. 227 *b*.

After the current passes through these coils it leaves the bell at the other binding post A and returns to the battery through the connecting wires and push button. This current energizes the magnet which by attracting the armature causes the clapper to strike the gong. As soon, however, as the armature moves toward the poles it is no longer in contact with screw S and the circuit is broken. The magnet then ceases to attract the armature and spring T brings it back again into contact with screw S, when the operation is repeated so long as the push button is being pressed.

If the wire from the battery, instead of being connected

at B, is joined to the support of screw S, the circuit is not broken when the magnet attracts the armature and the gong is struck but the once, the armature being released only at the will of the operator. A bell so connected may be called a single-stroke bell in distinction from the "clatter" bell as it is ordinarily connected. Such a bell is used in fire-alarm circuits.

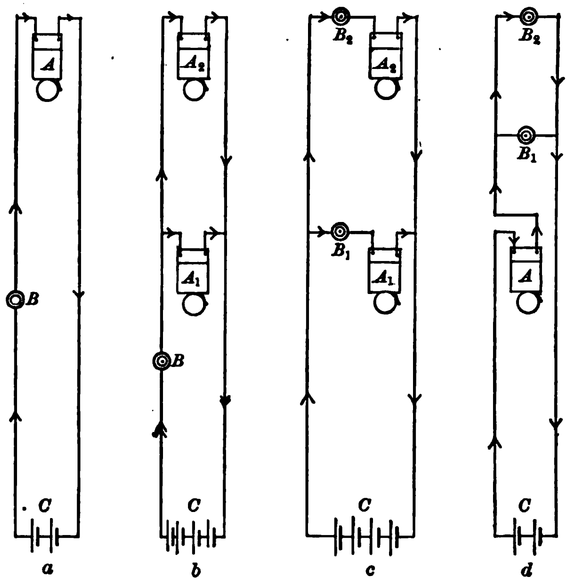


FIG. 228.

443. Electric Bell Circuits. — In Fig. 228 are shown electric bell circuits: (*a*) of one bell in circuit with a push button and a battery; (*b*) of two bells in circuit with one push button and a battery so that both bells ring when contact is made at the button; (*c*) of two bells, two push buttons, and a battery so that but one bell is rung when each button is pushed; (*d*) of one bell, two buttons, and a

battery so that the bell may be rung by pressing either button.

444. Electric Telegraph.—A telegraph instrument consists of two parts: the *transmitter* or the *key*, and the *receiver* or the *sounder*.

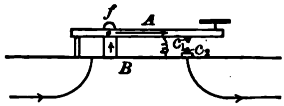


FIG. 229 *a*.

Figure 229 *a* and *b* is a diagram of a telegraph key which consists of a metal lever A supported at fulcrum f. C_1 and C_2 are two platinum tips, C_1 being attached to the lever A and C_2 being supported by the base B but insulated from it by rubber washers. The wires leading to the key connect with the base B and with the platinum tip C_2 as shown. S is a switch which, if pushed over against C_2, short circuits the key.

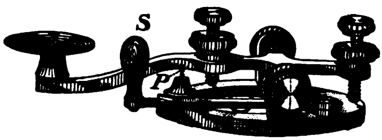


FIG. 229 *b*.

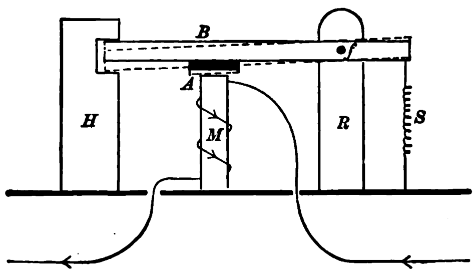


FIG. 230 *a*.

In Fig. 230 *a*, *b*, is shown a telegraph sounder which consists of an electro-magnet M over which is a metal lever B to which is attached at a point directly

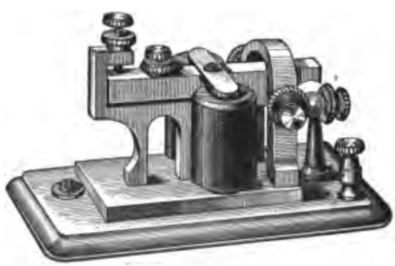


FIG. 230 *b*.

over the magnet the armature *A*; consisting of a piece of soft iron. The lever is supported by *R* at the fulcrum *f*. *S* is a spiral spring. When current flows through the electro-magnet *M*, the armature *A* is attracted, drawing the lever *B* down so that it strikes against the metal projecting piece *H* with a clicking sound. When the circuit is broken, the magnet releases *A* and the spring *S* pulls the lever up causing the end to strike against the upper side of the notch, making a similar click. According as the interval between these two clicks, one on closing, the other on opening the circuit, is short or long, the signal is considered a dot or a dash, combinations of which make up the telegraphic code.

Figure 231 shows the connection of two simple telegraph

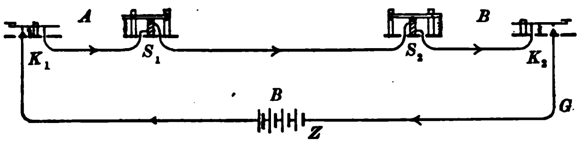


FIG. 231.

instruments in circuit with a battery. The return wire *GZ* may be eliminated by connecting both of these points to the earth which then acts as the return conductor.

In order that an operator at *A* may be able to telegraph signals to an operator at *B*, the key at *B* must be short circuited by closing its switch or by holding the lever down; the circuit is then under the complete control of the operator at *A*, and both the sounder at *A* and that at *B* respond to his interruptions of the current in the circuit.

445. Telegraph Relay. — If the distance between stations *A* and *B* is great, the resistance of the long line wire diminishes the current so much that it will not energize

the magnets of the sounder sufficiently to move the lever and give the signals. To obviate this difficulty a *relay* is employed. A relay is a delicately adjusted electro-magnet which when energized attracts its armature *A* and thus closes the circuit at points *b* and *c of a local battery* B^2 (Fig. 232) through the sounder *S*. The armature *A* is of small mass and easily moved, so that the weak current coming in through the line wire when passed many thousands of times around its iron core magnetizes it sufficiently to move *A*.

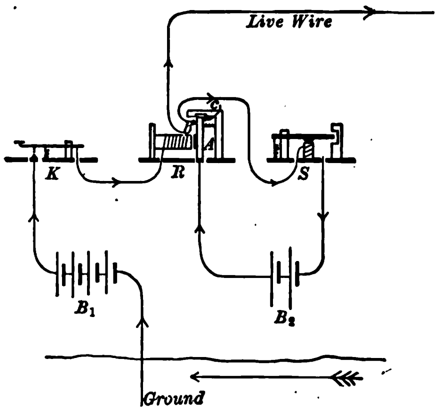


FIG. 232 *a*.

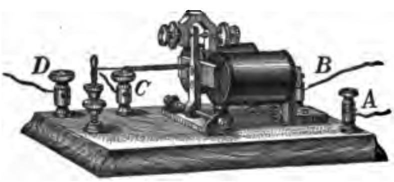


FIG. 232 *b*.

446. Electric Telephone. — A telephone consists of two parts: the *receiver* and the *transmitter*.

The receiver, shown in Fig. 233, consists of a permanent steel magnet *A*, around one pole of which is wound a coil *B* of fine insulated wire, the terminals of which are the binding posts *M* and *N*. Within the earpiece *G*, close to but not touching the magnet, is a thin iron diaphragm *E*.

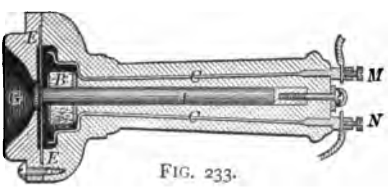


FIG. 233.

The *transmitter* (Fig. 234) consists of a mouthpiece, A, one end of which is closed by a thin diaphragm, d, which is connected at the center of its interior surface to a small insulated metallic disk, forming the cover of a small box, B, containing carbon granules. The insulated disk is connected to one pole of the battery in the primary circuit, P. The bottom of this box is a metallic plate connected to a wire from one terminal of the primary, P, of an induction coil, PS, the other terminal of which is connected to the other pole of the battery. Any agitation of the carbon granules in this box varies the resistance between the disks constituting the top and bottom of this box, and thus varies the current in the primary circuit. The agitation of the carbon granules, caused by sound waves proceeding from a speaker's mouth at A and impinging on the diaphragm at the end of the mouthpiece, varies the primary circuit, and consequently an induced E. M. F. is generated in the secondary coil which is connected through the receiver with the line wire. The current produced by this induced E. M. F. passes through the coil of wire around the magnet in B's receiver and strengthens or weakens that pole, according to its direction.

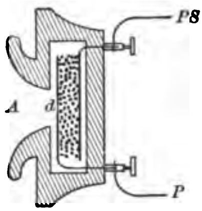


FIG. 234.

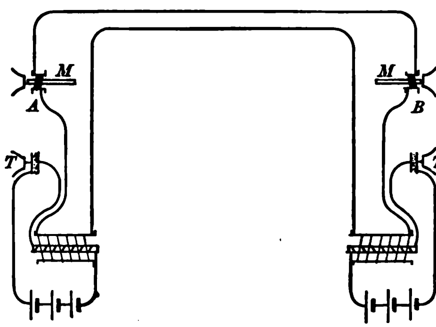


FIG. 235.

With a changing strength of pole the iron diaphragm of

the earpiece moves back and forth in exactly the same manner as the diaphragm of the transmitter is made to vibrate by the sound waves produced by the voice. Hence the effect on the ear is as if one were speaking directly into it. The connections of a telephone circuit of two sets of instruments are shown in Fig. 235.

447. Wireless Telegraph. — The action of this form of telegraph depends upon the following facts: (1) that the discharge of a high-tension spark coil is oscillatory, *i.e.* surges to and fro; (2) that if a tall conducting mast is connected with one terminal of the secondary coil, the electric oscillations in this mast will form ether waves which radiate in all directions. The length of these waves is much greater than of those which produce a maximum effect of heat or of light. These electric waves, called Hertz waves, from the scientist who first made a thorough investigation as to their character, reproduce electric oscillations in a similar mast located at a distance. Connected in circuit with this receiving mast is a tube of metal filings which, by the effect of these electric oscillations, changes from an electrical conductor of high resistance to one of low resistance. If this tube of filings, called a *coherer*, is also connected in circuit with a battery and a telegraph sounder, at the instant the electric oscillations make it conducting, the current from the battery passes through it and through the sounder, and the armature of the sounder being attracted, the usual click is heard. In circuit with the sounder is also a *decoherer*, which may be an electric bell with the coherer placed in the position of the gong, so that the instant the current passes through the coherer it also passes through the electro-magnet of the decoherer causing the clapper to strike the coherer, whose filings are disarranged by the blow. The tube is thus

once more made practically nonconducting, and the armature of the sounder is consequently released, giving the

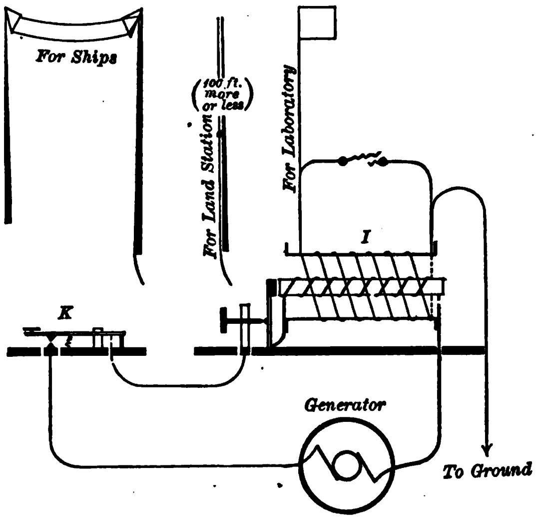


FIG. 236 *a*.

usual return click. A diagram of a wireless telegraph set up for use in the laboratory where, on account of the short

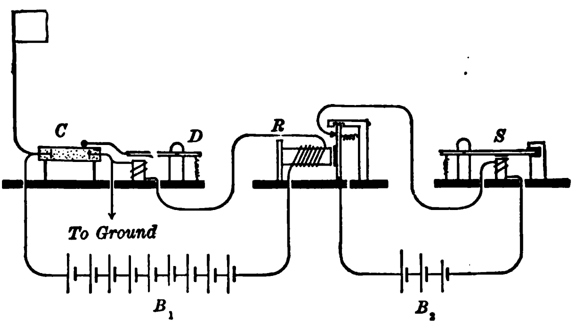


FIG. 236 *b*.

distance and absence of obstructions, no high mast is needed, is given in Fig. 236 *a*. *I* is the spark coil, *K* is the key, in the primary circuit, controlling the interval during which the sparks shall occur between the knobs connected with the secondary coil, and *G* is the generator in the primary circuit. The receiving instruments are represented in Fig. 236 *b*: the coherer *C*, the decoherer *D*, the relay *R*, the sounder *S*, and the batteries B_1 and B_2 for energizing the electro-magnet of the relay and sounder respectively.

This is called a wireless telegraph because no wires connect the transmitting and receiving instruments, the latter being actuated by ether waves generated by the oscillatory discharge of the spark coil.

448. Electric Lamp. — The construction of an incandescent electric lamp is shown in Fig. 237. The fine carbon filament *e*, placed within the glass bulb *y*, from which the air has been exhausted, is connected at one end with the outer threaded brass casing *d* at the lower end of the bulb; the other end of the filament is connected with the small metal plug *g*, which is insulated from the brass casing by the filling in of the intervening space with nonconducting cement. When the brass casing of the bulb is screwed into the key socket, as shown, electrical contact is made between the threaded portion of the bulb and that of the socket, and also between the metal plug *g* and the metal tongue *h*, which is

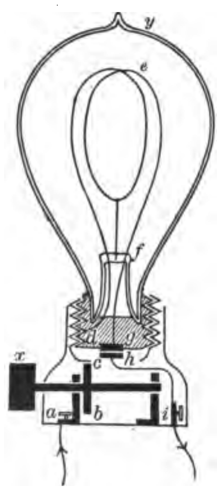


FIG. 237.

insulated by a sheet of mica from other parts of the socket except screw *i*, with which it is connected. If the key *x* is turned so that the metal piece *b* is touching the part *c* of the socket, a current entering at *a* flows to *b*, to *c*, to *d*, through *e*, to *f*, to *g*, to *h*, through *i* to the metal support connecting it with the screw, and out. If the key *x* be turned at right angles to its indicated position, it turns the part *b* around so that it is no longer in contact with *c* and the circuit is broken.

The filament *e* is a very thin piece of carbon, which is infusible and a conductor of high resistance; the length of the filament is so adjusted that when the lamp is connected to the points *a* and *i*, between which there is a P. D. of 110 volts, the current flowing through the filament will be sufficient to raise its temperature to white heat, thus giving as much light as is compatible with a fairly long life of the lamp. The heat produced by a current passing through a wire is .24 C^2Rt calories. If the current in a lamp is $\frac{1}{2}$ amp. and its resistance is 220 ohms, the heat produced per second $= .24 \times \frac{1}{4} \times 220 = 13.2$ calories. This heat equals the quantity of energy radiated by the incandescent filament per second in order that its temperature shall remain constant at the point of incandescence. The air is exhausted from the bulb to prevent combustion of the filament when heated.

449. Wiring for Lamp Circuits. — In incandescent lamp circuits (Fig. 238) the P. D. of the main wires, or *leads*, is kept fairly constant at 110 to 115 volts. Since *each*

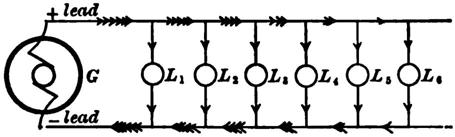


FIG. 238.

lamp requires that much P. D. to make it incandescent, the lamps must be connected in parallel, as shown in Fig. 238. The current flowing through each lamp is independent of that flowing through the others, since the P. D. of the leads is kept constant by an adjustment of the generator in the power house.

In order to economize in the quantity of copper wire needed for a given installation of lamps, the *three-wire system* may be used. In this system (Fig. 239) two 110-volt generators are joined in series, and therefore, between the two outer leads, A and C, there is a P. D. of 220 volts. The intermediate lead B is joined at m to the wire which connects the two generators, G_1 and G_2, in series. When there are the same number of lamps between the leads A and B as between B and C, there is no current flowing in the wire B, since there is no P. D. between m and n (the resistance A–B = resistance B–C, and the internal resistance G_2 = internal resistance G_1). If, however, the number of lamps between B and C is greater than that between A and B, the potential of m is lowered, and enough current flows in the wire to furnish that required for the excess of the lamps and to keep the potential of m midway between that of A and C.

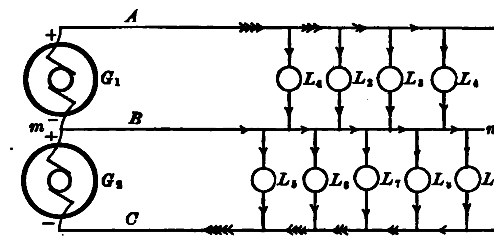


FIG. 239.

The wire B may be made smaller than either A or C, since it carries only the difference between the currents in these wires.

450. Arc Lamp. — In Fig. 240 is shown an electric arc

lamp with a mechanism for its automatic control. If the carbons are not in contact the current entering at one of the lamp terminals cannot flow from carbon *e* to carbon *f* and out through the coil C_1 so it flows through the shunt circuit C_2, which is a sucking coil. This draws up its iron core *n*, pulling up that end of lever *l* to which the iron core is attached, and loosens the clutch *c* on the upper carbon carrier, which then, by its own weight, moves down until the carbons come in contact. At this instant, the circuit being complete through the carbons and the second sucking coil C_1, whose resistance is much less than that of the shunt coil C_2, the greater part of the current flows through the sucking coil C_1 and causes its iron core n^1, to which the upper carbon carrier is attached, to be drawn up into the coil, slightly separating the carbons, thus "striking the arc." On account of the resistance of this arc, the points of the carbons are heated to incandescence, and the gases and solid particles moving across the space between the carbon points are also rendered luminous.

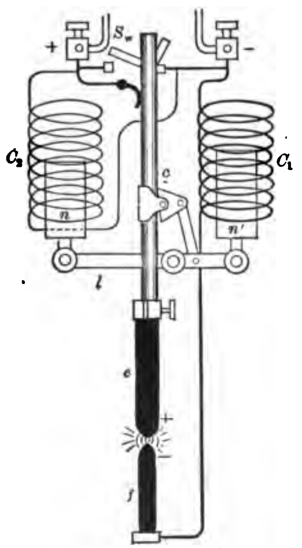


FIG. 240.

As the current continues to flow from the + carbon *e* to the − carbon *f*, the carbons wear away by their combustion with the oxygen of the air, the + carbon wearing away about twice as fast as the − carbon. This increases the space between *e* and *f*, and thereby lengthens the arc. Lengthening the arc increases the resistance of the circuit through

coil C_1, and causes more of the current to flow through the shunt coil C_2 until the coil is able to draw up its core *n*, and by means of lever *l* loosen the clutch *c*, allowing the carbon carrier to move down by its own weight, thus shortening the arc and reducing its resistance to a normal value.

Arc lamps are usually used in connection with alternating current generators, the E. M. F. of the circuit often being as high as 3000 volts. Since the P. D. necessary at the terminals of each lamp is but 50 volts, arc lamps are usually connected in series, each lamp having an automatic cutout that short-circuits the lamp whenever it is not in working condition, thus preventing the breaking the circuit through the other lamps in the circuit.

In a more recent type of lamp, the carbons are inclosed in a tight-fitting globe so that there is no free access of air. The combustion of the carbons being then slower, the carbons need to be renewed only every two or three days instead of every day, as is necessary in the ordinary arc lamp. These inclosed arc lamps are commonly so constructed as to require a P. D. at their terminals of 110 volts, and are connected in parallel with the ordinary 110-volt direct-current leads, in the same way as incandescent lamps are arranged.

Illustration. — Electric power for lamp or motor installations is purchased from power stations at a price varying from 5 to 15 cents per kilowatt-hour.

If a lamp or motor expends electric energy at the rate of a kilowatt for the period of an hour, the total electric energy expended is a kilowatt-hour.

For instance, an ordinary incandescent lamp with a P. D. of 110 volts at its terminals carries a current of about $\frac{1}{2}$ amp. The electric power $EC = 110 \times \frac{1}{2} = 55$ watts $= .055$ kilowatt. If this lamp is used for an hour, the energy expended is .055 kilowatt-hour. At 15 cents per kilo-

watt-hour, the cost of using the lamp for one hour is $.055 \times 15 = .825$ cents.

If the P. D. at the terminals of a street-car motor is 500 volts, and if a 20 amp. current flows through it, the expenditure of energy is $500 \times 20 = 10{,}000$ watts = 10 kilowatts. At 5 cents per kilowatt-hour, it would cost 50 cents to run the motor for 1 hour.

PROBLEMS

1. A 30 horse-power engine drives a dynamo; if the dynamo gives out 90% of its intake of energy, how many kilowatts does it develop? *Ans.* 20.142 kw.

2. An elevator weighing 2 metric tons (2×1000 kgm.) is to be lifted at the rate of 5 meters per second. If the output of the driving motor is 80% of its intake of energy: (*a*) How many kilowatts must be supplied to the motor? (*b*) How many joules of work must it do per second? *Ans.* (*a*) 24.5 kw. (*b*) 19600 joules per second.

3. The resistance of the line wire leading from a dynamo to a given installation of lamps is .05 ohm. If the current delivered to the lamps is 200 amp.: (*a*) What is the loss of power in the transmission? (*b*) What is the drop in voltage between the dynamo and the lamps? *Ans.* (*a*) 2 kw. (*b*) 10 volts.

APPENDIX

451. Standard Fortin Barometer.—A standard mercurial cistern barometer consists of the mercurial barometer constructed as described in § 70, inclosed in a metal tube that serves not only as a support and protector of the glass tube, but also to carry the scale by which the height of the mercury in the tube is read.

The bottom of the cistern is made of flexible leather and against the bottom presses a screw, *A* (Fig. 21 *b*), by the adjustment of which the level of the mercury in the cistern may be raised or lowered. The zero of the scale, *S*, fixed on the outside tube is the point, *O*, of the ivory pin which projects downward from the roof of the cistern.

To read the barometer, first adjust the screw, *A*, at the bottom of the cistern until the mercury surface in the cistern just touches the point, *O*, of the ivory pin; then the reading of the scale, *S*, on a level with the surface of the mercury in the tube is the height of the mercury in the tube above the surface of the mercury in the cistern. To obtain this reading on the scale, *S*, first raise or lower by means of a screw, *B*, on the side of the tube, the sliding vernier scale, *V*, until its lower or zero edge is tangent to the convex mercury surface in the tube. The position of this lower edge of the vernier scale on the fixed scale is next to be determined. Read the nearest whole number of inches on the scale, *S*, *below* this vernier scale, then the number of tenths and twentieths of an inch between this inch division and the lower edge of the vernier. If the zero edge of the vernier does not coincide exactly with any twentieth of an inch division, the fraction of the twentieth division remaining to be found is determined in the following manner: The vernier scale (Fig. 241) consists of 25 divisions

which are equivalent to 24 twentieths of an inch division of scale, S; consequently each vernier division is $\frac{1}{25}$ of $\frac{1}{20}$ or $\frac{1}{500}$ in. shorter than each twentieth inch division. $\frac{1}{500}$ in. =..002 in.

Look along the vernier scale and find which of its divisions coincides exactly with one of the twentieth of an inch divisions of scale, S. Suppose, for instance, it is the 7th vernier division. Then the 6th division is .002 in. above the twentieth division on scale, S, next below it; the 5th division is. 004 in. above the next twentieth division, the 4th division .006 inch above, and so on, the zero division being .014 in. above the next twentieth division below it.

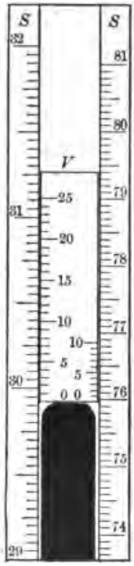


FIG. 241.

If, for example, the whole number of inches read is 29, the nearest number of tenths below the vernier, 9, the number of twentieths, 0, and the 7th vernier division is the coinciding division, the height of the mercury in the tube is 29 + .90 + .014 = 29.914 in.

If the fixed scale, S, is graduated in centimeters and millimeters, the vernier scale (Fig. 241) is constructed so that 10 vernier divisions equal 9 millimeter divisions or 1 vernier division = .9 mm.

Then each vernier division is .1 mm. = .01 cm. shorter than a millimeter division.

If the position of the vernier scale is as shown in the diagram, the height of the barometer is 75 + .9 + .06 = 75.96 cm.; because the 6th vernier division is the *coinciding* division, and as each vernier division falls .01 cm. short of the millimeter division next below it, the zero division of the vernier scale is .01 × .6 = .06 cm. above the nearest millimeter division below.

452. Determination of Pressure in Air-Pump Receiver. — If the volume of the receiver, R, is 2000 cc. and the volume of the pump cylinder, A, when the piston is at the top of the barrel is

1000 cc., at the end of the first upstroke, the 2000 cc. of air in R has expanded to fill the joint space $R + A$, which is 3000 cc., the pressure is, by Boyle's law, reduced to $\frac{2}{3}$ of an atmosphere. At the end of the second upstroke the 2000 cc. of air which was left in R under a pressure of $\frac{2}{3}$ of an atmosphere has again expanded to 3000 cc., thus reducing the pressure to $\frac{2}{3}$ of $\frac{2}{3}$ or $\frac{4}{9}$ of an atmosphere. At the end of each stroke the pressure will be $\frac{2}{3}$ of that at the end of the preceding stroke.

Stated in general terms, if $V_R =$ the volume of the receiver, R, $V_A =$ the maximum volume of space, A, at the end of the 1st stroke the pressure in $R = \frac{V_R}{V_R + V_A}$ of an atmosphere; at the end of the 2d stroke, the pressure is $\frac{V_R}{V_R + V_A}$ of $\frac{V_R}{V_R + V_A} = \left(\frac{V_R}{V_R + V_A}\right)^2$ of 1 atmosphere; at the end of the 3d stroke the pressure in $R = \left(\frac{V_R}{V_R + V_A}\right)^3$ of 1 atmosphere; at the end of the nth stroke, the pressure $= \left(\frac{V_R}{V_R + V_A}\right)^n$ of 1 atmosphere.

453. The Sine of an Angle. — The problem of the derrick, § 108, can be more readily solved by making use of the following property of right triangles.

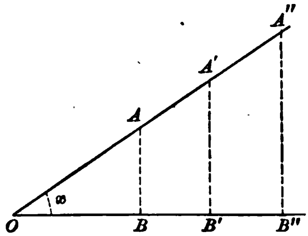


FIG. 242.

If from points A, A', A'' (Fig. 242), in one side of a given angle, x, the perpendiculars AB, $A'B'$, $A''B''$, are drawn upon the other side; since the ⊿s OAB, $OA'B'$, $OA''B''$ are similar,

$$\frac{AB}{OA} = \frac{A'B'}{OA'} = \frac{A''B''}{OA''} = \text{a constant ratio for the same angle, } x.$$

The name of this ratio is the *sine* of the $\angle x$. The sine of an angle is therefore the ratio of the length of the perpendicular drawn from one side of the angle upon the other to the length of the hypotenuse of the right triangle thus formed.

The values of the sines of angles from 0° to 90° are to be found in the table given in the appendix.

The sine of the $\measuredangle AOB$ (Fig. 243) $= \dfrac{AB}{OA}$.

The sine of the $\measuredangle DOH$ $= \dfrac{DH}{OD}$.

The sine of the $\measuredangle EOK$ $= \dfrac{EK}{OE}$.

The sine of the $\measuredangle FOB$ $= \dfrac{FL}{OF}$.

The sine of the $\measuredangle FOL$ $= \dfrac{FL}{OF}$.

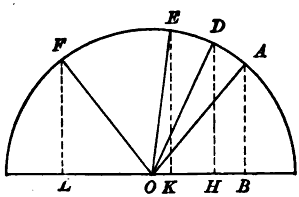


FIG. 243.

It is seen from the last two values given that the sine of an obtuse angle, such as FOB, is the same as the sine of is supplement, FOL. If it is required in solving a triangle to know the sine of an obtuse angle, say 130°, look in the table of sines for the sine of its supplement, 50°.

454. Cosine of an Angle. — In the $\triangle$ OAB (Fig. 243) $\dfrac{AB}{OA}$ = the sine of $\measuredangle AOB$, and $\dfrac{OB}{OA}$ = the sine of $\measuredangle OAB$. But OAB is the complement of AOB; therefore $\dfrac{OB}{OA}$ is called the *cosine* of AOB, *i.e.* the sine of the complement of AOB.

455. Solution of the Derrick Problem by Sines. — Returning to the problem of the derrick it is seen that $\dfrac{BS}{RS}$ = the sine of 30°. From the table of sines the sine of 30° = .500. Also $\dfrac{BR}{RS}$ = the sine of 60°. From the table of sines, sine 60° = .866,

$$\frac{2 \text{ tons}}{RS} = .866, \text{ or } RS = \frac{2}{.866} = 2.309.$$

$$\therefore \frac{BS}{2.309} = .500, \text{ or } BS = 2.309 \times .5 = 1.15 \text{ tons.}$$

456. Two Propositions for the Solution of Triangles. — The following two propositions will be found extremely useful in solving triangles.

I. Any two sides of a triangle are to each other as the sines of the opposite angles.

II. The square of any side of a triangle equals the sum of the squares of the other two sides, $\pm$ twice their product times the cosine of the angle included by them ($+$, if the included angle is obtuse; $-$, if acute).

Proof of Prop. I. — To prove $BC : AC = \text{sine } A : \text{sine } B$.

Draw $CD \perp$ to AB.

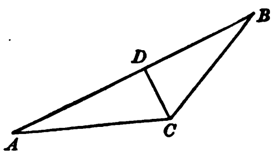


FIG. 244.

$$\frac{CD}{AC} = \text{sine } \measuredangle A$$

$$CD = AC \cdot \text{sine } A$$

$$\frac{CD}{BC} = \text{sine } \measuredangle B.$$

$$CD = BC \cdot \text{sine } B.$$

$$\therefore AC \cdot \text{sine } A = BC \cdot \text{sine } B,$$

or $BC : AC = \text{sine } A : \text{sine } B$. (If the product of two quantities equals the product of two other quantities, either two may be the means or extremes of a proportion.) Q.E.D.

Proof of Prop. II.

(*a*) To prove $\overline{AB}^2 = \overline{AC}^2 + \overline{BC}^2 + 2 \cdot AC \cdot BC \cdot \text{cosine } ACB$.

Draw $BD \perp$ to AC produced.

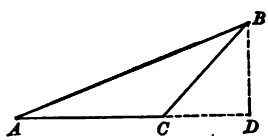


FIG. 245 *a*.

$$\overline{AB}^2 = (AC + CD)^2 + \overline{BD}^2,$$

$$\overline{BD}^2 = \overline{BC}^2 - \overline{CD}^2.$$

$$\therefore \overline{AB}^2 = \overline{AC}^2 + \overline{CD}^2 + 2\, AC \cdot CD + \overline{BC}^2 - \overline{CD}^2$$

$$= \overline{AC}^2 + \overline{BC}^2 + 2\, AC \cdot CD.$$

$$\frac{CD}{BC} = \text{sine } \measuredangle CBD = \text{cosine } \measuredangle BCD = \text{cosine } ACB,$$

or $\quad CD = BC \cdot \text{cosine}, ACB.$

$$\therefore \overline{AB}^2 = \overline{AC}^2 + \overline{BC}^2 + 2\, AC \cdot BC \cdot \text{cosine } ACB. \qquad \text{Q.E.D.}$$

(*b*) To prove $\overline{AB}^2 = \overline{AC}^2 + \overline{BC}^2 - 2 \cdot AC \cdot BC \cdot \cos ACB.$
Draw $BD \perp$ to AC.

$$\overline{AB}^2 = (AC - CD)^2 + \overline{BD}^2,$$

$$\overline{BD}^2 = \overline{BC}^2 - \overline{CD}^2.$$

$$\begin{aligned}\therefore \overline{AB}^2 &= \overline{AC}^2 + \overline{CD} - 2\, AC \cdot CD \\ &\quad + \overline{BC} - \overline{CD}^2 \\ &= \overline{AC}^2 + \overline{BC} - 2\, AC \cdot CD.\end{aligned}$$

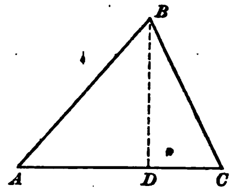


FIG. 245 *b*.

$$\frac{CD}{BC} = \text{sine } CBD = \text{cosine } ACB,$$

or $\quad CD = BC \cdot \text{cosine } ACB.$

$$\therefore \overline{AB}^2 = \overline{AC}^2 + \overline{BC}^2 - 2 \cdot AC \cdot BC \cdot \cos ACB. \qquad \text{Q.E.D.}$$

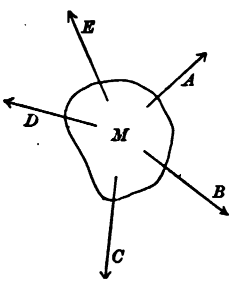
The problem of the derrick may be solved very simply by means of Prop. I.

$$BS : BR = \text{sine } 30° : \text{sine } 60°,$$

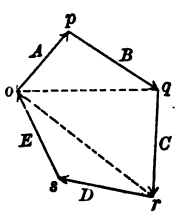
or $\quad BS : 2 \text{ tons} = .500 : .866.$

$$BS = \frac{2 \times .500}{.886} = 1.15 \text{ tons.}$$

457. Equilibrium of Any Number of Concurrent Forces. — Let a body, M, be in equilibrium under the action of any number of forces, A, B, C, D, and E. Starting from any point o (Fig. 246 *b*) draw a line op representing force A in magnitude and direction, and from p a line pq representing force B, and from q a line qr representing force C, and similarly the lines rs and so representing forces D and E; then a closed polygon is formed.



a



b

FIG. 246.

The resultant of *op* and *pq* is the diagonal *oq*; the resultant of the diagonal *oq* and the force *qr* is the diagonal *or*, and this diagonal forms with forces *rs* and *so* a triangle with sides taken in order, hence represents a state of equilibrium according to the principle of the triangle of forces.

Hence, in general, any number of concurrent forces in equilibrium may be represented by the sides of a closed polygon *taken in order*.

458. Equilibrium of Parallel Forces a Special Case of Concurrent Forces. — From experimental results it has been shown that as the angle between two forces, A and B, of three forces in equilibrium grows less, the more nearly does the sum of A and B equal their equilibrant C, and that in the limit when the angle is zero, $A + B = C$.

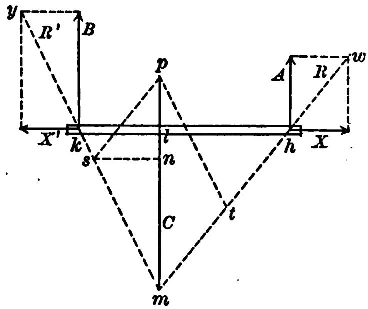


FIG. 247.

Suppose that a body is acted upon by two parallel forces A and B in the same direction, as shown in Fig. 247, applied at points h and k respectively, and if at point h a force X is applied and at point k, a force X' equal in magnitude, but opposite in direction, the addition of these two forces will not affect the joint effect of A and B, for X and X' neutralize each other.

The resultant of forces A and X is the force R represented by the diagonal *hw*. The resultant of forces B and X' is the force R' represented by the diagonal *ky*.

The lines of direction of R and R' meet at m. From this point lay off *mt* equal to R and *ms* equal to R'; then the diagonal *mp* represents their resultant. Draw the line *sn* parallel to force X'; then *ns* and *mn* are the original components of R', *viz.* X' and B. Similarly, *sn* and *np* are the original components of R, *viz.* X and A.

Since the resultant of X and X' is zero, this diagonal mp, or force C, must represent the resultant of forces A and B, and can be proved equal to their sum. For ⊿s msn and kBy are equal; therefore, mn equals force B; also, ⊿s snp and hAw are equal; therefore, np equals force A. Therefore,

$$A + B = mn + np = mp = \text{force } C.$$

⊿s mns and mlk are similar; therefore,

$$mn : ns = ml : kl \quad (1).$$

⊿s nsp and mlh are similar; therefore,

$$np : ns = ml : hl \quad (2).$$

From equation (1)

$$mn \cdot kl = ns \cdot ml.$$

From (2) $$np \cdot hl = ns \cdot ml.$$

Therefore, $$mn \cdot kl = np \cdot hl.$$

But $$mn = \text{force } B \text{ and } np = \text{force } A.$$

Therefore, $$B \cdot kl = A \cdot hl,$$

or $$A : B = kl : hl.$$

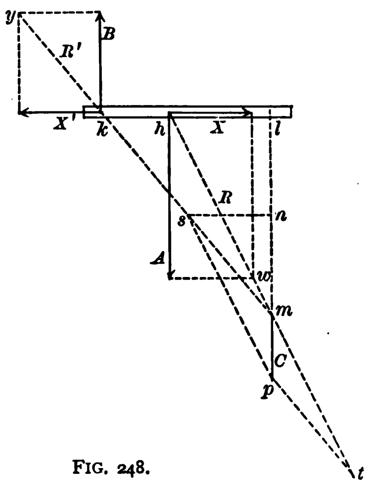


FIG. 248.

Hence, the point l, where the diagonal mp cuts the line hk, is the point of application of force C, the resultant of forces A and B, and is so situated that the magnitudes of the forces A and B are inversely proportional to their distances from it.

If the two parallel forces A and B are in opposite directions, as shown in Fig. 248, and if two equal and opposite forces X and X' are applied respectively at points h and k,

the line of direction of the resultant R of forces A and X meets the line of direction of the resultant R' of forces B and X' at the point m. From this point lay off mt representing R and ms representing R'; then the diagonal mp is their resultant. The forces X and X' being equal and opposite have a zero resultant; hence, mp or force C is the resultant of A and B, and can be proved to equal their difference.

Draw line sn parallel to force X. Then sn and np are the original components of $sp = mt = R$, and therefore equal X and A respectively; and ns and mn are the original components of R', and therefore equal X' and B respectively.

But $$mp = np - mn.$$

Therefore, $$\text{force } C = \text{force } A - \text{force } B.$$

Further, the $\triangle$ nps and lmh are similar.

$$\therefore\ ns : np = hl : lm. \qquad (1)$$

Also, $\triangle$ nms and lmk are similar.

$$\therefore\ ns : nm = kl : lm. \qquad (2)$$

From (1) and (2),

$$ns \cdot lm = np \cdot hl = nm \cdot kl,$$

or $$np : nm = kl : hl,$$

but $$np = \text{force } A \text{ and } nm = \text{force } B.$$

Therefore $$A : B = kl : hl.$$

Hence the point l is the point of application of the resultant C of the forces A and B, and is so situated that the magnitudes of A and B are inversely proportional to their distances from it.

Corollary. — If force A = force B and they act in opposite directions, the $\triangle$ hAw and kBy are equal, and the $\measuredangle Ahw = \measuredangle Bky$. Hence R is parallel to R', their lines of direction will never meet, and the resultant of A and B is zero. The forces, however, will not be in equilibrium, since they are not acting in the same straight

line; consequently rotation will be produced about a point midway between h and k. The value of the moment will be $A \cdot \frac{hk}{2} + B \cdot \frac{hk}{2} = A \cdot hk$, since $A = B$.

Such a combination of two equal parallel forces acting in opposite directions, but not in the same straight line, is called a mechanical couple. The effect of a mechanical couple is always to produce rotation, the value of the moment being the product of either force times the distance between their points of application, measured perpendicular to the direction of the forces.

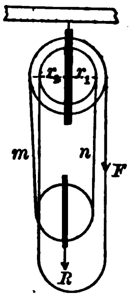


FIG. 249.

459. The Differential Pulley. — This pulley (Fig. 249) has a very general use nowadays in raising very great masses a short distance. The two sheaves of the fixed pulley instead of being of equal radii, as in the other pulleys heretofore described, are of slightly *unequal* radii, r_1 and r_2. The sheaves are *toothed* wheels to avoid slipping, and an endless chain passes around the sheaves and hangs in a loop below the movable pulley, as shown in the diagram. The tensions of m and of n are each $\frac{1}{2} R$. The moment of tension m about the axis of the fixed pulley $= \frac{R}{2} \times r_2$. The moment of tension n about this axis $= \frac{R}{2} \times r_1$. The moment of the force $F = F \times r_2$.

When the pulley is in equilibrium, the moments of F and of tension $n =$ the moment of tension m,

$$Fr_2 + \frac{Rr_1}{2} = \frac{Rr_2}{2},\ Fr_2 = \frac{R}{2}(r_2 - r_1),\ F = R \times \frac{r_2 - r_1}{2\, r_2}.$$

The difference in the radii of the sheaves of the fixed pulley $(r_2 - r_1)$ can be made very small, making the mechanical advantage of this pulley very great.

For example, if $r_2 = 8$ in., and $r_1 = 7\frac{3}{4}$ in., the force F to support a resistance R equal to 1 ton (2000 lb.) $= \frac{2000(8 - 7\frac{3}{4})}{2 \cdot 8}$ $= \frac{2000 \cdot \frac{1}{4}}{16} = \frac{500}{16} = 31\frac{1}{4}$ lb.

The mechanical advantage $R:F=\frac{2000}{31\frac{1}{4}}=64$.

The distance F moves in one revolution of the fixed pulley $=2\pi r_2$. Since the chain winds up on the larger sheave and unwinds on the smaller sheave, it winds up $2\pi r_2$ and unwinds $2\pi r_1$, so that the resistance R rises $\frac{1}{2}(2\pi r_2-2\pi r_1)=\pi(r_2-r_1)$.

The work done upon the resistance $=R\pi(r_2-r_1)$.

The work done by the force $F=F\cdot 2\pi r_2$.

Since $F=R\times\frac{r_2-r_1}{2r_2}$, the work done by the force

$$F=R\cdot 2\pi r_2\cdot\frac{r_2-r_1}{2r_2}=R\pi(r_2-r_1),$$

which is the same as that done upon the resistance. Therefore, neglecting the weight of the chain and pulleys and the friction of the bearings, the efficiency is 100%.

460. Coefficient of Cubical Expansion. — Let α = coefficient of linear expansion, then if the temperature of a cube 1 cm. on a side is raised from 0° to 1° C., each side becomes $1+\alpha$ cm. long, and the volume of the cube at 1° C. is $(1+\alpha)^3=1+3\alpha+3\alpha^2+\alpha^3$.

Since the coefficient α is a very small fraction, *e.g.* .000018, its square and cube will be still smaller, *e.g.* .000000000324 and .000000000000005832 respectively, hence, in the above expression, the last two terms may be neglected and $(1+\alpha)^3$ taken as equal to $1+3\alpha$. Since 3α is the increase of unit volume at 0° C. for a rise in temperature of 1°, 3α is the coefficient of cubical expansion, and is 3 times the linear coefficient.

461. Relation between Conjugate and Principal Focal Distances of a Convex Mirror. — To determine the relation between the distances of conjugate foci from a convex mirror and its principal focal distance.

Let MN (Fig. 250) be a convex mirror, O the object, I its virtual image, and C the center of curvature of the mirror. OP and OQ are two rays from O incident on the mirror.

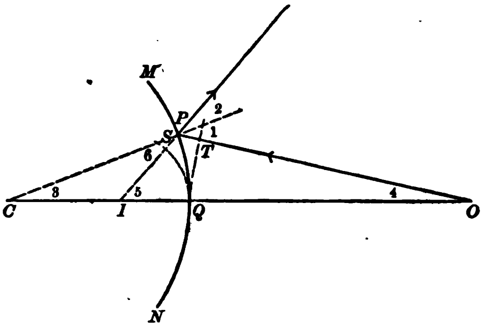


FIG. 250.

$$\measuredangle 1 = \measuredangle 2 \text{ (law of reflection).}$$
$$\measuredangle 1 = \measuredangle 3 + \measuredangle 4 \text{ (ext. } \measuredangle \text{ of } \triangle).$$
$$\measuredangle 5 = \measuredangle 3 + \measuredangle 6 \text{ (ext. } \measuredangle \text{ of } \triangle).$$
$$\measuredangle 6 = \measuredangle 2 \text{ (vert. } \measuredangle\text{s).}$$
$$\therefore \measuredangle 6 = \measuredangle 1 \text{ (Ax. 1).}$$
$$\therefore \measuredangle 5 = \measuredangle 3 + \measuredangle 1;$$
but
$$\measuredangle 1 = \measuredangle 3 + \measuredangle 4.$$
$$\therefore \measuredangle 5 = 2 \cdot \measuredangle 3 + \measuredangle 4. \qquad (1)$$

A *radian* is an angle which at the center of a ⊙ is subtended by an arc as long as its radius.

Since the length of the circumference of a circle is 2π times as long as its radius, the total angle about a point must equal 2π radians.

$2\pi = \frac{2\pi r}{r}$, or an angle measured in radians equals the arc of a circle which it subtends divided by the radius.

With C, I, and O as centers respectively draw the arcs PQ, SQ, and TQ, which if the angle is small, or P is near to Q, may be considered equal without making any great error.

Therefore, $\angle 5 = \frac{SQ}{D_i}$, $\angle 3 = \frac{PQ}{R}$, $\angle 4 = \frac{TQ}{D_o}$.

Then by eq. (1) $$\frac{SQ}{D_i} = \frac{2 \cdot PQ}{R} + \frac{TQ}{D_o}.$$

Dividing through by SQ ($= PQ = TQ$),

$$\frac{1}{D_i} = \frac{2}{R} + \frac{1}{D_o},$$

or $$\frac{1}{D_o} - \frac{1}{D_i} = -\frac{2}{R} = -\frac{1}{f},$$

which is the equation for conjugate focal distances for a convex mirror.

462. Relation between Conjugate and Principal Focal Distances of a Convex Lens. — In Fig. 251, let L represent a section of half of a double convex lens with C and C' as its centers of

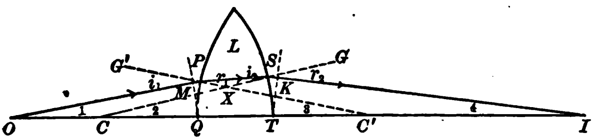


FIG. 251.

curvature. Let the radii CT and $C'Q$ equal R_1 and R_2 respectively. Let O be the luminous point and the path of the ray OP be $OPSI$. $\angle OPG' = \angle i_1$, the 1st $\angle$ of incidence; $\angle SPC' = \angle r_1$, the 1st $\angle$ of refraction; $\angle PSC = \angle i_2$, the 2d $\angle$ of incidence; $\angle ISG = \angle r_2$, the 2d $\angle$ of refraction. The $\angle$s at O, C, C', and I are $\angle$s 1, 2, 3, and 4 respectively. Let n be the index of refraction from air to glass. The relation between the conjugate focal distances, OQ and IT, and the radii of curvature of the lens is determined as follows: —

$$\angle i_1 = \angle 1 + \angle 3 \text{ (ext. } \angle \text{ of a } \triangle\text{)};$$

also $$\angle r_2 = \angle 2 + \angle 4 \text{ (ext. } \angle \text{ of a } \triangle\text{)};$$

then $$i_1 + r_2 = 1 + 2 + 3 + 4. \qquad (1)$$

$$\text{sine } i_1 = n \text{ sine } r_1 \text{ and sine } r_2 = n \text{ sine } i_2.$$

If the angles are small, the sines are $\propto$ to the angles, and we may assume $i_1 = nr_1$ and $r_2 = ni_2$, $i + r_2 = nr_1 + ni_2$.

Since $\measuredangle X$ is common to the two triangles, CXC^1 and PSX,

$$r_1 + i_2 = 2 + 3.$$

Substituting in eq. (1) $\quad nr_1 + ni_2 = 1 + 4 + r_1 + i_2$,

or $$(n-1)(r_1 + i_2) = 1 + 4 = (n-1)(2+3). \quad (2)$$

Draw arcs QM and TK with OQ and IT as radii respectively. An angle (measured in radians) equals the $\dfrac{\text{subtended arc}}{\text{radius}}$. If the angles 1, 2, 3, and 4 are very small, these arcs are practically equal.

Therefore, $\quad \measuredangle 1 = \dfrac{QM}{D_o}, \ \measuredangle 2 = \dfrac{TS}{R_1}, \ \measuredangle 3 = \dfrac{QP}{R_2}, \ \measuredangle 4 = \dfrac{TK}{D_i}.$

Substituting in eq. (2), $\quad \dfrac{QM}{D_o} + \dfrac{TK}{D_i} = (n-1)\left(\dfrac{TS}{R_1} + \dfrac{QP}{R_2}\right).$

Dividing by $\quad QM (= TK = TS = QP)$,

$$\frac{1}{D_o} + \frac{1}{D_i} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right). \quad (3)$$

Equation (3) is the general equation for convex lenses. If the curvatures of both faces of the lens are equal, $R = R'$, and if the index $n = \frac{3}{2}$, the equation becomes

$$\frac{1}{D_o} + \frac{1}{D_i} = \left(\frac{3}{2} - 1\right)\left(\frac{2}{R}\right) = \frac{1}{2} \cdot \frac{2}{R} = \frac{1}{R}.$$

$$\frac{1}{D_o} + \frac{1}{D_i} = \frac{1}{R}. \quad (4)$$

463. Principal Focal Length. — 1. *Of a Double Convex Lens.* $D_i = f$ when the incident rays are parallel or $D_o = \infty$.

Therefore, $\dfrac{1}{\infty} + \dfrac{1}{D_i} = \dfrac{1}{R}$, or $D_i = R$. This means that when the incident rays are parallel, the rays after passing through the lens converge at the center of curvature, which is therefore the principal focus.

2. *Of a Plano-Convex Lens.*—When one surface of the lens is plane, $R_1 = \infty$ and $\frac{1}{R_1} = 0$.

Equation (3) for a plano-convex lens becomes

$$\frac{1}{D_o} + \frac{1}{D_i} = (n-1)\left(\frac{1}{R}\right).$$

If $D_o = \infty$ and $n = \frac{3}{2}$, $\frac{1}{D_i} = \frac{1}{2} \cdot \frac{1}{R} = \frac{1}{2R}$, or $D_i = 2R$; *i.e.* the focal length of a plano-convex lens is equal to twice the radius of curvature of the curved face.

464. Relative Positions of Object and Image.—*If* $D_o > 2R$ *but* $< \infty$.—Let $D_o = 3R$.

$$\frac{1}{3R} + \frac{1}{D_i} = \frac{1}{R}, \text{ or } \frac{1}{D_i} = \frac{2}{3R}.$$

$\therefore D_i = \frac{3}{2}R$; *i.e.* the image will be formed at a point whose distance from the lens $< 2R$ but $> r$.

2. *If* $D_o < 2R$ *but* $> R$, $D_i > 2R$ but $< \infty$. This is the converse of 1.

3. *If* $D_o = 2R$, $D_i = 2R$, since $\frac{1}{2R} + \frac{1}{2R} = \frac{1}{R}$. Cases 1, 2, and 3, together with the case where $D_o = \infty$, represent all the conditions for the production of *real* images by lenses.

4. *If* $D_o < R$.—Let $D_o = \frac{1}{2}R$.

$$\frac{1}{\frac{1}{2}R} + \frac{1}{D_i} = \frac{1}{R}, \quad \frac{1}{D_i} = -\frac{\frac{1}{2}}{\frac{1}{2}R} = -\frac{1}{R}, \quad D_i = -R.$$

This means that if the object is placed within the focal length, the image distance is negative, *i.e.* on the same side of the lens as the object, and the image is virtual. In the above case the image is twice as far as the object from the lens and is twice as large in each dimension, or the amount of magnification is 2 diameters. As the object approaches the principal focus, the virtual image moves farther from the lens and the amount of magnification increases.

To illustrate: let $D_o = \frac{7}{8}R$.

Then $\frac{1}{\frac{7}{8}R} + \frac{1}{D_i} = \frac{1}{R}$, or $\frac{1}{D_i} = \frac{-\frac{1}{8}}{\frac{7}{8}R} = -\frac{1}{7R}$, or $D_i = -7R$.

Since the image is 8 times as far as the object from the lens, the magnification is 8 diameters.

465. Winding of a Drum Armature. — Suppose in Fig. 252 the cylindrical core of the armature to have 12 longitudinal grooves deep enough to hold two layers of wire each. There are then 12 segments of the commutator, *a–m*, respectively.

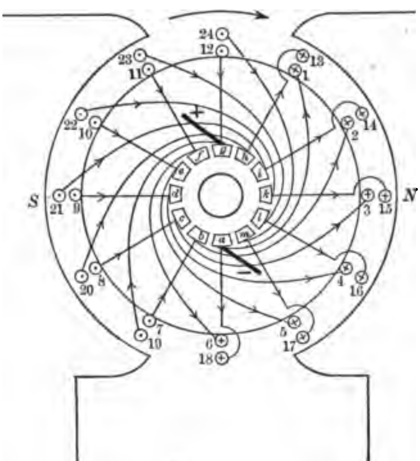


FIG. 252.

To show the winding: starting at segment *g* of the commutator the wire runs up to 12, then along the groove in the core, down the rear end to the groove in which is wire 6, then across the front end connecting at segment *f*. Running also from segment *f* is the wire leading to 11, thence around the drum to 5, connecting across the front end with segment *e*. From *e* also to 10, thence around the drum to 4, and then to segment *d*. From *d* also to 9, thence to 3, and then to segment *c*. From *c* also to 8, thence to 2, and then to segment *b*. From *b* also to 7, thence to 1, and then to segment *a*. Now there has been wound one layer of wire in the grooves of the drum. Then *a*, for the second layer, is connected with 18, thence to 24, and then to segment *m*. From *m* also a wire runs to 17, thence to 23, and then to segment *l*. From *l* also a wire runs to 16, around the drum to 22, and then to segment *k*. From *k* also a wire runs to

15, around the drum to 21, and then to segment *i*. From *i* also a wire runs to 14, around the drum to 20, and then to segment *h*. From *h* also a wire runs to 13, around the drum to 19, and then to segment *g*. Now there have been wound two layers of wire in each of the 12 grooves of the drum and these coils are all connected as shown. The brushes are placed on the segments at the end of the vertical diameter of the commutator.

When the armature is revolved in the direction indicated by the large arrow, the E. M. F. induced in those coils on the right moving down in front of the N pole has, according to the three-finger rule, a direction *from* the observer, hence the current in the connecting wires across the front end will be from the commutator as shown. The direction of the E. M. F. in the coils on the left moving up in front of the S pole will be, by the three-finger rule, *toward* the observer, and will run toward the commutator across the front. Since the current in both wires connected with segment *g*, for the given instant represented, is running toward it, the brush on *g* is the + brush. Similarly, the currents in both wires connected to *a* are running *from* it, hence the brush on *a* is the — brush.

466. Table of Sines and Tangents

Angle	Sine	Tangent	Angle	Sine	Tangent	Angle	Sine	Tangent
0°	0.000	0.000	31°	0.515	0.601	61°	0.875	1.804
1	0.017	0.017	32	0.530	0.625	62	0.883	1.881
2	0.035	0.035	33	0.545	0.649	63	0.891	1.963
3	0.052	0.052	34	0.559	0.675	64	0.899	2.050
4	0.070	0.070	35	0.574	0.700	65	0.906	2.145
5	0.087	0.087	36	0.588	0.727	66	0.914	2.246
6	0.105	0.105	37	0.602	0.754	67	0.921	2.356
7	0.122	0.123	38	0.616	0.781	68	0.927	2.475
8	0.139	0.141	39	0.629	0.810	69	0.934	2.605
9	0.156	0.158	40	0.643	0.839	70	0.940	2.747
10	0.174	0.176	41	0.656	0.869	71	0.946	2.904
11	0.191	0.194	42	0.669	0.900	72	0.951	3.078
12	0.208	0.213	43	0.682	0.933	73	0.956	3.271
13	0.225	0.231	44	0.695	0.966	74	0.961	3.487
14	0.242	0.249	45	0.707	1.000	75	0.966	3.732
15	0.259	0.268	46	0.719	1.036	76	0.970	4.011
16	0.276	0.287	47	0.731	1.072	77	0.974	4.331
17	0.292	0.306	48	0.743	1.111	78	0.978	4.705
18	0.309	0.325	49	0.755	1.150	79	0.982	5.145
19	0.326	0.344	50	0.766	1.192	80	0.985	5.671
20	0.342	0.364	51	0.777	1.235	81	0.988	6.314
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